Modeling Renewable Electricity Purchasing for Sustainable Management of Clarkson University's Energy Portfolio

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Abstract. We present a dynamical system model to show the Potsdam, NY campus of Clarkson University is 100% renewable with the university's new contract as of 2019 with Brookfield Renewable. The model creates periodic functions simulating energy inputs which can be used to generate alternative past and future scenarios. Clarkson University changed their source of electrical energy to Brookfield Renewable in July 2019 as they moved towards their 2025 goal of being 100% renewable. To claim that a MWh of electricity used by campus is renewably generated, a Renewable Energy Credit (REC) has to be purchased or generated and applied to it. The new contract with Brookfield Renewable provides each supplied MWh with its own REC. Combined with Clarkson University's other renewable energy sources, 95% of electricity consumed by campus can be certified as renewable. To model the remaining portion of electricity consumed by off-campus properties, we rely on data that accounts for consumed and delivered electricity, prices for that electricity, and monetary credits generated by local energy sources. Our developed dynamical system models the monetary credit generation, debt accumulation, and REC accrual over 34 months using real university data. As not all data parameters were explicitly available, we explore estimating parameters in three ways: directly from the data as time-varying functions, as constants, and stochastically, as random variables with distributions consistent with the provided data. We validate the alternative models against the data and estimate sensitivity to parameters.

1. Introduction.

In July 2019, Clarkson University switched their main energy provider from a local energy service company to Brookfield Renewable, believing this would be cheaper for the university long-term and help them reach their goal of supplying the campus with 100% renewable energy by 2025. Through the contract with Brookfield Renewable and the existing partnership with the New York Power Authority (NYPA), 95% of the campus's electricity needs could be provided from local renewable energy sources. Further, each MWh of energy provided by Brookfield Renewable has a renewable energy credit (REC) attached to it, and since NYPA's energy supply comes to campus as 100% renewable hydropower, with these two sources 95% of the campus is considered to be supplied with certified renewable energy. This led to the question of whether or not the last 5% of the energy used by the Potsdam campus was covered by enough RECs as well. Each MWh of electricity used by the campus needed to have a REC applied to it if the campus was to be considered 100% renewable.

The main campus of Clarkson University is in the town of Potsdam, located in northern New York state, and has the majority of student housing and academic buildings. Outside of the main campus, Clarkson University owns other buildings around Potsdam, and operates a satellite campus in Schenectady, NY, and the Beacon Institute for Rivers and Estuaries in Beacon, NY. In this paper only the properties in Potsdam, NY are analyzed as they all obtain electric energy from the same local sources, including renewable solar and hydro energy

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facilities.

The renewable energy credits (or renewable energy certificates), also referred to as RECs, described in this paper are associated with every MWh produced from a renewable energy source that is sent to the electricity grid [2]. These RECs can be generated locally or purchased from the market in order for an institution to claim they run on renewable energy. In our paper, the RECs Clarkson University brings in will have to cover all MWh of electricity brought to campus to ensure the campus is running fully on renewable energy. For ease of understanding and transferring data into the model, units of electricity in this paper will be in units of kWh instead of MWh since the dataset records electricity purchased and delivered in kWh. RECs cannot be applied to any bills as they are not any form of currency and while in some situations RECs may have a monetary value, this is not the case for those supplied to the university from Brookfield Renewable and Clarkson Solar. Therefore Clarkson University's RECs cannot be sold for any monetary value and can only be applied to offset electricity brought to campus.

Clarkson University also receives monetary credits each month from Potsdam Hydro and Clarkson Solar in exchange for using actual dollars to help with maintenance costs of each source. When Clarkson University pays to help upkeep each electricity source, at the end of the month the monetary credits that are generated from each source at individual rates are given back to the university. In some cases, Clarkson University gets back more in monetary credits than they originally paid (usually from Potsdam Hydro) and in some cases they get back less (usually from Clarkson Solar). These monetary credits act as a type of rebate that the university can apply to National Grid bills to offset any costs they owe and the amount can accumulate indefinitely (until applied to a bill). Whatever bills are owed to sources other than National Grid must be paid using actual dollars from the university's funds. Further, whatever is owed to National Grid that cannot be covered by the monetary credits generated monthly from these two sources must also be paid using money from the university's accounts.

The model developed in this paper uses data obtained from Clarkson University's various electricity sources to determine the % renewability of the campus and simulate past and future electricity use for the university. To understand the variability of the data used and the likelihood of fluctuation in the results we compare three different versions of the model: (a) constant parameter model, (b) data-driven parameter model, where parameters are re-estimated at every time step, and (c) stochastic model, where parameters are drawn from a probability distribution at every step. The comparison will allow us to understand the rates at which monetary credits and RECs are generated versus the rates at which they are applied, which can help the university predict its spending habits and renewability in yearly cycles. Understanding these cycles will allow Clarkson University to better allocate their resources as they balance renewability and cost. Further, with unknowns existing in the dataset, the model comparison is important as it addresses how we could best estimate and model the parameters to answer the question of renewability and visualize energy usage on campus.

This paper models the performance of the electricity purchasing portfolio at Clarkson University. Section 2 presents the problem description along with the assumptions and data used to create the model. Section 3 describes how state equations of the model, input functions, and parameters were determined. The purpose of the model was to evaluate if the proposed purchasing strategy results in a sustainable portfolio, and if the campus can be certified as fully renewable. This section also presents scenarios in which the model was evaluated, with detailed discussion of outcomes given in Section 4. Future work for this model is presented in Section 5 while conclusions are presented in Section 6.

A generalized version of the presented model can be used for other campuses or institutions to help them reach renewabilitiy or financial goals as well. For an individual institution, data on all electricity sources and transfer of resources must be available to apply a similar model. By creating specific assumptions, developing parameters, and defining equations for resource relationships, a facility could obtain similar results. These results can show historical trends in energy usage or provide predictive modeling so that real-world steps can be taken to adjust portfolio behavior and achieve a desired outcome.

2. Problem Description. Clarkson University works with many local electricity sources to purchase and obtain energy to power the campus. These include a local solar farm (Clarkson Solar), local hydro plants and renewable energy businesses (Brookfield Renewable, NYPA, and Potsdam Hydro), and National Grid. With these local sources they transfer and exchange electricity, money, monetary credits, and RECs. Table 1 provides a brief overview of each of these sources and the commodities exchanged. Our model is based on understanding the relationships between these agents and the exchanges they are a part of. Quantitative data about these components was obtained from Clarkson University records.

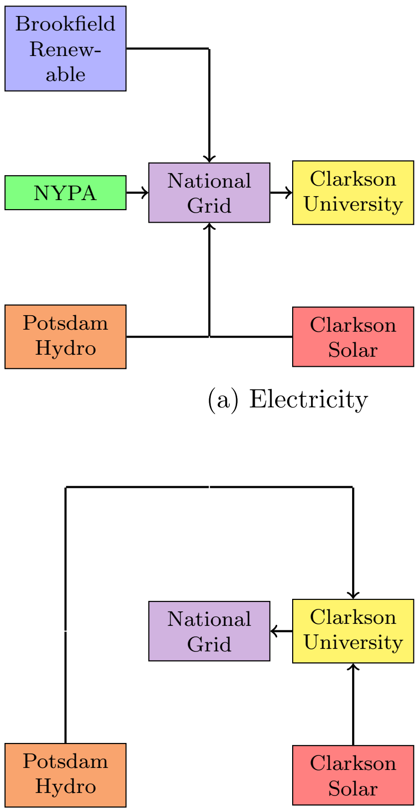
Agent	Description
Clarkson University	University in upstate NY consuming electricity
Brookfield Renewable	Power company operating hydroelectric plants
National Grid	Electricity and gas utility and distribution company
NYPA	New York Power Authority; provides renewable hydroelectric energy
Clarkson Solar	Solar farm on university-owned land in Potsdam, NY
Potsdam Hydro	Hydroelectric plant in Potsdam, NY

Resource	Description
Electricity	Generated by power plants, distributed through National Grid to the campus
Money	Real dollars spent by the University and National Grid
Monetary Credits	Generated by (some) power plants and used by the University as a rebate to National Grid
RECs	Generated by (some) power plants and used to certify the renewability of campus

Table 1: Agents and resources involved in the electricity trade model.

Clarkson University pays each of the local sources separately for the electricity they produce except for Brookfield Renewable, which is paid by Clarkson University through National

Grid. The university accumulates the monetary credits generated at Potsdam Hydro and Clarkson Solar each month when they pay each site to help with monthly maintenance. These monetary credits are useful because they can offset any bill given to Clarkson University by National Grid and can be used the month they are generated or at a later time. All electricity is delivered through National Grid; therefore all delivery fees can be paid for using monetary credits. Further, the actual electricity purchased from Brookfield Renewable for consumption is charged through National Grid so monetary credits can be used for this bill as well. These exchanges of resources are demonstrated by Figure 1.



