Maximizing Harvest Yields in a Three-Species System

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Abstract

Management decisions on sustainable harvesting of any species in our marine ecosystems benefit from mathematical modeling and simulations due to the underlying complex ecological interactions between species. Using basic mathematical analysis and numerical simulation tools, we consider the problem of investigating the maximum sustainable yield (MSY) and the maximum economic yield (MEY) when harvesting in a fishery system consisting of one predator and two competing prey species. Results show that the harvesting effort required to achieve MEY is less than what is needed to achieve MSY. This implies that increasing harvesting effort beyond what is needed to reach MEY will not necessarily deliver more profits but may run the risk of driving some of the species of the system into extinction. Furthermore, results show that under the MEY management policy, a predator-oriented harvesting approach is recommended when harvesting single-species only. For double-species harvesting in a system with weak interspecific competition and weak predation, a prey-oriented harvesting approach is recommended, but when there is strong interspecific competition and strong predation, a predator-oriented harvesting approach is recommended.

1 Introduction

Species of fish in fisheries, areas where fish are harvested for commercial purposes, are often part of a complex ecological community. In these marine communities, species interact primarily through competition and predation. This project analyzes a system made of three species: two prey and one predator. It is assumed that the predator preys on both species of prey within the system. Further, it is assumed that the two prey are in competition [9]. For example, consider a hawk that preys on both mice and

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squirrels. If the mice population increases, the squirrel population may be positively affected since more mice will be available as prey for hawks. However, an increased mouse population may eventually lead to a higher population of hawks, requiring more prey, thus, negatively impacting squirrels through increased predation pressure. An example of a fishery system with two prey species and one predator is sketched below. In the figure, the solid arrows represent the positive effect of predation on the predator,

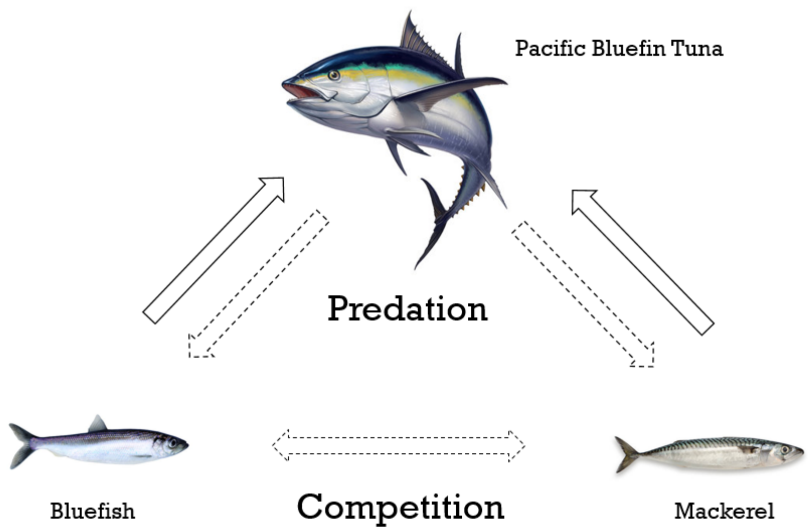

Figure 1: Predation and competition between Pacific bluefin tuna, bluefish, and mackerel.

while the single-direction dotted arrows represent the negative effect of predation on the prey. The two-direction dotted arrow between the prey represents the negative impact of the competition that this inter-specific interaction has on the two prey.

This project investigates two harvesting management approaches on a theoretical two-prey one-predator fishery system, namely, the maximum sustainable yield (MSY) and the maximum economic yield (MEY). While both are theoretical top catch or yield that can be harvested in the long-term on a particular species, they differ in perspectives depending on the underlying goal. MSY considers harvesting for yield to guarantee that species are not driven to extinction (an environment sustainability goal) while MEY considers harvesting for yield to guarantee profitability (an economic goal). MEY is the value of the largest positive difference between total revenues and total costs of fishing. In other words, MEY represents the harvesting effort at which economic rent is maximized. Economic rent is an amount of money earned that exceeds the required economic input into a particular system. Thus, in fishery science, we can consider economic rent to be profit.

The history of the work to restrict harvesting by means of harvest control rules is summarized in [8]. Harvest control rules have been described as the analytical basis for the tactics used to combat the issue of over-exploitation and provide a basis to create a way forward that translates ecological information into fishery management information like total allowable catch limit [1]. This research is in line with international efforts being made for the sustainable management of fishery resources. Goal 14 is one of the 17 Sustainable Development Goals established by the United Nations (UN) in 2015. The official wording of Goal 14 is to "Conserve and sustainably use the oceans, seas and marine resources for sustainable development" [12]. This effort encapsulates maintaining fish stocks at levels that produce a maximum sustainable yield [13].

The techniques and issues associated with mathematical bioeconomic modeling of the exploitation of biological resources such as fisheries have been discussed in detail [3, 4, 7]. Historically, modeling of the harvesting conducted in fisheries involved only one species. However, since most marine fisheries are multi-species in nature, exploitation of mixed-species fisheries has begun to gain more exposure and study from researchers [6, 15, 10, 11, 14]. The task of creating a realistic model of a multi-species community is difficult, and even with the success of formulating such a model, it is very likely that the model may not be analytically realistic since marine environments are extremely complex and interdependent [7]. Thus, how best to harvest ecologically interdependent populations in the sense of maximizing revenue, while maintaining biological existence and ecological balance, is a necessary and significant optimal control problem for fisheries [7]. This paper aims to address the MSY and MEY investigation in a multi-species system that considers all possible harvesting scenarios–this includes eight different variants of harvesting options: prey-focused, predator focused, or some amalgam of the two. To the best of our knowledge, the MSY and MEY analysis on a three-species system with two competing prey species and one predator that looks at all possible harvesting scenarios have not been carried out. The author's advisor published work on MSY and MEY analyses on a different three-species system ([5]) and as expected, different dynamic interactions between species affect the MSY and MEY management recommendations.

The mathematical processes used in this paper (differential equations and linear algebra) provide the basis for the numerical simulation codes. In particular, we investigated the MSY and MEY questions on all eight harvesting scenarios. Results show that in this three-species system, the harvesting effort required to realize MEY is less than what is needed to reach MSY. This desirable result informs managers and decision makers in the fishery community that with actual and current data to determine MEY, both economic and ecological goals of profit and sustainability may be achieved. Moreover, depending on the strength of the inter-specific competition and the strength of the predation effect on the prey species, the harvesting approach to reach MEY level may be predator-oriented or prey-oriented.

The paper is organized as follows. Section 2 discusses our model and its solutions. Section 3 lays out the equilibria of the system. Section 4 and 5 delve into the MSY and MEY for all eight harvesting cases, respectively. Section 6 uses numerical simulations to illustrate theoretical results. Finally, Section 7 presents a summary of our investigation.

2 Mathematical Model and its Solutions

Consider the ecological system where there are two competing prey species and one predator species:

$$\begin{cases} \dfrac{dx_1}{dt} &= \lambda_1 x_1(1-\frac{x_1}{K_1}) - \alpha_{12}x_1x_2 - \alpha_{13}x_1x_3 - q_1E_1x_1 \\ \dfrac{dx_2}{dt} &= \lambda_2 x_2(1-\frac{x_2}{K_2}) - \alpha_{21}x_1x_2 - \alpha_{23}x_2x_3 - q_2E_2x_2 \\ \dfrac{dx_3}{dt} &= \alpha_{31}x_1x_3 + \alpha_{32}x_2x_3 - x_3^2 - q_3E_3x_3 \end{cases} \quad (1)$$

with initial values $x_1(0) > 0, x_2(0) > 0, x_3(0) > 0$.

The interpretation of the ecological parameters is as follows: The parameters λ_1 and λ_2 are the respective growth parameters for prey x_1 and prey x_2; K_1 and K_2 are the environmental carrying capacities of x_1 and x_2, respectively; α_{12} and α_{21} are the competition effects of x_2 on x_1 and x_1 on x_2, respectively; α_{12} and α_{21} are effects of competition; α_{13} and α_{23} are predation effects of x_3 on x_1 and x_2, respectively; α_{31} and α_{32} are the predation effects on x_3 by preying on x_1 and on x_2, respectively. The parameters $E_1, E_2, E_3 \geq 0$ are the respective harvesting efforts placed upon species x_1, x_2, x_3. Likewise, q_1, q_2, q_3 are the respective catchability coefficients. A catchability coefficient is a parameter used in fisheries science and related fields to describe the efficiency of a fishing method or gear in capturing fish. The catchability coefficient is a dimensionless value that represents the proportion of fish in a population that are caught by a unit of effort (such as the amount of fishing gear used or the time spent fishing). The catchability coefficient is influenced by a variety of factors, including the behavior of the fish species being targeted, the characteristics of the fishing gear, and the skill and experience of the fishermen. Catchability coefficients may vary widely between different fishing methods and gear types, and may also change over time as fish populations and fishing practices evolve. Table 1 and Table 2 below summarize descriptions of the ecological and economic parameters, respectively:

λ_1	growth parameter of x_1
λ_2	growth parameter of x_2
K_1	environmental carrying capacity of x_1
K_2	environmental carrying capacity of x_2
α_{12}	competition effect of x_2 on x_1
α_{21}	competition effect of x_1 on x_2
α_{13}	predation effect of x_3 on x_1
α_{23}	predation effect of x_3 on x_2
α_{31}	predation effect on x_3 by preying on x_1
α_{32}	predation effect on x_3 by preying on x_2

Table 1: The ecological parameters in our model. All parameters are positive.

c_1	fishing cost per unit effort for x_1
c_2	fishing cost per unit effort for x_2
c_3	fishing cost per unit effort for x_3
p_1	price per unit biomass of x_1
p_2	price per unit biomass of x_2
p_3	price per unit biomass of x_3
q_1	catchability coefficient of x_1
q_2	catchability coefficient of x_2
q_3	catchability coefficient of x_3
E_1	harvesting effort on x_1
E_2	harvesting effort on x_2
E_3	harvesting effort on x_3

Table 2: The economic parameters in our bioeconomic model. All parameters are positive.

3 Coexistence Equilibrium

In this section, we compute the coexistence equilibrium for the system (1). The coexistence equilibrium is the solution (x_1, x_2, x_3) such that each component is positive. Observe that there are boundary equilibria, that is, where at least one of the components is zero. We do not consider these boundary equilibria. We are interested in the coexistence equilibrium only because we address sustainability of the three species in the system. Moreover, since we aim to address sustainability in terms of harvesting efforts, the goal is to compute each x_i as a function of the harvesting effort E_i. The main result of this section is contained in equation (5); but, in preparation for Sections 4 and 5, further details on M_1, M_2, M_3 are computed here. In particular, it is important to express x_i as a linear combination of the E_1, E_2, E_3.

Equilibrium or steady-states are obtained by algebraically solving for the dependent variables x_1, x_2, x_3 when the derivatives are set to zero. In other words, consider the system of equations

$$\left\{\begin{array}{lcl} \lambda_1 x_1(1-\frac{x_1}{K_1}) - \alpha_{12}x_1x_2 - \alpha_{13}x_1x_3 - q_1E_1x_1 &=& 0\\ \lambda_2 x_2(1-\frac{x_2}{K_2}) - \alpha_{21}x_1x_2 - \alpha_{23}x_2x_3 - q_2E_2x_2 &=& 0\\ \alpha_{31}x_1x_3 + \alpha_{32}x_2x_3 - x_3^2 - q_3E_3x_3 &=& 0. \end{array}\right.$$

This is equivalent to

$$\left\{\begin{array}{lcl} x_1 = 0 \text{ or } \lambda_1(1-\frac{x_1}{K_1}) - \alpha_{12}x_2 - \alpha_{13}x_3 - q_1E_1 &=& 0\\ x_2 = 0 \text{ or } \lambda_2(1-\frac{x_2}{K_2}) - \alpha_{21}x_1 - \alpha_{23}x_3 - q_2E_2 &=& 0\\ x_3 = 0 \text{ or } \alpha_{31}x_1 + \alpha_{32}x_2 - x_3 - q_3E_3 &=& 0 \end{array}\right. \tag{2}$$

To investigate maximum sustainable yield (MSY) and maximum economic yield (MEY), we need to compute the coexistence equilibrium, which is the nonzero population levels x_1, x_2, x_3 that solve (2). Since we would like to examine the efforts needed to reach

MSY or MEY, population levels for each species should be expressed as functions of the harvesting efforts E_1, E_2, E_3. Hence, consider the sub-system consisting of non-boundary values:

$$\begin{cases} \lambda_1(1-\frac{x_1}{K_1}) - \alpha_{12}x_2 - \alpha_{13}x_3 - q_1E_1 & = & 0 \\ \lambda_2(1-\frac{x_2}{K_2}) - \alpha_{21}x_1 - \alpha_{23}x_3 - q_2E_2 & = & 0 \\ \alpha_{31}x_1 + \alpha_{32}x_2 - x_3 - q_3E_3 & = & 0 \end{cases} \tag{3}$$

This is a system of three linear equations in three unknowns x_1, x_2, x_3. Suppose M is the determinant given by

$$\begin{aligned} M &= \begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \frac{\lambda_2}{K_2} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & -1 \end{vmatrix} \\ &= -\frac{\lambda_1\lambda_2}{K_1K_2} + \alpha_{12}\alpha_{23}\alpha_{31} + \alpha_{13}\alpha_{21}\alpha_{32} - \alpha_{13}\alpha_{31}\frac{\lambda_2}{K_2} - \alpha_{23}\alpha_{32}\frac{\lambda_1}{K_1} + \alpha_{21}\alpha_{12}. \end{aligned} \tag{4}$$