(a) Electricity

(b) Money

(c) Monetary Credits

(d) RECs

Figure 1: Transfer of resources between local agents

Looking at REC flow into campus is essential to answer the question of how renewable the campus is right now and this exchange is shown by Figure 1d . Brookfield Renewable and Clarkson Solar provide RECs that Clarkson University can use to offset non-renewable energy brought to campus. RECs come from Brookfield Renewable as they are attached to every MWh of electricity the university consumes under the Brookfield Renewable contract and additional RECs are generated at a 1:1 ratio for the energy produced at the solar farm. As the RECs from the solar farm no longer have monetary worth, the university is able to receive them in exchange for the cost they pay to the farm each month. This is an easy way for Clarkson to obtain more RECs that they can then apply to the non-renewable energy used by the campus.

2.1. Qualitative Modeling. To develop a model, we needed to create equations that would demonstrate how the transfer of money, monetary credits, and RECs were affected over time.

Assumptions underlying this model, explained in the previous section, can be summarized as follows:

(A1) Generation charges and/or upkeep charges are paid to NYPA, and local power plants directly in real dollars.
(A2) Generation charges to Brookfield Renewable can be paid in monetary credits.
(A3) National Grid delivers all electricity, so all delivery costs can be paid by monetary credits.
(A4) Clarkson is billed at the end of the month for the power used during that month.
(A5) Monetary credits are generated by Clarkson Solar and Potsdam Hydro.
(A6) Monetary credits can be used the same month they are generated or at a later time.
(A7) RECs are received from the solar farm and Brookfield Renewable at a 1:1 ratio for the energy the farm produces.

These assumptions were essential in creating the model presented in this paper. It is important to understand the relationship between Clarkson University and the electricity sources as well as how payments are made via dollars or monetary credits.

Constraints we had to incorporate include:

(C1) Purchased electricity must match the consumed energy,
(C2) Minimize real-dollar spending (i.e. maximize use of monetary credits).

This model does not include any potential increases in production capacity or renovations in structures that would reduce their energy footprint.

2.2. Preparation of Data. The dataset used contains energy prices, consumption and delivery amounts for all local sources Clarkson University is working with, and monetary credits generated from the local solar farm and hydro plant between June 2014 and October 2018. Since RECs are assumed to be generated at a 1:1 ratio with the energy produced by Clarkson Solar, this is sufficient to infer all relationships in Figure 1. All values in the dataset are monthly totals.

Since the obtained data was not complete, only the 34 month range of September 2015 to June 2018 could provide the record of variables needed to identify parameters in the model. The price parameters that were calculated using the dataset are presented in Table 2 along with their mean μ and standard deviation σ values. These parameters will be used in the model to help us better understand the relationships between monetary credits, dollars, RECs, and electricity.

To determine which off-campus properties best represented Clarkson's current situation, any accounts that were no longer present in 2019 were immediately disregarded as they would be unnecessary when modeling future spending. Accounts with inconsistent or seemingly inaccurate data were disregarded as well. After eliminating non-usable accounts, there were 21 off-campus properties left to analyze. These smaller off-campus properties were aggregated to determine energy usage outside the main Potsdam campus. It was important to determine

Price	μ	σ
Hydroelectric sale β_h	11.74	0.9789
Hydroelectic purchase ρ_h	10.57	0.8810
Solar sale β_s	11.55	0.9065
Solar purchase ρ_s	12.39	0.2072
NYPA purchase ρ_N	3.37	0.3052
Brookfield purchase ρ_B	6.18	0.2856
Energy delivery δ	2.05	0.1787

Table 2: Summary statistics (mean μ and standard deviation σ) for various prices in the model. All quantities are in cents (10^{-2} dollars) per kWh.

both monthly and yearly sums for the energy consumed by the smaller accounts because monthly values could be directly input into the model while yearly values could immediately answer the question of whether or not the Potsdam section of Clarkson University's campus was 100% renewable (described in Section 4).

3. Mathematical model. The mathematical model of the energy portfolio specifies the evolution of state variables based on relationships between accounts. The representation of the input and output flows and the estimation of parameters were based on the available historical data.

3.1. State equations. The state equations capture how monthly increments of state variables, that is monetary credit accumulation $C[n]$, debt $D[n]$, and REC accrual $R[n]$, depend on the values of these variables and on variation in energy production and consumption, represented by time-varying functions derived from data.

For two of the input energy sources, Brookfield Renewable (E_B) and NYPA (E_N), the value used in the model is the energy the campus consumes from these sources each month. For Potsdam Hydro (E_H) and Clarkson Solar (E_S), the value is the energy generated by these sources each month. All energy values appear as $E_*[n]$ in the model and they have units of $[kWh]$. RECs are given units of energy, as they are applied in a 1:1 proportion to certify renewability of each consumed kWh of power; however, RECs do not represent an additional usable electricity, but only a certificate of renewability for a certain amount of energy. All coefficients β_*, ρ_*, δ can be assumed to vary over time but brackets are omitted for brevity until Subsection 3.4.

The monthly change in monetary credits is given by

$$\Delta C[n] = \beta_H E_H[n] + \beta_S E_S[n] \underbrace{-\omega\delta E_N[n] - (\mu\delta + \gamma\rho_B)E_B[n]}_{\text{bill payment using monetary credits}} \tag{3.1}$$

where the first two terms account for earnings from Clarkson-affiliated power plants and the last two terms indicate expenses toward power companies (NYPA and Brookfield Renewable). Coefficients β_* and ρ_* are prices of energy (accounted as monetary credits here), while δ is the delivery cost of energy (which we assume to be the same for each source as all energy is delivered through National Grid).

This model assumes a payment strategy that first uses any available monetary credits to pay eligible bills, therefore eliminating the carryover of credits between months. Such modeling choice was made based on discussions with stakeholders at Clarkson University. The time variations in $\omega[n]$, $\mu[n]$, and $\gamma[n]$ may be used to determine any particular strategy of portfolio management; optimization of these values is discussed in Subsection 3.2 and may be the topic for any follow-up work.

The monthly value of debt (in real dollars) is given by

$$\begin{aligned} \Delta D[n] = &- \rho_H E_H[n] - \rho_S E_S[n] - \rho_N E_N[n] \\ &- (1-\omega)\delta E_N[n] - (1-\mu)\delta E_B[n] - (1-\gamma)\rho_B E_B[n] \end{aligned} \tag{3.2}$$

where the first three terms represent what is paid for the consumption of energy from NYPA and what Clarkson pays to the local solar farm and hydro plant to help with monthly maintenance. Again, δ is the delivery cost for energy and ρ_* values are prices of energy from the different sources.

The last three terms in (3.2) represent the dollar cost to deliver energy from NYPA along with the dollar cost to purchase and receive energy from Brookfield Renewable. The portion of bills paid using monetary credits are

- ω for NYPA's delivery cost,
- μ for Brookfield Renewable's delivery cost, and
- γ for Brookfield Renewable's consumption cost.

The portfolio is managed by the monthly choice of control parameters $\omega[n]$, $\mu[n]$, and $\gamma[n]$ that determine the portion of bills that are paid for by money vs. monetary credits. The management strategy evaluated in this paper is described in Subsection 3.2, although the model could certainly accommodate different strategies implemented in an analogous way.

The monthly increment of RECs is given by

$$\Delta R[n] = E_S[n] + (E_B[n] - E_B[n]) - E_{SA}[n] \tag{3.3}$$

where the first term accounts for the energy gained from the local solar farm. The second and third term represent that energy consumed from Brookfield Renewable is at a 1:1 ratio with the RECs applied to that energy as stated in the contract between the provider and Clarkson University. Therefore, the energy from Brookfield Renewable comes to campus already completely renewable and thus (3.3) can be simplified to

$$\Delta R[n] = E_S[n] - E_{SA}[n]. \tag{3.4}$$

This means there are no leftover RECs from Brookfield Renewable that can be applied to energy purchased from other sources. The last term in (3.3) is the energy consumed by the small off-campus properties associated with Clarkson University.

To determine if the campus is covered 100% by renewable energy, $R[n]$ in (3.4) must be positive at the end of each fiscal year. If $R[n]$ is negative, the local solar farm does not generate enough energy over the course of a year to provide campus with the RECs needed to cover the energy consumed by the off-campus properties; the solution may be to purchase RECs from another local renewable source or on the open market (see Section 4 for further discussion).