Observe that M computes the determinant of the coefficients of x_1, x_2, x_3 when solving (3). We consider the case $M \neq 0$. For any given set of parameter values, the sign of M may be positive or negative: $M > 0$ happens when interspecific competition between the prey species and predation effect are both strong; $M < 0$ happens when interspecific competition between the prey species and predation effect are both weak. What is important is that the parameter values must be chosen such that $M \neq 0$. By Cramer's Rule, the population levels x_1, x_2, x_3 as functions of E_1, E_2, E_3 are given by

$$x_1 = \frac{M_1}{M}, \quad x_2 = \frac{M_2}{M}, \quad x_3 = \frac{M_3}{M}, \tag{5}$$

where

$$\begin{aligned} M_1 = M_1(E_1,E_2,E_3) &= \begin{vmatrix} \lambda_1 - q_1E_1 & \alpha_{12} & \alpha_{13} \\ \lambda_2 - q_2E_2 & \frac{\lambda_2}{K_2} & \alpha_{23} \\ q_3E_3 & \alpha_{32} & -1 \end{vmatrix}, \\ M_2 = M_2(E_1,E_2,E_3) &= \begin{vmatrix} \frac{\lambda_1}{K_1} & \lambda_1 - q_1E_1 & \alpha_{13} \\ \alpha_{21} & \lambda_2 - q_2E_2 & \alpha_{23} \\ \alpha_{31} & q_3E_3 & -1 \end{vmatrix}, \\ M_3 = M_3(E_1,E_2,E_3) &= \begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} & \lambda_1 - q_1E_1 \\ \alpha_{21} & \frac{\lambda_2}{K_2} & \lambda_2 - q_2E_2 \\ \alpha_{31} & \alpha_{32} & q_3E_3 \end{vmatrix}. \end{aligned} \tag{6}$$

In preparation for the next two sections' analysis on MSY and MEY, let us rewrite M_1, M_2, M_3 as functions of the harvesting efforts E_1, E_2, E_3. Expanding the determinant for M_1 about the first column and simplifying, we see that M_1 is linear in E_1, E_2, E_3:

$$\begin{aligned} M_1 &= (\lambda_1 - q_1E_1)\begin{vmatrix} \frac{\lambda_2}{K_2} & \alpha_{23} \\ \alpha_{32} & -1 \end{vmatrix} - (\lambda_2 - q_2E_2)\begin{vmatrix} \alpha_{12} & \alpha_{13} \\ \alpha_{32} & -1 \end{vmatrix} + q_3E_3\begin{vmatrix} \alpha_{12} & \alpha_{13} \\ \frac{\lambda_2}{K_2} & \alpha_{23} \end{vmatrix} \\ &= M_{11}E_1 + M_{12}E_2 + M_{13}E_3 + M_{10} \end{aligned} \tag{7}$$

where

$$M_{11} = -\begin{vmatrix} \frac{\lambda_2}{K_2} & \alpha_{23} \\ \alpha_{32} & -1 \end{vmatrix} q_1, \quad M_{12} = \begin{vmatrix} \alpha_{12} & \alpha_{13} \\ \alpha_{32} & -1 \end{vmatrix} q_2, \quad M_{13} = \begin{vmatrix} \alpha_{12} & \alpha_{13} \\ \frac{\lambda_2}{K_2} & \alpha_{23} \end{vmatrix} q_3$$

and

$$M_{10} = -\lambda_1(\frac{\lambda_2}{K_2} + \alpha_{23}\alpha_{32}) + \lambda_2(\alpha_{12} + \alpha_{13}\alpha_{32}).$$

Here, M_{1i} denotes the coefficient of E_i in the expansion of M_1 for $i = 1, 2, 3$, while M_{10} consists of the parameters in M_1 that do not involve E_i. Observe that M_{11} is a positive number. Similarly, expanding the determinant for M_2 about the second column and simplifying, we see that M_2 is linear in E_1, E_2, E_3:

$$\begin{aligned} M_2 &= -(\lambda_1 - q_1E_1)\begin{vmatrix} \alpha_{21} & \alpha_{23} \\ \alpha_{31} & -1 \end{vmatrix} + (\lambda_2 - q_2E_2)\begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{13} \\ \alpha_{31} & -1 \end{vmatrix} - q_3E_3\begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{13} \\ \alpha_{21} & \alpha_{23} \end{vmatrix} \\ &= M_{21}E_1 + M_{22}E_2 + M_{23}E_3 + M_{20} \end{aligned} \tag{8}$$

where

$$M_{21} = \begin{vmatrix} \alpha_{21} & \alpha_{23} \\ \alpha_{31} & -1 \end{vmatrix} q_1, \quad M_{22} = -\begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{13} \\ \alpha_{31} & -1 \end{vmatrix} q_2, \quad M_{23} = -\begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{13} \\ \alpha_{21} & \alpha_{23} \end{vmatrix} q_3$$

and

$$M_{20} = \lambda_1(\alpha_{21} + \alpha_{23}\alpha_{31}) - \lambda_2(\frac{\lambda_1}{K_1} + \alpha_{13}\alpha_{31}).$$

Here, M_{2i} denotes the coefficient of E_i in the expansion of M_2 for $i = 1, 2, 3$, while M_{20} consists of the parameters in M_2 that do not involve E_i. Observe that M_{22} is a positive number.

Finally, expanding the determinant for M_3 about the third column and simplifying, we see that M_3 is linear in E_1, E_2, E_3:

$$\begin{aligned} M_3 &= (\lambda_1 - q_1E_1)\begin{vmatrix} \alpha_{21} & \frac{\lambda_2}{K_2} \\ \alpha_{31} & \alpha_{32} \end{vmatrix} - (\lambda_2 - q_2E_2)\begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} \\ \alpha_{31} & \alpha_{32} \end{vmatrix} + q_3E_3\begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} \\ \alpha_{21} & \frac{\lambda_2}{K_2} \end{vmatrix} \\ &= M_{31}E_1 + M_{32}E_2 + M_{33}E_3 + M_{30} \end{aligned} \tag{9}$$

where

$$M_{31} = -\begin{vmatrix} \alpha_{21} & \frac{\lambda_2}{K_2} \\ \alpha_{31} & \alpha_{32} \end{vmatrix} q_1, \quad M_{32} = \begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} \\ \alpha_{31} & \alpha_{32} \end{vmatrix} q_2, \quad M_{33} = \begin{vmatrix} \frac{\lambda_1}{K_1} & \alpha_{12} \\ \alpha_{21} & \frac{\lambda_2}{K_2} \end{vmatrix} q_3$$

and

$$M_{30} = \lambda_1\left(\alpha_{21}\alpha_{32} - \alpha_{31}\frac{\lambda_2}{K_2}\right) - \lambda_2\left(\frac{\lambda_1}{K_1}\alpha_{32} - \alpha_{12}\alpha_{31}\right).$$

Here, M_{3i} denotes the coefficient of E_i in the expansion of M_3 for $i = 1, 2, 3$, while M_{30} consists of the parameters in M_3 that do not involve E_i.

4 MSY in different harvesting scenarios

The system (1) considers three species, two competing prey species and a predator, and so there are eight possible harvesting scenarios. The table summarizes these eight harvesting scenarios.

	E_1	E_2	E_3	Harvesting Scenario
Case 1	$= 0$	$\neq 0$	$\neq 0$	Prey 1 is not harvested.
Case 2	$= 0$	$= 0$	$\neq 0$	Prey 1 and prey 2 are not harvested.
Case 3	$\neq 0$	$= 0$	$\neq 0$	Prey 2 is not harvested.
Case 4	$\neq 0$	$\neq 0$	$\neq 0$	All species are harvested.
Case 5	$= 0$	$= 0$	$= 0$	No harvesting.
Case 6	$\neq 0$	$\neq 0$	$= 0$	Predator is not harvested.
Case 7	$= 0$	$\neq 0$	$= 0$	Prey 1 and predator not harvested.
Case 8	$\neq 0$	$= 0$	$= 0$	Prey 2 and predator not harvested.

Table 3: The eight harvesting scenarios for a system that has three species; E_1 = harvesting effort on Prey 1, E_2 = harvesting effort on Prey 2, and E_3 = harvesting effort on the Predator.

To investigate MSY, we will look at maximizing the yield function defined below. Depending on the harvesting scenario being considered, the yield function $Y = Y(E)$ is computed by using the formula

$$Y(E) = (\text{Population of harvested species as a function of } E) \times E, \tag{10}$$

where E is the common harvesting effort on the species being harvested. For example, if the first and second species are harvested while the third is not, we set $E_1 = E_2 = E$ and set $E_3 = 0$. Now, observe that the populations x_1, x_2, x_3 as given in (4), (5), with (6), give linear functions of E_1, E_2, E_3. Since these are linear functions of E, we can express each of them in the form $f(E) = mE + f(0)$ where m is the slope of the linear function and $f(0)$ is the y-intercept. Note that m and $f(0)$ are independent of E.

The yield function (10) for the three-species ecological system (1) is a quadratic function in E. Moreover, the yield function Y as defined in (10) with populations x_1, x_2, x_3 as given in (5) result to

$$Y(E) = f(E)E = (mE + f(0))E = mE^2 + f(0)E.$$

In other words, the yield function will only have a maximum value when $m = f'(E)$ is negative and $f(0) > 0$. Moreover, the harvesting effort E^* that maximizes yield is $E^* = \frac{-f(0)}{2f'(E)}$.

Let us look at Case 1 in more detail. Here, $E_1 = 0$ and $E_2 = E_3 = E \neq 0$ (the second and third species are being harvested), then

$$f(E)_{\text{Case 1}} = \underbrace{x_2(0, E, E)}_{\text{species 2 is harvested}} + \underbrace{x_3(0, E, E)}_{\text{species 3 is harvested}}.$$

Moreover, using notations from the previous section,

$$\begin{aligned} f(E)_{\text{Case 1}} &= \frac{M_2(0,E,E)}{M} + \frac{M_3(0,E,E)}{M} \\ &= \tfrac{1}{M}\left((M_{22}+M_{23}+M_{32}+M_{33})E + M_{20} + M_{30}\right), \end{aligned}$$

so that the yield function is

$$\begin{aligned} Y(E) &= \tfrac{1}{M}\left((M_{22}+M_{23}+M_{32}+M_{33})E + M_{20} + M_{30}\right)E \\ &= \tfrac{1}{M}\left(M_{22}+M_{23}+M_{32}+M_{33}\right)E^2 + \tfrac{1}{M}(M_{20}+M_{30})E, \end{aligned} \tag{11}$$

which has a maximum at

$$E^* = \frac{M_{20}+M_{30}}{-2(M_{22}+M_{23}+M_{32}+M_{33})},$$

provided the coefficient of E^2 is negative and the coefficient of E is positive in (11). The latter requirement is needed to make sure that E^* is positive.

Finally, the yield function, being quadratic, does not have a maximum if the coefficient of E^2 is zero, as the function would be linear; this particularly occurs in Case 5 below. Let us summarize the yield functions for each of the harvesting scenarios, where f is replaced by the corresponding population level of species being harvested. By definition of MSY, it is required that the coexistence equilibrium $(x_1, x_2, x_3) = \left(\frac{M_1}{M}, \frac{M_2}{M}, \frac{M_3}{M}\right)$ exists.

- **Case 2.** $E_1 = E_2 = 0, E_3 = E \neq 0$. Only the predator is harvested, so the yield function is

$$\begin{aligned} Y(E) &= x_3(0,0,E)E \\ &= \tfrac{1}{M}M_{33}E^2 + \tfrac{1}{M}M_{30}E. \end{aligned}$$

If maximum is reached, it occurs at harvesting effort

$$E^* = \frac{M_{30}}{-2M_{33}}.$$

- **Case 3.** $E_1 = E_3 = E \neq 0, E_2 = 0$. Only the first prey and the predator are harvested, so the yield function is

$$\begin{aligned} Y(E) &= (x_1(E,0,E) + x_3(E,0,E))E^2 \\ &= \tfrac{1}{M}\left(M_{11}+M_{13}+M_{31}+M_{33}\right)E^2 + \tfrac{1}{M}(M_{10}+M_{30})E. \end{aligned}$$

If maximum is reached, it occurs at harvesting effort

$$E^* = \frac{M_{10}+M_{30}}{-2(M_{11}+M_{13}+M_{31}+M_{33})}.$$

- **Case 4.** $E_1 = E_2 = E_3 = E \neq 0$. All three species are harvested, so the yield function is

$$\begin{aligned} Y(E) &= (x_1(E,E,E) + x_2(E,E,E) + x_3(E,E,E))E \\ &= \tfrac{1}{M}\left(M_{11}+M_{12}+M_{13}+M_{21}+M_{22}+M_{23}+M_{31}+M_{32}+M_{33}\right)E^2 \\ &\quad + \tfrac{1}{M}(M_{10}+M_{20}+M_{30})E. \end{aligned}$$

If maximum is reached, it occurs at harvesting effort

$$E^* = \frac{M_{10} + M_{20} + M_{30}}{-2(M_{11} + M_{12} + M_{13} + M_{21} + M_{22} + M_{23} + M_{31} + M_{32} + M_{33})}$$

- **Case 5.** $E_1 = E_2 = E_3 = 0$. None of the three species is harvested so the yield function is not a quadratic function. In this case, vacuously, there is no MSY.

- **Case 6.** $E_1 = E_2 = E \neq 0, E_3 = 0$. Only the two prey species are harvested, so the yield function is

$$\begin{aligned} Y(E) &= (x_1(E, E, 0) + x_2(E, E, 0))E \\ &= \tfrac{1}{M}\left(M_{11} + M_{12} + M_{21} + M_{22}\right)E^2 + \tfrac{1}{M}(M_{10} + M_{20})E. \end{aligned}$$

  If maximum is reached, it occurs at the harvesting effort

$$E^* = \frac{M_{10} + M_{20}}{-2(M_{11} + M_{12} + M_{21} + M_{22})}.$$

- **Case 7.** $E_2 = E \neq 0, E_1 = E_3 = 0$. Only the second prey is harvested, so the yield function is

$$\begin{aligned} Y(E) &= x_2(0, E, 0)E \\ &= \tfrac{1}{M}M_{22}E^2 + \tfrac{1}{M}M_{20}E. \end{aligned}$$

  If the maximum is reached, it occurs at the harvesting effort

$$E^* = \frac{M_{20}}{-2M_{22}}.$$

  Now, having previously noted that M_{22} is a positive number, this implies that the maximum is reached provided $M < 0$ and $M_{20} < 0$. There is no maximum when $M > 0$.

- **Case 8.** $E_1 = E \neq 0, E_2 = E_3 = 0$. Only the first prey is harvested, so the yield function is

$$\begin{aligned} Y(E) &= x_1(E, 0, 0)E \\ &= \tfrac{1}{M}M_{11}E^2 + \tfrac{1}{M}M_{10}E. \end{aligned}$$

  If maximum is reached, it occurs at harvesting effort

$$E^* = \frac{M_{10}}{-2M_{11}}.$$

  Since M_{11} is always a positive number, the maximum occurs only when $M < 0$ and $M_{10} < 0$. There is no maximum when $M > 0$.