As all state variables contain the same basic parameters and input, the system of equations for our model can be written as:

(3.5)

$$\begin{bmatrix} C[n+1] \\ D[n+1] \\ R[n+1] \end{bmatrix} = \begin{bmatrix} C[n] \\ D[n] \\ R[n] \end{bmatrix} + \begin{bmatrix} \Delta C[n] \\ \Delta D[n] \\ \Delta R[n] \end{bmatrix}$$

$$\begin{bmatrix} \Delta C[n] \\ \Delta D[n] \\ \Delta R[n] \end{bmatrix} = \begin{bmatrix} \beta_H & \beta_S & -\omega\delta & -(\mu\delta + \gamma\rho_B) & 0 \\ -\rho_H & -\rho_S & -(\rho_N + \delta(1-\omega)) & -(\delta(1-\mu) + \rho_B(1-\gamma)) & 0 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} E_H[n] \\ E_S[n] \\ E_N[n] \\ E_B[n] \\ E_{SA}[n] \end{bmatrix}$$

3.2. Control variables. Generated monetary credits from Clarkson Solar and Potsdam Hydro can only be applied to bills that go through National Grid. In our model, these bills are for delivery of energy purchased from NYPA and for the purchase and delivery of energy from Brookfield Renewable to Clarkson University. This model determines control parameters $\omega[n]$, $\mu[n]$, and $\gamma[n]$ under the assumption that monthly generated monetary credits are first applied to NYPA delivery costs (δE_N), then Brookfield Renewable delivery (δE_B) and finally to Brookfield Renewable consumption ($\rho_B E_B$). Any reasonable strategy must ensure that all bills are paid on a monthly basis; additionally, the considered strategies use as much monetary credits as possible in each month.

In the analyzed period, the monthly income of monetary credits was always smaller than the portion of the total bill that could be paid using these credits, that is

$$\beta_H E_h[n] + \beta_S E_S[n] \leq \delta E_N[n] + (\delta + \rho_B) E_B[n]. \tag{3.6}$$

The actual portion of the bill paid using monetary credits is controlled by parameters ω, μ, γ (see (3.1)).

The priority is always to apply the credits to the NYPA bill (δE_N), then Brookfield Renewable delivery (δE_B) and then Brookfield Renewable consumption ($\rho_B E_B$). For the analyzed data, the monthly income was sufficient to always cover the NYPA bill ($\omega[n] \equiv 1$) but the remaining two variables fluctuate. When the income is insufficient, $\gamma = 0$ as in realization shown in Figure 8.

To show that number of monetary credits generated must equal number of monetary credits used we can write: $\beta_H E_h[n] + \beta_S E_S[n] = \omega\delta E_N[n] + \mu\delta E_B[n] + \gamma\rho_B E_B[n]$. The parameters μ and γ can easily be determined based on the monetary credits available for the month and the assumed order the monetary credits are applied (knowing $\omega = 1$ for this model). To determine what monetary credits do not cover and therefore what portion of the National Grid bill must still be paid using dollars, we calculate: $(1-\omega)\delta E_N[n] + (1-\mu)\delta E_B[n] + (1-\gamma)\rho_B E_B[n]$ which become the last three terms subtracted in (3.2). Note that since $\omega = 1$, the first of the three terms subtracted in (3.2) is actually 0, meaning only the $\mu[n]$ and $\gamma[n]$ terms will contribute to the bill and affect results of (3.2).

For a more general strategy, similar steps can be rearranged to align with adjusted model assumptions. The three bill components that may be paid with monetary credits can be reordered and control parameters can be recalculated as needed. If monetary credits are applied to Brookfield Renewable consumption ($\rho_B E_B$) first, it may be the case that both $\omega[n]$ and $\mu[n]$ are always 0, but this strategy would need to be explored to determine if that is true. The model's complexity would also increase if none of the three control parameters equaled 0 or 1 and all changed each month. Further, $\omega[n]$, $\mu[n]$, and $\gamma[n]$ could be manually selected (i.e. not determined from other parameters using code) to apply a specific strategy to manage the portfolio.

The investigation of this model uses the monetary credit application strategy presented in this section to determine how monetary credit accumulation $C[n]$, debt $D[n]$, and REC accrual $R[n]$, change over time.

3.3. Inputs: Energy Consumption and Generation. Flows in electrical energy tend to have strong cyclic components. Energy production in solar plants is driven by the duration and intensity of sunlight throughout the year and production in hydro plants is driven by the consistency and strength of water flowing through the plant, both of which change with the seasons in upstate New York. Similarly, consumption contains cyclic components due to need for cooling in summer, heating in winter, and higher use of campus facilities while semesters are in session.

Each of the five energy inputs (Potsdam Hydro, Clarkson Solar, Clarkson's Small Accounts, NYPA, and Brookfield Renewable) was modeled as a noisy periodic function that was then used to create alternative histories and extrapolate from the data:

$$x[n] = A_0 + \sum_{k=1}^{K} \left[A_k \cos \frac{2\pi n}{P_k} + B_k \sin \frac{2\pi n}{P_k} \right] + \mathcal{N}(\mu, \sigma^2) \tag{3.7}$$

where x stands in for any of the $E_*[n]$ signals, A_k and B_k values are constant coefficients, P_k values are periods, and $\mathcal{N}(\mu, \sigma^2)$ is a normally-distributed random number. In principle, a linear term could be included to account for any prominent linear trends in the data, but after evaluating this possibility, we found that results were not greatly affected if the term was omitted.

In summary, the parameters in (3.7) for each signal $E_*[n]$ were determined as follows. First, the number of periodic components K and their periods P_k were determined based on expected seasonal variation and on results of spectral analysis of the data. Second, the coefficients A_k and B_k were determined by a linear least-squares regression of data onto the chosen number of periodic components. Finally, the mean and the variance of the random component were set to match the mean and the variance of the residual between the data and the periodic component with the parameters chosen in the previous step.

Spectral analysis. Spectral analysis of a time trace can be used to identify dominant periodic components in the signal. Discrete Fourier transform (DFT) [1, §11.9] is the most common transformation of the signal used for the purpose of spectral analysis. The input to DFT is a function sampled at equally-spaced time intervals $x[n]$. The DFT $\hat{x}[k]$ is a complex-valued sequence

$$\hat{x}[k] := \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-i \frac{2\pi}{NT} kn}, \tag{3.8}$$

where the index k indicates the integer multiples of the basic frequency $1/NT$, where $T =$ 1month is the sampling period of data. The importance of each frequency k/NT is measured by the Power Spectral Density (PSD) $|\hat{x}[k]|^2$; we choose the periods associated with peaks in PSD as those important for including in the model (3.7). To avoid $\hat{x}[0]$ overshadowing nontrivial periodic components, we first manually remove the mean value of input time series.

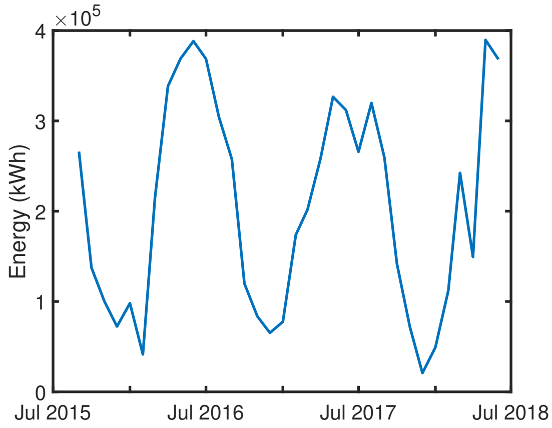


Figure 2: The record of Clarkson Solar energy input over 34 months visualized as (a) a time trace, and (b) Power Spectral Density (PSD). The dominance of the component with the yearly (12 month) periodicity is indicated by the maximum in the PSD.

As an example, the PSD for the solar power (Figure 2) shows that the yearly component, corresponding to the maximum in the PSD, dominates the data, matching the expectations about seasonal variation of the amount of sun in New York. Similar plots (see Figure 11 in Appendix A) were analyzed for all other time traces with results given in Appendix A. Depending on the relative magnitude of peaks, one or more dominant periods were used for each signal. When periods determined by spectral analysis were close to expected seasonal variations or their integer fractions (higher harmonics), rounded periods were used to simplify interpretation.