We record the necessary conditions that guarantee the existence of MSY in each harvesting scenario. Namely, we require $x_i > 0$, the coefficient of E^2 in $Y(E)$ must be negative, and the coefficient of E in $Y(E)$ must be positive. Since each x_i has a denominator M, we consider two cases when $M > 0$ and when $M < 0$. Finally, we have seen that MSY is not possible for Case 5, Case 7, and Case 8 whenever $M > 0$. The results are summarized in the next two tables.

Harvesting Scenario	Necessary Conditions for MSY, $M > 0$
Case 1 $E_1 = 0$ $E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{22} + M_{23})E + M_{20} > 0 \\ (M_{32} + M_{33})E + M_{30} > 0 \\ M_{22} + M_{23} + M_{32} + M_{33} < 0 \\ M_{20} + M_{30} > 0 \end{cases}$
Case 2 $E_1 = E_2 = 0$ $E_3 = E \neq 0$	$\begin{cases} M_{33}E + M_{30} > 0 \\ M_{33} < 0 \\ M_{30} > 0 \end{cases}$
Case 3 $E_1 = E_3 = E \neq 0$ $E_2 = 0$	$\begin{cases} (M_{11} + M_{13})E + M_{10} > 0 \\ (M_{31} + M_{33})E + M_{30} > 0 \\ M_{11} + M_{13} + M_{31} + M_{33} < 0 \\ M_{10} + M_{30} > 0 \end{cases}$
Case 4 $E_1 = E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{11} + M_{12} + M_{13})E + M_{10} > 0 \\ (M_{21} + M_{22} + M_{23})E + M_{20} > 0 \\ (M_{31} + M_{32} + M_{33})E + M_{30} > 0 \\ M_{11} + M_{12} + M_{13} \\ +M_{21} + M_{22} + M_{23} \\ +M_{31} + M_{32} + M_{33} < 0 \\ M_{10} + M_{20} + M_{30} > 0 \end{cases}$
Case 6 $E_1 = E_2 = E \neq 0$ $E_3 = 0$	$\begin{cases} (M_{11} + M_{12})E + M_{10} > 0 \\ (M_{21} + M_{22})E + M_{20} > 0 \\ M_{11} + M_{12} + M_{21} + M_{22} < 0 \\ M_{10} + M_{20} > 0 \end{cases}$

Table 4: Harvesting Scenario: Necessary Condition for MSY where M>0

Harvesting Scenario	Necessary Conditions for MSY, $M < 0$
Case 1 $E_1 = 0$ $E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{22} + M_{23})E + M_{20} < 0 \\ (M_{32} + M_{33})E + M_{30} < 0 \\ M_{22} + M_{23} + M_{32} + M_{33} > 0 \\ M_{20} + M_{30} < 0 \end{cases}$
Case 2 $E_1 = E_2 = 0$ $E_3 = E \neq 0$	$\begin{cases} M_{33}E + M_{30} < 0 \\ M_{33} > 0 \\ M_{30} < 0 \end{cases}$
Case 3 $E_1 = E_3 = E \neq 0$ $E_2 = 0$	$\begin{cases} (M_{11} + M_{13})E + M_{10} < 0 \\ (M_{31} + M_{33})E + M_{30} < 0 \\ M_{11} + M_{13} + M_{31} + M_{33} > 0 \\ M_{10} + M_{30} < 0 \end{cases}$
Case 4 $E_1 = E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{11} + M_{12} + M_{13})E + M_{10} < 0 \\ (M_{21} + M_{22} + M_{23})E + M_{20} < 0 \\ (M_{31} + M_{32} + M_{33})E + M_{30} < 0 \\ M_{11} + M_{12} + M_{13} \\ +M_{21} + M_{22} + M_{23} \\ +M_{31} + M_{32} + M_{33} > 0 \\ M_{10} + M_{20} + M_{30} < 0 \end{cases}$
Case 6 $E_1 = E_2 = E \neq 0$ $E_3 = 0$	$\begin{cases} (M_{11} + M_{12})E + M_{10} < 0 \\ (M_{21} + M_{21})E + M_{20} < 0 \\ M_{11} + M_{12} + M_{21} + M_{22} > 0 \\ M_{10} + M_{20} < 0 \end{cases}$
Case 7 $E_1 = E_3 = 0$ $E_2 = E \neq 0$	$\begin{cases} M_{22}E + M_{20} < 0 \\ M_{22} > 0 \\ M_{20} < 0 \end{cases}$
Case 8 $E_1 = E \neq 0$ $E_2 = E_3 = 0$	$\begin{cases} M_{11}E + M_{10} < 0 \\ M_{11} > 0 \\ M_{10} < 0 \end{cases}$

Table 5: Harvesting Scenario: Necessary Condition for MSY where M<0

5 MEY in different harvesting scenarios

To investigate MEY, we look at maximizing the economic yield function; this is the yield that comes from multiplying the harvesting effort E_i by the economic rent π_i:

$$\begin{aligned}\Pi(E) &= (\text{Total revenue due to harvesting as a function of } E) \times E \\ &= \pi_1 \times E_1 + \pi_2 \times E_2 + \pi_3 \times E_3,\end{aligned}$$

where $\pi_i = p_i q_i x_i - c_i$, for $i = 1,2,3$. Here, $x_i = x_i(E_i)$ is the population of the harvested species as a function of the harvesting effort.

The fishing cost per unit effort for each species is captured by c_i for $i = 1,2,3$. The price per unit biomass for each species is captured by p_i, for $i = 1,2,3$. The catchability coefficients are represented by q_i, for $i = 1,2,3$. A catchability coefficient relates biomass abundance to capture or fishing mortality. It involves various aspects of the fishery, such as individual and population biology, characteristics of the fishing gear, amount of fishing, fishing strategies, and environmental fluctuation [2].

Like in the MSY analysis, we will look at eight harvesting scenarios.

- **Case 1.** $E_1 = 0, E_2 = E_3 = E \neq 0$. Only the second prey and the predator are harvested, so the economic yield is

$$\begin{aligned}\Pi(E) &= \left(p_2q_2x_2(0,E,E) + p_3q_3x_3(0,E,E) - (c_2+c_3)\right) E \\ &= \left(\tfrac{p_2q_2}{M}((M_{22}+M_{23})E + M_{20}) - c_2\right) E + \left(\tfrac{p_3q_3}{M}((M_{32}+M_{33})E + M_{30}) - c_3\right) E \\ &= \left(\tfrac{p_2q_2}{M}(M_{22}+M_{23}) + \tfrac{p_3q_3}{M}(M_{32}+M_{33})\right) E^2 + \left(\tfrac{p_2q_2}{M} M_{20} + \tfrac{p_3q_3}{M} M_{30} - (c_2+c_3)\right) E\end{aligned}$$

  If maximum is achieved, it occurs at the harvesting effort

$$\bar{E} = \frac{p_2q_2M_{20} + p_3q_3M_{30} - (c_2+c_3)M}{-2\left(p_2q_2(M_{22}+M_{23}) + p_3q_3(M_{32}+M_{33})\right)}.$$

- **Case 2.** $E_1 = E_2 = 0, E_3 = E \neq 0$. Only the predator is harvested, so the economic yield is

$$\begin{aligned}\Pi(E) &= (p_3q_3x_3(0,0,E) - c_3)E \\ &= \tfrac{p_3q_3}{M}(M_{33}E + M_{30}) - c_3)E \\ &= \tfrac{p_3q_3}{M} M_{33}E^2 + (\tfrac{p_3q_3}{M} M_{30} - c_3)E\end{aligned}$$

  If maximum is achieved, it occurs at the harvesting effort

$$\bar{E} = \frac{p_3q_3M_{30} - c_3M}{-2p_3q_3M_{33}}$$

- **Case 3.** $E_1 = E_3 = E \neq 0, E_2 = 0$. Only the first prey and the predator are harvested, so the economic yield is

$$\begin{aligned}\Pi(E) &= \left(p_1q_1x_1(E,0,E) + p_3q_3x_3(E,0,E) - (c_1+c_3)\right) E \\ &= \left(\tfrac{p_1q_1}{M}(M_{11}+M_{13}) + \tfrac{p_3q_3}{M}(M_{31}+M_{33})\right) E^2 + \left(\tfrac{p_1q_1}{M} M_{10} + \tfrac{p_3q_3}{M} M_{30} - (c_1+c_3)\right) E\end{aligned}$$

If maximum is achieved, it occurs at harvesting effort

$$\bar{E} = \frac{p_1 q_1 M_{10} + p_3 q_3 M_{30} - (c_1 + c_3) M}{-2\left(p_1 q_1 (M_{11} + M_{13}) + p_3 q_3 (M_{31} + M_{33})\right)}$$

- **Case 4.** $E_1 = E_2 = E_3 = E \neq 0$. All three species are harvested, so the economic yield is

$$\begin{aligned} \Pi(E) &= \left((p_1 q_1 x_1 + p_2 q_2 x_2 + p_3 q_3 x_3)(E, E, E) - (c_1 + c_2 + c_3)\right) E \\ &= \left(\tfrac{p_1 q_1}{M}(M_{11} + M_{12} + M_{13}) + \tfrac{p_2 q_2}{M}(M_{21} + M_{22} + M_{23}) + \tfrac{p_3 q_3}{M}(M_{31} + M_{32} + M_{33})\right) E^2 \\ &\quad + \left(\tfrac{p_1 q_1}{M} M_{10} + \tfrac{p_2 q_2}{M} M_{20} + \tfrac{p_3 q_3}{M} M_{30} - (c_1 + c_2 + c_3)\right) E. \end{aligned}$$

If maximum is achieved, it occurs at harvesting effort

$$\bar{E} = \frac{p_1 q_1 M_{10} + p_2 q_2 M_{20} + p_3 q_3 M_{30} - (c_1 + c_2 + c_3) M}{-2\left(p_1 q_1 (M_{11} + M_{12} + M_{13}) + p_2 q_2 (M_{21} + M_{22} + M_{23}) + p_3 q_3 (M_{31} + M_{32} + M_{33})\right)}.$$

- **Case 5.** $E_1 = E_2 = E_3 = E = 0$. None of the three species is harvested. Like the MSY, we do not consider the MEY investigation in this case.

- **Case 6.** $E_1 = E_2 = E \neq 0, E_3 = 0$. Only the two prey species are harvested, so the economic yield is

$$\begin{aligned} \Pi(E) &= \left((p_1 q_1 x_1 + p_2 q_2 x_2)(E, E, 0) - (c_1 + c_2)\right) E \\ &= \left(\tfrac{p_1 q_1}{M}(M_{11} + M_{12}) + \tfrac{p_2 q_2}{M}(M_{21} + M_{22})\right) E^2 + \left(\tfrac{p_1 q_1}{M} M_{10} + \tfrac{p_2 q_2}{M} M_{20} - (c_1 + c_2)\right) E \end{aligned}$$

If maximum is achieved, it occurs at harvesting effort

$$\bar{E} = \frac{p_1 q_1 M_{10} + p_2 q_2 M_{20} - (c_1 + c_2) M}{-2\left(p_1 q_1 (M_{11} + M_{12}) + p_2 q_2 (M_{21} + M_{22})\right)}.$$

- **Case 7.** $E_2 = E \neq 0, E_1 = E_3 = 0$. Only the second prey is harvested, so the economic yield is

$$\begin{aligned} \Pi(E) &= (p_2 q_2 x_2(0, E, 0) - c_2) E \\ &= \tfrac{p_2 q_2}{M}((M_{22} E + M_{20}) - c_2) E \\ &= \tfrac{p_2 q_2}{M} M_{22} E^2 + (\tfrac{p_2 q_2}{M} M_{20} - c_2) E \end{aligned}$$

If maximum is achieved, it occurs at the harvesting effort

$$\bar{E} = \frac{p_2 q_2 M_{20} - c_2 M}{-2 p_2 q_2 M_{22}}.$$

Recalling that M_{22} is always a positive number, we see that the maximum occurs only when $M < 0$.

- **Case 8.** $E_1 = E \neq 0, E_2 = E_3 = 0$. Only the first prey is harvested, so the economic yield is

$$\begin{aligned}\Pi(E) &= (p_1 q_1 x_1(E,0,0) - c_1)E \\ &= \frac{p_1 q_1}{M}((M_{11}E + M_{10}) - c_1)E \\ &= \frac{p_1 q_1}{M} M_{11} E^2 + (\frac{p_1 q_1}{M} M_{10} - c_1)E\end{aligned}$$

If maximum is achieved, it occurs at the harvesting effort

$$\bar{E} = \frac{p_1 q_1 M_{10} - c_1 M}{-2 p_1 q_1 M_{11}}$$

Since $M_{11} > 0$, it follows that the MEY occurs only when $M < 0$.

Like in the MSY analysis, we need to look at two main cases: when M is positive and when M is negative. We also require that the x_i for the species being harvested is positive, the coefficient of E^2 in the economic yield $\Pi(E)$ is negative, and the coefficient of E in $\Pi(E)$ is positive. We summarize the results in the next two tables, which are quite similar in form to the MSY, but takes into consideration the price p_i per unit biomass, the catchability coefficients q_i, and the fishing cost c_i per unit effort for each harvested species. Finally, like in the MSY analysis, maximum does not occur for Cases 5, 7, 8 whenever $M > 0$.

For each harvesting scenario, it is interesting to determine how the harvesting effort E^* to achieve MSY is related to the harvesting effort $\bar{E}$ to achieve MEY. In the single-species case, it was established that the MSY and MEY occurs only when the predator is harvested (Case 2) when $M > 0$ and in all three single-species harvesting (Cases 2, 7, 8) when $M < 0$. Let us compare E^* and $\bar{E}$, for $i = 1, 2, 3$:

$$\begin{aligned}E^* - \bar{E} &= \frac{M_{i0}}{-2M_{ii}} - \frac{p_i q_i M_{i0} - c_i M}{-2 p_i q_i M_{ii}} \\ &= \frac{p_i q_i M_{i0} - p_i q_i M_{i0} + c_i M}{-2 p_i q_i M_{ii}} \\ &= \frac{c_i M}{-2 p_i q_i M_{ii}}.\end{aligned}$$

When $M < 0$, since $M_{ii} > 0$, it follows that $E^* > \bar{E}$. With this inequality, any single-species harvesting efforts beyond $\bar{E}$ but less than E^* do not contribute towards a higher economic profit, and any harvesting efforts beyond E^* neither support economic profitability goals nor environment sustainability goals. When $M > 0$, since $M_{33} < 0$ (Case 2), it follows that $E^* > \bar{E}$. Thus, in both cases, we have the environmentally-desirable situation that $\bar{E} < E^*$. In other words, in all cases, whether harvesting prey only or predator only, it is true that the harvesting effort needed to reach MEY is less than the harvesting effort to reach MSY.