Linear least-squares. Once the most relevant periods were determined, they were used in (3.7) to model periodic functions for each energy signal. The coefficients A_k, B_k are then estimated by linear least-squares regression of parameters in (3.7), written in a matrix form as

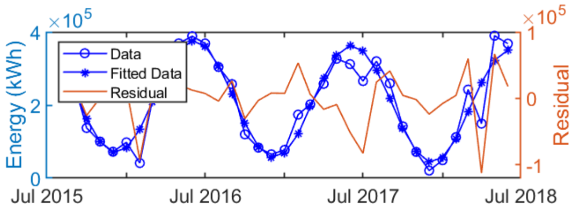
$$\overbrace{\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \end{bmatrix}}^{\vec{x}} = \overbrace{\begin{bmatrix} 1 & \cos\frac{2\pi t_0}{P_1} & \sin\frac{2\pi t_0}{P_1} & \cos\frac{2\pi t_0}{P_2} & \sin\frac{2\pi t_0}{P_2} & \dots \\ 1 & \cos\frac{2\pi t_1}{P_1} & \sin\frac{2\pi t_1}{P_1} & \cos\frac{2\pi t_1}{P_2} & \sin\frac{2\pi t_1}{P_2} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}}^{S} \overbrace{\begin{bmatrix} A_0 \\ A_1 \\ B_1 \\ \vdots \end{bmatrix}}^{\vec{\alpha}} \tag{3.9}$$

Since this equation is overdetermined (more time-series points than coefficients), it typically has no exact solution. Multiplying by $S^\top$ from the left results in the normal equation $S^\top\vec{x} =$

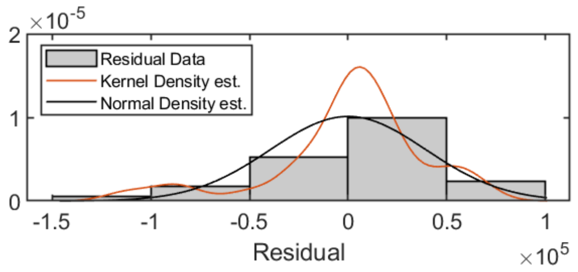
$(S^\top S)\vec{\alpha}$, which has a unique solution that amounts to a least-squares fit of the periodic model to data. Commonly, the resulting solution is written as

$$\vec{\alpha} = S^\dagger \vec{x}, \tag{3.10}$$

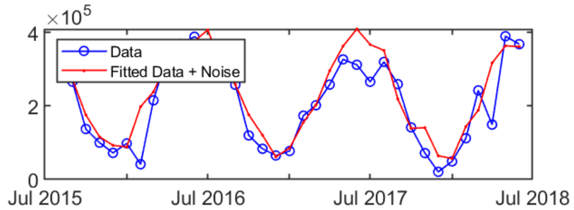
where $S^\dagger := (S^\top S)S^\top$ is the Moore–Penrose pseudoinverse.



(a) Fitted data overlaid on the original energy signal for Clarkson Solar (E_S). Residual values between the curves are also graphed



(b) Comparison of distribution of residuals: histogram, normal density estimate and nonparametric kernel density estimate. Estimated mean $\mu = -4.6652 \cdot 10^{-11}$ and standard deviation $\sigma = 4.08 \times 10^4$.



(c) Alternate history of Clarkson Solar energy trace generated using (3.7).

Figure 3: Demonstration of a periodic model for the time trace of the Clarkson Solar energy output.

After computing coefficients, we use the model (3.9) to produce realizations of models for each time trace and compare them with data. An example of the created model for the case of the Clarkson Solar farm is shown in Figure 3.

The original data (energy generated at the solar farm each month (E_S)), the periodic fit, and the residual (the difference between the data and the periodic fit) are displayed in Figure 3a. A histogram showing the range of the residual values over all the time steps is provided in Figure 3b, with a kernel density fit overlaid on top. Mean μ and standard deviation σ values of the residual were calculated as well so that a noise component could be included in the model (see last term of (3.7)). This noise component is added to the original fitted periodic curve of Figure 3a and the resulting fitted data with noise curve is plotted against the original data to compare similarities in Figure 3c.

The resulting curve of fitted data with noise included follows the original data curve very closely for Clarkson Solar (see Figure 3c). The other energy traces yielded similar analysis with the number of terms that provided a best fit for them, yet the solar farm showed its cyclical nature most clearly. A summary of the mean and standard deviation values of the residual, along with the R^2 values of the fitted curves compared to the original data for each energy signal are provided in Table 5 in Appendix A.

3.4. Estimating prices and conversion rates. The following prices and conversion coefficients vary from month to month: energy to monetary credit conversion coefficients for Clarkson Solar and Potsdam Hydro (β_*), the delivery price for energy by National Grid (δ), and the price for energy consumed from Brookfield Renewable, NYPA, Clarkson Solar, and Potsdam Hydro (ρ_*). Since we have access to all monetary data, we can compute values of these parameters on a monthly basis (Data-Driven Model); however, it can be useful to model the values of parameters as either constant-in-time (Constant Model), or as drawn randomly from an estimated distribution (Stochastic Model).

These models were used for different purposes, as explained below.

3.4.1. Data-Driven Model. The data-driven model treats the parameters in (3.5) as time-varying values, estimated from the available data at each time step. While some parameter values were directly a part of the data set, others had to be inferred. For example, the prices for the energy consumed (ρ_*) are determined by dividing the amount the university paid to that source by the amount of electricity produced by that source. Also, the monetary credit to \$ conversion factors (β_*) are determined by dividing the monthly monetary credits generated for a source by the kWh of energy produced by the source. The seven parameters are therefore represented in the data-driven model by time-traces of 34 monthly values.

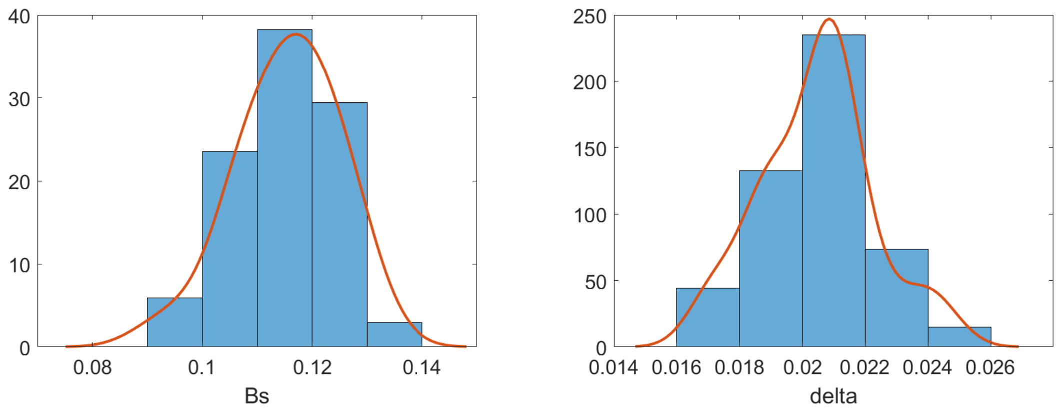
This model is the most accurate during the observed period, however it cannot be used to extrapolate. Therefore, it was used to judge the quality of the other two parameter models.

3.4.2. Constant Model. The simplest model treated the seven parameters as constant values, estimated as time-averages of their monthly values, resulting in a time-invariant matrix in the model Equation (3.5). The average values used for the constant model can be found in Table 2.

The constant model was used as the baseline for future projections of the energy portfolio. Additionally, we used it to estimate the impact of the hypothesized change in price ρ_B that Clarkson University pays to Brookfield Renewable for their electricity. This alternative scenario models the renegotiation of the contract that occurred in July 2019; at the time it was hypothesized that the average price of \$0.0618 per kWh could be negotiated to be as low as \$0.055 per kWh.

3.4.3. Stochastic Model. The stochastic model treats the parameters in (3.5) as stochastic processes, with values at each time step drawn independently from normal distributions whose means and variances match means and variances of the corresponding values over the available time period (*Gaussian white noise*). This model was used to generate alternative historical scenarios, used to estimate whether the historical data and the constant model could be considered outliers or not.

As before, we judged qualitatively whether each parameter was normally distributed or not during the observed time history by comparing the histogram and gaussian kernel density estimate of the distribution with the normal distribution, An example of one of the histograms is shown in Figure 4a for the β_S parameter (energy to monetary credit conversion factor for solar farm). All other parameters have similar estimates of their densities to that of β_S with the only slight deviation occurring for the δ parameter (price for energy delivery), shown in Figure 4b.



(a) Solar sale price β_S (b) Energy delivery price δ

Figure 4: Histograms of two time-varying parameters calculated during the 34 month period. Overlaid is a Gaussian-kernel density estimate of the PDF. Other parameter distributions are similar to β_S, while δ is the parameter distribution with the strongest deviation from the gaussian.

4. Results. A configuration of the energy portfolio model comprises the four components (state equations, control variables, input time traces, and parameterization) described in Section 3. Relationships between state equations and control variables are fixed; however, for any particular simulation one can choose to use historic record for input time traces, or traces simulated using (3.7). Similarly, historic values can be used for the price and conversion parameters (Subsection 3.4.1), or we can use constant values (Subsection 3.4.2) or Gaussian white noise (Subsection 3.4.3). In order to analyze the behavior of the electricity portfolio, we have simulated the models in the following configurations over 34 months:

(a) 4 versions of the model with historic input traces and parameters represented by:
   - (i) time-varying historic values (1 run) (Subsection 3.4.1),
   - (ii) mean of historic parameter values (Subsection 3.4.2) and the energy price of the old energy service provider (1 run),
   - (iii) mean of historic parameter values and the theoretical energy price of Brookfield Renewable (1 run),
   - (iv) stochastically time-varying parameters (1000 runs) (Subsection 3.4.3),

(b) 1 version of the model with stochastically time-varying parameters *and* input traces

(1000 runs).

The following subsections use these simulations to

- evaluate the potential for 100% renewability of the electricity portfolio and study trends in REC accrual,
- analyze consequences of cyclic behavior in inputs on RECs and control parameters,
- compare quantitative models,
- evaluate the effect of randomness in inputs and parameters on monetary credit and debt trends.

4.1. Determining Clarkson University's Campus Renewability. To determine whether or not the Potsdam portion of Clarkson University's campus was 100% renewable we had to look at REC usage over the course of the year (as month by month basis was unnecessary). The automatic offsets of Brookfield Renewable energy, as modeled in (3.3), combined with the 100% renewable hydropower purchased from NYPA meant that 95% of the campus energy was certified renewable. To certify the remaining 5% of energy used, we needed to know if the RECs associated with power generated at the local solar farm offset the amount consumed by the remaining smaller, off-campus properties not part of the main campus in Potsdam, NY.

We point out again that only the Potsdam area campus for Clarkson University was analyzed. We did not have data for the Capitol Hill Region and Beacon campuses so no estimates were made for these areas. However, Clarkson University should be able to apply any excess RECs from this model to energy used by these campuses to help offset that energy and aid them in becoming more renewable as well. Without understanding the sources these campuses exchange resources with, analogously to the flowchart in Figure 1, it is very difficult to assess whether they can be classified as renewable.

Our model assumes that each kWh of power generated at the solar farm equated to a REC which could be applied to certify each kWh consumed by the off-campus properties ("small accounts") as renewable. Therefore, we had to look at the yearly balance of RECs generated by the solar farm (E_S) compared to energy consumed by the smaller accounts (E_{SA}), as demonstrated by (3.4). Two full fiscal years worth of data are provided by the 34 month range analyzed and two more 9 month and 6 month time periods could be used as well. The results, showing $R[n]$ RECs left at the end of each fiscal year (EOY), are presented in Table 3.

	Generation [kWh]	Consumption [kWh]	Excess at EOY [kWh]
Sep '15 to May '16	1,637,950	1,017,149	620,801
June '16 to May '17	2,625,216	1,882,678	742,538
June '17 to May '18	2,334,150	2,205,082	129,068
June '18 to Nov '18	1,316,167	1,208,139	108,028

Table 3: Solar generated RECs (E_S), consumption by smaller accounts (E_{SA}), and excess RECs ($R[n]$)

This table shows an excess of RECs for each calculated year, which means the Potsdam campus has enough RECs to apply to each kWh they use. We can now claim that with Clarkson University in contract with Brookfield Renewable, the Potsdam campus is 100%

renewable. This is equivalent to saying that the $R[n]$ presented in (3.4) is positive at the end of each fiscal year.

It is difficult to gauge long term trends from only 34 months of complete data. Obtaining monthly energy consumption values for the small off-campus properties (E_{SA}) was also difficult because of inconsistencies and the potential for human error in transcribing the data. For this reason, the presented information could serve to give credence, but not prove definitively that Clarkson University is operating with complete renewability in Potsdam. Longer and more reliable data would certainly improve the confidence in these results.

The REC surplus between 9/15 and 5/17 in Table 3 is relatively high, likely because not all energy used in off-campus properties was accounted for in that period; it is therefore difficult to estimate whether the trends discussed will continue or not.

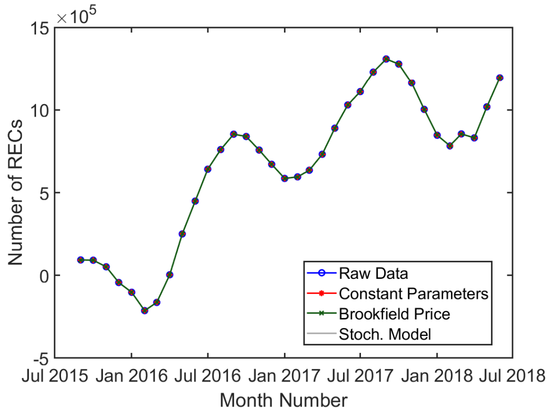


Figure 5: Running sum of REC accrual $R[n]$ over 34 months, accounting for how many are generated and used each months.

The REC accrual $R[n]$ is a running sum of the monthly surplus RECs $\Delta R[n]$ (3.4); it demonstrates that more RECs are generated than needed to certify renewability of the off-campus properties each month and that these RECs accumulate over time (shown in Figure 5). It is easy to see in this figure that no changes in parameters of Table 2 affected the REC accrual. All realizations of simulation Sim. (a) plot on the same line. This is because (3.4) only depends on the energy produced by the solar farm (E_S) and energy used by the smaller accounts (E_{SA}) and the energy input traces for all realizations of simulation Sim. (a) remained unchanged for the modeling as the parameters were adjusted.

An increasing trend of REC accrual is seen when we look at the data on a month by month basis in Figure 5. It appears Clarkson University will continue to gain excess RECs if they are consistent with their energy usage on their Potsdam campus. However, Clarkson University could see that while REC generation may appear to increase, consumption could increase as well to balance this out and excess could be very minimal in the future. Further, Table 3 appears to show a decreasing trend in excess RECs when looked at yearly and if there were no excess RECs eventually, this could affect the portfolio management.

An extrapolation of the data presented in Figure 5 was accomplished by running a version of the model similar to Sim. (b). In this version, a stochastic model simulates what the value of REC accrual could reach over the next 12 months past the original 34 month period. Though previous factors discussed support that these results are not definitive, Figure 6 shows that RECs will continue to increase rapidly. Note this graph only accounts for accrual values and does not predict future consumption. With the limited data available we cannot confidently anticipate the energy usage by the off-campus properties and therefore cannot foresee whether or Clarkson University campus will remain renewable.

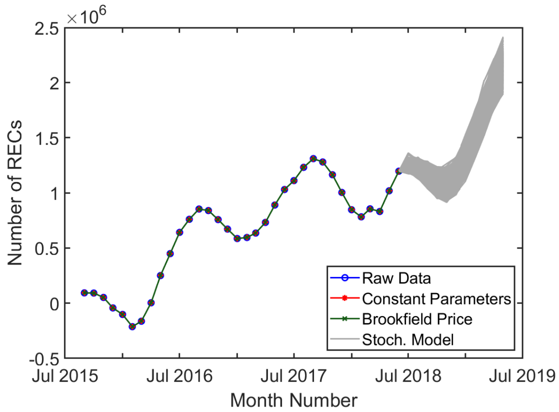


Figure 6: Simulated REC accrual $R[n]$ extended 12 months

The histogram for this data indicates that the distribution of values at the end of the extended 12 month period is symmetric and approximately normal. The REC accrual values reach a mean of 177,196 total RECs accrued with a variability of 3.97%.

4.2. Effect of Cyclic Energy Signals on RECs and Control Parameters. The amount of excess RECs the campus has at any given month ($\Delta R[n]$) in the analyzed time frame can be viewed in Figure 7 (blue line). We can see how these excess RECs Clarkson University gains each month vary as the energy inputs (orange lines of Figure 7) vary. It is easy to see that some sources are largely cyclical over the course of a year, like Brookfield Renewable (E_B) and Clarkson Solar (E_S), whereas others appear more steady.

Excess RECs $\Delta R[n]$ are strongly associated with solar production during the year. The data indicates that the solar farm operates at a monetary loss of around \$1600 per month. The expense is justified as the solar farm provides educational opportunities for Clarkson University and the opportunity for campus to reach its 100% renewability goal.

The cyclic nature of energy sources drives the cyclic nature of RECs. It is therefore expected that the portfolio parameters μ and γ, which control the portion of the Brookfield Renewable's bill paid by monetary credits vs. money, could likely have a cyclic nature. To check this, we graphed the inputs with strongest cyclic nature, the Brookfield Renewable E_B and Clarkson Solar E_S energy, along with parameters μ and γ on a plot in Figure 8.