To tackle the question, how is $\bar{E}$ related to E^* in case two species or three species are being harvested, we will use numerical observations in the next section.

Harvesting Scenario	Necessary Conditions for MEY, $M > 0$
Case 1 $E_1 = 0$ $E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{22} + M_{23})E + M_{20} > 0 \\ (M_{32} + M_{33})E + M_{30} > 0 \\ p_2q_2(M_{22} + M_{23}) + p_3q_3(M_{32} + M_{33}) < 0 \\ p_2q_2M_{20} + p_3q_3M_{30} - (c_2 + c_3)M > 0 \end{cases}$
Case 2 $E_1 = E_2 = 0$ $E_3 = E \neq 0$	$\begin{cases} M_{33}E + M_{30} > 0 \\ M_{33} < 0 \\ p_3q_3M_{30} - c_3M > 0 \end{cases}$
Case 3 $E_1 = E_3 = E \neq 0$ $E_2 = 0$	$\begin{cases} (M_{11} + M_{13})E + M_{10} > 0 \\ (M_{31} + M_{33})E + M_{30} > 0 \\ p_1q_1(M_{11} + M_{13}) + p_3q_3(M_{31} + M_{33}) < 0 \\ p_1q_1M_{10} + p_3q_3M_{30} - (c_1 + c_3)M > 0 \end{cases}$
Case 4 $E_1 = E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{11} + M_{12} + M_{13})E + M_{10} > 0 \\ (M_{21} + M_{22} + M_{23})E + M_{20} > 0 \\ (M_{31} + M_{32} + M_{33})E + M_{30} > 0 \\ p_1q_1(M_{11} + M_{12} + M_{13}) \\ +p_2q_2(M_{21} + M_{22} + M_{23}) \\ +p_3q_3(M_{31} + M_{32} + M_{33}) < 0 \\ p_1q_1M_{10} + p_2q_2M_{20} + p_3q_3M_{30} - (c_1 + c_2 + c_3)M > 0 \end{cases}$
Case 6 $E_1 = E_2 = E \neq 0$ $E_3 = 0$	$\begin{cases} (M_{11} + M_{12})E + M_{10} > 0 \\ (M_{21} + M_{22})E + M_{20} > 0 \\ p_1q_1(M_{11} + M_{12}) + p_2q_2(M_{21} + M_{22}) < 0 \\ p_1q_1M_{10} + p_2q_2M_{20} - (c_1 + c_2)M > 0 \end{cases}$

Table 6: Harvesting Scenario: Necessary Condition for MEY where M>0

Harvesting Scenario	Necessary Conditions for MEY, $M < 0$
Case 1 $E_1 = 0$ $E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{22} + M_{23})E + M_{20} < 0 \\ (M_{32} + M_{33})E + M_{30} < 0 \\ p_2q_2(M_{22} + M_{23}) + p_3q_3(M_{32} + M_{33}) > 0 \\ p_2q_2M_{20} + p_3q_3M_{30} - (c_2 + c_3)M < 0 \end{cases}$
Case 2 $E_1 = E_2 = 0$ $E_3 = E \neq 0$	$\begin{cases} M_{33}E + M_{30} < 0 \\ M_{33} > 0 \\ p_3q_3M_{30} - c_3M < 0 \end{cases}$
Case 3 $E_1 = E_3 = E \neq 0$ $E_2 = 0$	$\begin{cases} (M_{11} + M_{13})E + M_{10} < 0 \\ (M_{31} + M_{33})E + M_{30} < 0 \\ p_1q_1(M_{11} + M_{13}) + p_3q_3(M_{31} + M_{33}) > 0 \\ p_1q_1M_{10} + p_3q_3M_{30} - (c_1 + c_3)M < 0 \end{cases}$
Case 4 $E_1 = E_2 = E_3 = E \neq 0$	$\begin{cases} (M_{11} + M_{12} + M_{13})E + M_{10} < 0 \\ (M_{21} + M_{22} + M_{23})E + M_{20} < 0 \\ (M_{31} + M_{32} + M_{33})E + M_{30} < 0 \\ p_1q_1(M_{11} + M_{12} + M_{13}) \\ +p_2q_2(M_{21} + M_{22} + M_{23}) \\ +p_3q_3(M_{31} + M_{32} + M_{33}) > 0 \\ p_1q_1M_{10} + p_2q_2M_{20} + p_3q_3M_{30} - (c_1 + c_2 + c_3)M < 0 \end{cases}$
Case 6 $E_1 = E_2 = E \neq 0$ $E_3 = 0$	$\begin{cases} (M_{11} + M_{12})E + M_{10} < 0 \\ (M_{21} + M_{22})E + M_{20} < 0 \\ p_1q_1(M_{11} + M_{12}) + p_2q_2(M_{21} + M_{22}) > 0 \\ p_1q_1M_{10} + p_2q_2M_{20} - (c_1 + c_2)M < 0 \end{cases}$
Case 7 $E_1 = E_3 = 0$ $E_2 = E \neq 0$	$\begin{cases} M_{22}E + M_{20} < 0 \\ M_{22} > 0 \\ p_2q_2M_{20} - c_2M < 0 \end{cases}$
Case 8 $E_1 = E \neq 0$ $E_2 = E_3 = 0$	$\begin{cases} M_{11}E + M_{10} < 0 \\ M_{11} > 0 \\ p_1q_1M_{10} - c_1M < 0 \end{cases}$

Table 7: Harvesting Scenario: Necessary Condition for MEY where M<0

6 Numerical Simulations

In this section, numerical simulations were generated using MATLAB. These serve to illustrate and explore the theoretical results obtained in the previous two sections. We consider two cases: $M > 0$ and $M < 0$. The parameter values and initial values used in the simulations are recorded below:

Parameters	Values for $M > 0$	Values for $M < 0$
$\alpha_{12} = \alpha_{21}$	1.9	0.01
$\alpha_{13} = \alpha_{23}$	0.7	0.1
$\alpha_{31} = \alpha_{32}$	1.1	0.5
(λ_1, λ_2)	(8.5, 9.5)	(8.5, 9.5)
$K_1 = K_2$	10	10
E	1	1
$p_1 = p_2 = p_3$	3	3
$q_1 = q_2 = q_3$	1	1
$c_1 = c_2 = c_3$	1	1

For numerical comparison purposes, we have chosen equal values for the ecological parameters that measure growth (λ_i) of the prey species and their carrying capacities K_i; we also assume a toggle value of $E = 1$ or $E = 0$ when species are harvested. All economic parameters p_i, q_i, c_i are chosen to be equal, too. It is important to note, however, that we are assuming that the catchability coefficient for all three species are all equal to 1; results may be different if the catchability coefficients are not the same.

Furthermore, when applying or confirming the theoretical results, it is important to look at how parameters must be chosen in order to determine the sign of the determinant M of our three-species system. The choice of the sign of M highly depends on the ecological parameters that measure interspecific competition between the preys $(\alpha_{12}, \alpha_{21})$, the predation effect of the predator on the two prey species $(\alpha_{13}, \alpha_{23})$, and the predation effect of the predator on itself $(\alpha_{31}, \alpha_{32})$. In other words, when the interspecific competition between prey species and the predation are strong, the system will lean towards a positive value for M; otherwise, the sign of M will be negative.