The third control parameter ω was assumed to be consistent at $\omega = 1$, although an alternative management strategy may allow ω to change. Figure 8 shows that the majority of the time $\mu \approx 1$. Since μ and ω both control the portion of the delivery cost paid by monetary

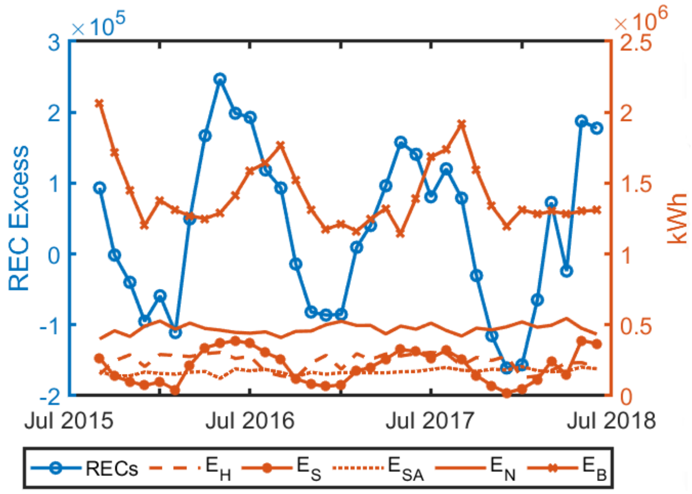


Figure 7: Comparing cyclic nature of excess RECs $\Delta R[n]$ to the cycles of the energy input signals $E_*[n]$

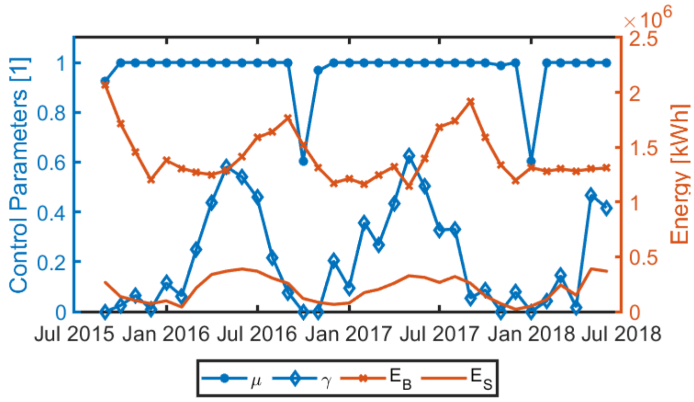
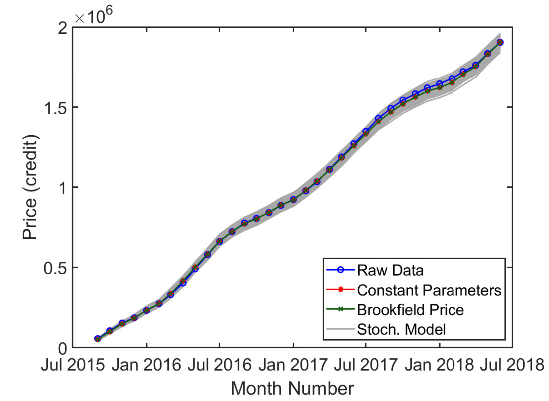


Figure 8: Visualization of the change in control parameters μ and γ over the 34 month time period. Energy signals from Brookfield Renewable E_B and Clarkson Solar E_S included to compare cyclic trends in control parameters with that of inputs

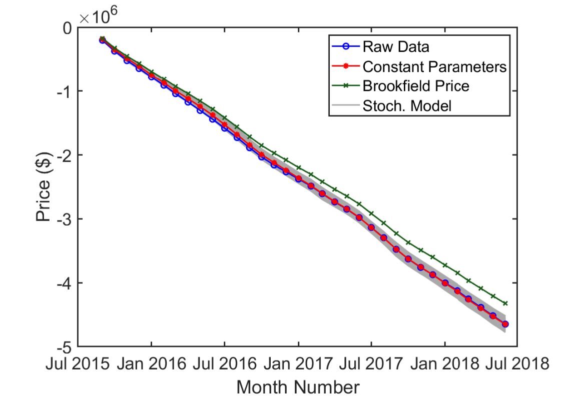
credits, it would likely be possible to simplify this model by setting both $\omega = \mu = 1$.

On the other hand, the parameter controlling the consumption cost γ remains largely cyclic due to the cyclic nature of energy production. As described in Subsection 3.2, further research could be done showing how these control parameters can be manually adjusted to align bill payment and monetary credit usage with a particular portfolio.

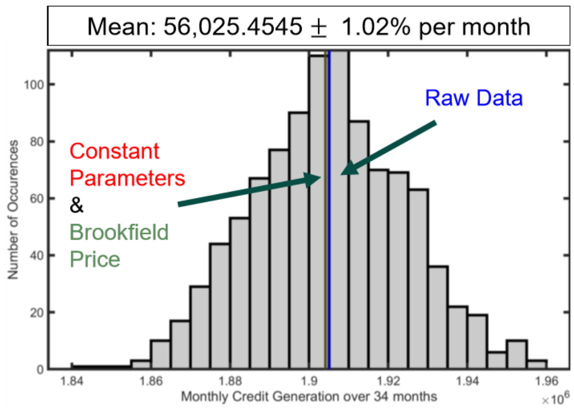
4.3. Comparing Quantitative Models. The results of the different realizations of the simulation with historic energy inputs, but different choices of parameter models Sim. (a) are shown in Figure 9. The figure describes the trends for accumulation of monetary credits $C[n]$



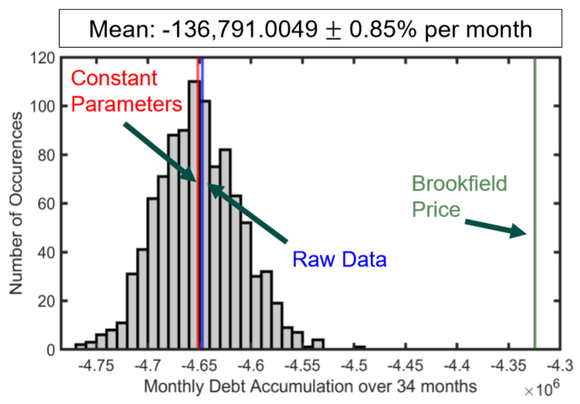
(a) Running sum of credit generation $C[n]$ (assumes monetary credits not applied to any bills during 34 mo. time period)



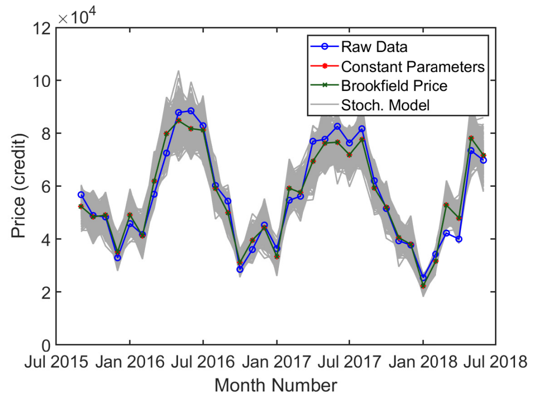
(b) Running sum of debt accumulation $D[n]$ (assumes no bills paid off during 34 mo. time period)



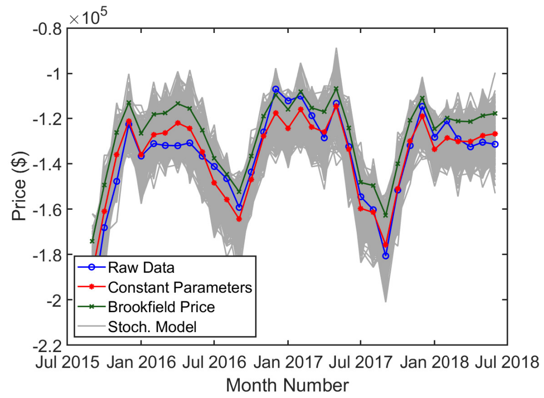
(c) Histogram showing distribution of values at month 34 ($C[34]$) from each of the 1000 randomized runs from Figure 9a



(d) Histogram showing distribution of values at month 34 ($D[34]$) from each of the 1000 randomized runs from Figure 9b



(e) Monetary credits generated each monthly increment ($\Delta C[n]$)



(f) Debt accumulated each monthly increment ($\Delta D[n]$)

Figure 9: Monetary credit generation $C[n]$ and debt accumulation $D[n]$ trends

and debt $D[n]$ with three subfigures providing details of the analysis for each.

To look at the differences in credit and debt accumulation over time more closely, running sums were graphed with the assumption no monetary credits were used and no bills were paid. These graphs are shown in Figures 9a and 9b. The running sum of monetary credit accumulation $C[n]$ in Figure 9a plots (3.1) but without including the negative terms that would take away from the growth (the terms containing E_N and E_B). This allowed us to actually see how the parameters were affecting the outcomes. To model the growth of debt $D[n]$ as if it was never paid off, we plotted (3.2) as it is presented on Figure 9b. In reality, at the end of each monthly credit cycle, Clarkson University pays the bills to National Grid and other partners. However, the resulting zeroing-out of the balances masks the difference between model alternatives.

All realizations of simulation Sim. (a) align closely with one another as can be seen on the running sum plots, Figures 9a and 9b. The gray band in the figures is the result of the stochastic model Sim. (a.iv) which treats the values of each parameter as a normal distribution and generated random variables using the same calculated mean and variance for each parameter. This essentially generated a new historical dataset for the seven parameters of thirty-four unique monthly values to be used in the model. The range produced by simulations with stochastically-varied parameters Sim. (a.iv) tells us the existing history, simulation Sim. (a.i), is not an outlier based on its position within the range of randomized outcomes. Further, the simulation with mean historic values (Sim. (a.ii)) appears centered in the randomized band from the stochastic model Sim. (a.iv) in both Figures 9a and 9b. This is to be expected as the stochastic model uses normal distributions centered around historic means and simulation Sim. (a.i) uses essentially averages of those values.

A clearer view of the distribution of the randomized data from simulation Sim. (a.iv) can be seen in Figures 9c and 9d. These graphs take the final value of each randomized curve (month 34) plotted in Figures 9a and 9b, respectively, and plot them on a histogram to show where the distribution could lay. Also, known values from the other versions of simulation Sim. (a) are graphed and labeled to see how they compare in the theoretical range.

The variability in the outcomes of the model in the final month of the simulation (month 34) is only about 1% as shown by the histograms of Figures 9c and 9d. This small variability can also be seen by how thin the gray band formed by stochastic simulation Sim. (a.iv) is on Figures 9a and 9b. With such a consistency of energy usage and prices, we can claim the low variability in the parameters is not enough to significantly affect monetary credit generation $C[n]$ and debt accumulation $D[n]$. Also, we can now clearly see that with the lower suggested price for Brookfield Renewable (simulation Sim. (a.iii)), as opposed to the average price previously paid to the energy service company (simulation Sim. (a.ii)), there is a clear savings in overall price paid for energy. From the historical data, the debt would be at least \$300,000 less at the end of this specific 34 month range than with the old energy provider as shown by the green Brookfield Price vertical line in Figure 9d.

Lastly, to demonstrate how monetary credits are generated and debt is accumulated month to month ($\Delta C[n]$ and $\Delta D[n]$, respectively), the 4 versions of simulation Sim. (a) are plotted at monthly n increments. These monthly increments do not take into account the monetary credit and debt values of the previous month so no running sum values are used. Again, the gray band is the range at which the curves are likely to appear at each monthly increment and are generated using the stochastic simulation Sim. (a.iv).

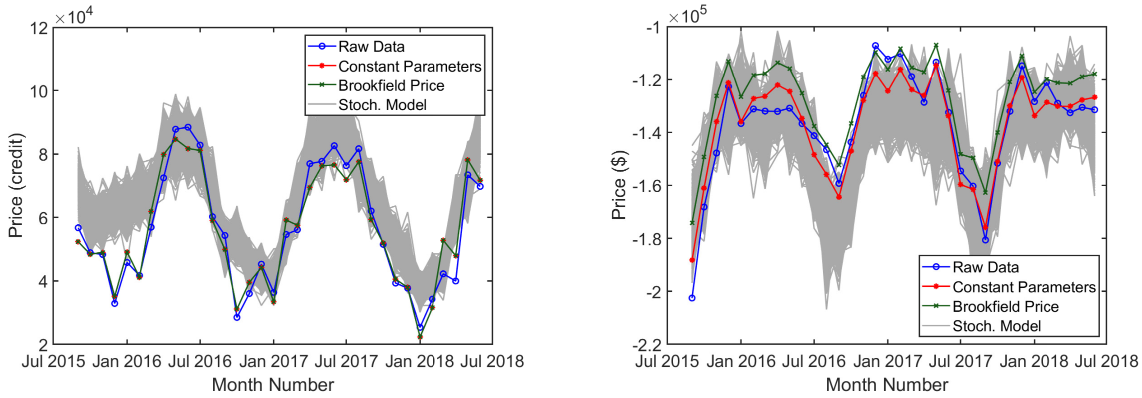
Based on current energy usage and agreements, Clarkson University gains monetary credits from local sources every month. This will continue indefinitely unless agreements with local partners change. The credits generated each month during the 34 month timeframe of this model are presented in Figure 9e for all realizations of Sim. (a). We can see that monetary credit generation occurs in cycles and that some times of the year have high variability compared to others. This is similar to the bills Clarkson University must pay each month as well. Energy usage changes over the year and some months have more variability than others as shown by Figure 9f. As previously described for the running sum plots, the monthly increment plots also demonstrate that the historical data simulation Sim. (a.i) is not an outlier and the simulation with mean historic values Sim. (a.ii) is centered in the stochastically generated range.

To summarize, Figure 9 displays that with Clarkson University's current energy usages, monetary credits are constantly being generated for the university to apply to National Grid bills and with Brookfield Renewable as a new, cheaper power source, Clarkson University will certainly be able to save money on bills moving forward.

4.4. Modeling Monetary Credit and Debt Trends Including Noise Components. We began exploring the cyclic trends of the energy inputs by developing and modeling equations that describe the periodic nature of each signal and include an additional noise factor. The stochastic modeling was repeated to determine how well the data was represented by the simulated curves with a periodic model with noise, calculated for each energy input E_* using (3.7). Parameters were still randomized, but using the simulated curves, the inputs were now randomized as well. This resulted in simulation Sim. (b). The simulated plots of monetary credit generation and debt accumulation incremented monthly ($\Delta C[n]$ and $\Delta D[n]$) were developed and are presented in Figure 10. These plots are calculated the same as those shown in Figure 9e and Figure 9f, except the energy inputs are now estimated stochastically using the periodic equation (3.9) instead of using values directly from the dataset.

The simulated curves appear to overestimate both the monetary credits generated and debt accumulated over time in Figure 10. To try to account for this trend of overestimation, a linear trendline was added to (3.7) (included as a $B_0 t$ term). This additional coefficient was intended to improve the fit of the simulated curves but it results in only slight, if any, differences. Therefore, it was determined that the B_0 coefficient was not significant in modeling the periodic trends of the energy signals. While the period and noise curves do similarly represent the data, further work could be done to improve the model.

5. Future Work and Applications. While the results in this paper support that Clarkson University's Potsdam campus is 100% renewable, this work should be extended to account for the Capitol Hill Region and Beacon campuses that Clarkson University runs as well. It is important to apply this model to the current purchasing situation of these campuses to be able to understand how much renewable energy each campus uses and how much of their energy consumed does not yet have RECs applied to it. Any excess RECs that the university can be certain will not be needed in Potsdam should be applied to the other campuses to help them reach 100% renewability as well. More modeling will have to be completed to determine which campus these RECs should be applied to first or if that answer changes based on other



(a) Simulated credit generation for each monthly increment $\Delta C[n]$

(b) Simulated debt accumulation for each monthly increment $\Delta D[n]$

Figure 10: Simulated monthly behavior of credit and debt accretion using fitted periodic curves with noise for energy inputs

factors. Further, if any of the campuses undergo renovations or additions this could affect results positively or negatively and should be included in the model as well. For example, only the west hydro plant that is a part of Potsdam Hydro is considered in this model. The east hydro plant was still under renovation for the time period the data was collected so there is no data for this plant. In the future, the monetary credits coming from Potsdam Hydro will likely be much greater as both west and east plants come online full time.

Another aspect that could be explored in this model is the adjustment of the control parameters ω, μ, and γ. In this model we assume any monetary credits gained must be used before dollars can be applied to a bill. We also assume the order in which these monetary credits are applied to bills (NYPA delivery, Brookfield Renewable delivery, and then Brookfield Renewable consumption). Any changes in these assumptions would lead to developing different strategies for redistribution of bill payment. This would allow one to explore many alternative scenarios as ω, μ, and γ will change based on the model assumptions. This topic is briefly explored in Subsection 3.2.

An interesting extension to this model would involve looking at the data seasonally. Solar energy clearly varies on a seasonal basis, as shown in Subsection 3.3, and we determined the other energy inputs behaved similarly, but it would be interesting to see exactly how much seasonal changes affect these sources. As we are able to model the solar energy production as a periodic curve, we can expand into predictive modeling for our system. By developing and showing periodic trends of the inputs, we can model and then predict and test future possible scenarios. Understanding periodic trends will allow Clarkson University to better manage their campus financially and adjust energy usage to coordinate with peak and low times as well. In regards to the existing model, further sensitivity analysis could be performed to better understand the influence of each parameter and the importance of each frequency in the periodic functions. The significance of the 2.333 period could be explored as well. With this analysis we could determine which parameters affect outcomes the most or if any have little to no effect, we can assume that they may be treated as constants or disregarded in the model, leading to further simplification.

This model, or a similar one, could be replicated for other campuses or institutions as well. Any institution attempting to reach 100% renewability could work to understand all of their infrastructure that consumes energy. Then to reach renewability, they could look for sources that may be able to supply energy with RECs applied or sources that could supply RECs individually. To support local infrastructure and economy, it would be great to look for these sources locally to the institution.

6. Conclusion. Based on yearly generation versus consumption values, the Potsdam area campus for Clarkson University can truly be considered 100% renewable after the recent switch to Brookfield Renewable as their main energy provider. Model outcomes demonstrate little variability in parameters, providing predictable monetary credit and debt growth moving forward. Accumulation of RECs appears to be increasing, but as the first seven months on Figure 5 shows, if Clarkson University ever starts with no RECs in their "bank" there may be concern for a month or so that energy used is not covered by RECs they have. However, if we continue to look at this data on a yearly level, enough excess RECs are generated during summer months to minimize this concern as data shows each kWh at the end of the fiscal year to be covered. As stated, the data used for this model was not taken over a long time period and due to the lack of data, it is difficult to truly determine long term trends.

Constant, data-driven, and stochastic models all provided very similar outcomes for this particular dataset and the equations used. There is limited variability in the data and figures containing the modified Brookfield Renewable price support the claim that this will be a cheaper energy provider long term for Clarkson University. Fitted periodic curves with noise provide a reasonable, yet overestimated, look at model outcomes and further work could be done to improve their fit with the model data.

A similar model could be completed for other campuses or institutions. If data were collected on local agents for the transfer of electricity, money, monetary credits, and RECs for the institution, assumptions could be applied similar to Subsection 2.1 and state equations generated following the process of Subsection 3.1. Every facility is unique, and by adapting the approach used in this model to a more generalized form, each facility could achieve individually beneficial results. Adjustment of control variables, analyzation of renewability, and comparison of quantitative models all can contribute to understanding historical and predictive trends in a facility's portfolio behavior. These results can help institutions adjust their energy usage to reach renewability and financial goals.

Acknowledgements. I would like to thank Dr. Marko Budišić for all of his support and guidance both as a professor and as a research advisor. I would also like to thank Mr. Michael Tremper for providing data to work with and for answering any questions as well as Dr. Susan Powers for her feedback and suggestions for the research. Finally, I would like to acknowledge the members of the PICMath team I was previously part of and the work we accomplished together which was essential for continuing this research. Thank you to Ben Brooks, Michael Roome, and Silas Lin, all students at Clarkson University at the time.

Funding: A portion of this work was sponsored by PICMath, a Mathematical Association of America (MAA) program supported by the National Science Foundation (NSF grant DMS-1722275) and the National Security Agency (NSA).

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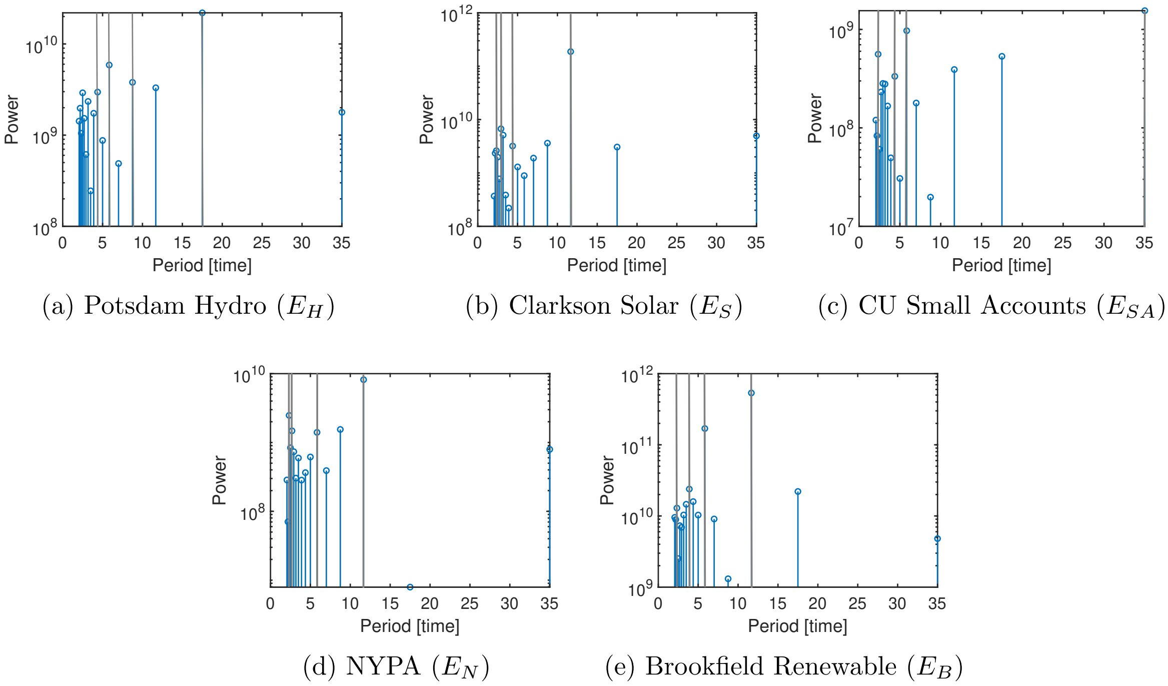
Appendix A. Energy Signal Data. Tables 4 and 5 contain information about each of the five energy signals that serve as inputs in the model. Table 4 lists the values of the top four periods seen for each energy input when performing Fast Fourier Transform on each signal. These Fourier transform plots help determine the top periods because the most relevant periods are those with the highest points on the graph. The plots are shown in Figure 11 and the most relevant periods are noted with vertical lines passing through each of the top represented period values. Values are recorded and listed in order of relative importance from left to right in Table 4.



(a) Potsdam Hydro (E_H)

(b) Clarkson Solar (E_S)

(c) CU Small Accounts (E_{SA})

(d) NYPA (E_N)

(e) Brookfield Renewable (E_B)

Figure 11: Fourier transforms of energy signals showing relevance of periods along with vertical lines showing top four most relevant periods of each signal (listed in Table 4)

Columns 2 through 5 of Table 5 present these same periods, but as rounded values instead of the raw estimated value. These rounded values are what are used in the model. This is because the R^2 values are very similar when comparing rounded and non-rounded periods

	Period 1	Period 2	Period 3	Period 4	R^2
Potsdam Hydro E_H	17.5	5.833	8.75	4.375	0.5782
Clarkson Solar E_S	11.67	2.917	4.375	2.333	0.8655
CU Small Accounts E_{SA}	35	5.833	2.333	4.375	0.6137
NYPA E_N	11.67	2.333	2.692	5.833	0.5709
Brookfield Renewable E_B	11.67	5.833	3.889	2.333	0.8635

Table 4: Top four periods associated with each energy signal E_* (seen in Figure 11) and R^2 values describing fit of periodic curve to original signal using raw data for periods. Period 1 is most represented by the data and Period 4 is least represented by the data out of the top four periods listed.

and we assume some calculation error due to the small dataset used to determine outputs for this model, so the simpler rounded values may be acceptable. The underlined values in columns 2 through 5 of Table 5 are the periods deemed most well-represented by the signal (from looking at the plots in Figure 11) and are used for simulating alternative histories as periodic functions in Subsection 3.3. Of course, frequencies are needed in the actual model equation, so the inverses of these periods are used in the actual calculations.

	P1	P2	P3	P4	$\mu(10^{-10})$	$\sigma(10^4)$	R^2
Potsdam Hydro E_H	<u>18</u>	<u>6</u>	<u>9</u>	4	-1.207	3.9863	0.5234
Clarkson Solar E_S	<u>12</u>	3	4	2.333	-0.4665	4.0810	0.8868
Small Accounts E_{SA}	<u>35</u>	<u>6</u>	<u>2.333</u>	4	-0.6078	1.3381	0.5845
NYPA E_N	<u>12</u>	<u>2.333</u>	3	6	-1.1813	2.3874	0.5820
Brookfield Renewable E_B	<u>12</u>	<u>6</u>	4	2.333	-3.5609	8.2981	0.8756

Table 5: Signal information about each of the five energy inputs E_*. Includes four most relevant periods (rounded values with most relevant listed as P1, least relevant listed as P4), which periods are actually used in the periodic modeling (underlined values), mean μ and standard deviation σ of residual for each signal, and R^2 coefficient describing fit of data to original signal using rounded values for periods.

While not all periods are used in the model, they are provided in this table for the possibility that someone may wish to perform additional modeling with them in the future. Columns 6 and 7 contain values of the mean and standard deviation of the residual for each energy signal and column 8 is the R^2 coefficient, representing the overall fit of the data.