Single-species harvesting

Although different species are being harvested at each harvesting scenario, let us compare the amount of harvesting effort E^* to reach maximum sustainable yield and their corresponding yield $Y(E^*) = MSY(E^*)$ amounts using numerical simulations. We will also look at the analogous point $(\bar{E}, MEY(\bar{E}))$ for the maximum economic yield. The results are organized as follows: single-species, double-species, and three-species harvesting for $M > 0$ and $M < 0$. Recall that there is only one case for single-species harvesting when $M > 0$:

Single-species $M > 0$	$(E^*, \text{MSY}(E^*))$	$\left(\bar{E}, \text{MEY}(\bar{E})\right)$
Case 2 $(E_1, E_2, E_3) = (0, 0, 1)$	(3.5424, 8.0983)	(3.2841, 20.8817)

There are three cases for single-species harvesting when $M < 0$:

Single-species $M < 0$	$(E^*, \text{MSY}(E^*))$	$\left(\bar{E}, \text{MEY}(\bar{E})\right)$
Case 2 $(E_1, E_2, E_3) = (0, 0, 1)$	$(4.9449, 22.0242)$	$(4.7598, 61.2201)$
Case 7 $(E_1, E_2, E_3) = (0, 1, 0)$	$(4.4667, 20.0312)$	$(4.3007, 55.7100)$
Case 8 $(E_1, E_2, E_3) = (1, 0, 0)$	$(3.9650, 17.5382)$	$(3.8156, 48.7242)$

Refer to Figures 2 and 3 for an illustration of the numbers in this table with $M < 0$. Figure 2 shows the different yield functions when analyzing MSY for Cases 2, 7, and 8. Figure 3 shows the different yield functions when analyzing MEY for Cases 2, 7, and 8.

Observe that in all single-harvesting cases for both signs of M, we have $\bar{E} < E^*$; this means that the economic profitability goal of achieving MEY requires (slightly) less effort than the environmental sustainability goal of achieving MSY. Moreover, the yields for MEY are at least two times higher when harvesting at MEY level. In the case that $M < 0$, the highest economic yield and the highest sustainability yield occur in single-species harvesting when only the predator is harvested. Thus, theoretical analyses and numerical simulations suggest that when harvesting one species only, an approach that is predator-oriented with an economic profitability goal is recommended due to the high yield amount that will come from a slightly higher effort than harvesting one of the prey species only. Comparing this predator-oriented harvesting approach when harvesting single-species only with the results in ([5]), we see that harvesting recommendations clearly depend on the dynamic interactions between species in a system.

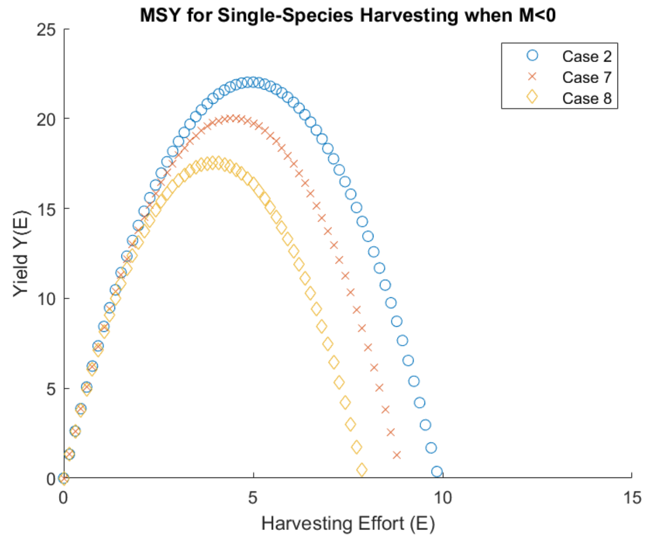

Figure 2: MSY for single-species harvesting in a system with weak interspecific competition and predation

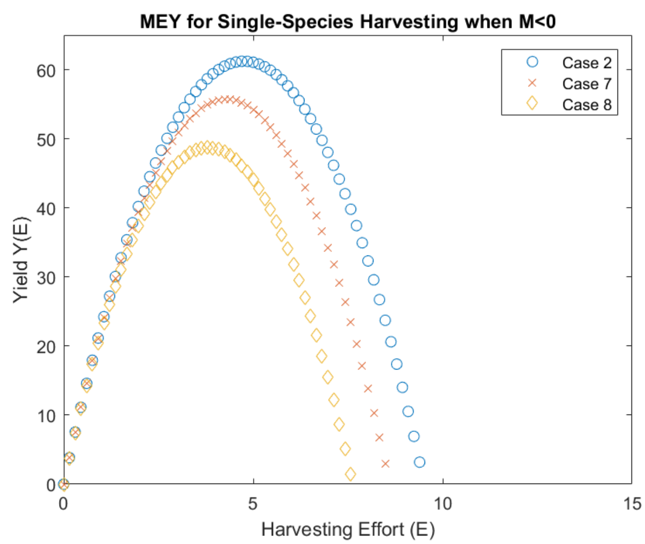

Figure 3: MEY for single-species harvesting in a system with weak interspecific competition and predation

Double-species harvesting

Let us now look at the MSY and MEY results for both $M > 0$ and $M < 0$ in case two species are being harvested. First, we observe the results numerically in the provided tables. We then provide graphical representations as can be seen in Figures 4 and 5 for $M > 0$ and Figures 6 and 7 for $M < 0$. As noted previously, double-species harvesting analysis are Cases 1, 3, and 6.

Double-species $M > 0$	$(E^*, \text{MSY}(E^*))$	$\left(\bar{E}, \text{MEY}(\bar{E})\right)$
Case 1 $(E_1, E_2, E_3) = (0, 1, 1)$	$(8.4743, 26.5010)$	$(7.5710, 63.4577)$
Case 3 $(E_1, E_2, E_3) = (1, 0, 1)$	$(10.4615, 36.8593)$	$(9.4718, 90.6446)$
Case 6 $(E_1, E_2, E_3) = (1, 1, 0)$	$(4.5125, 9.3783)$	$(3.7888, 19.8337)$

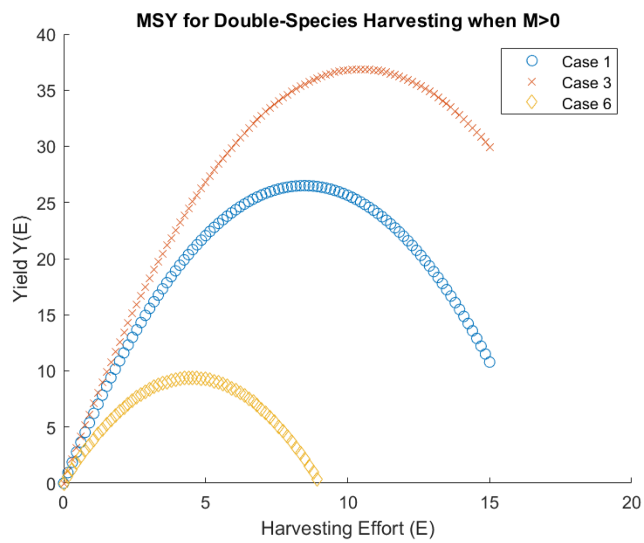

Figure 4: MSY for double-species harvesting in a system with strong interspecific competition and predation

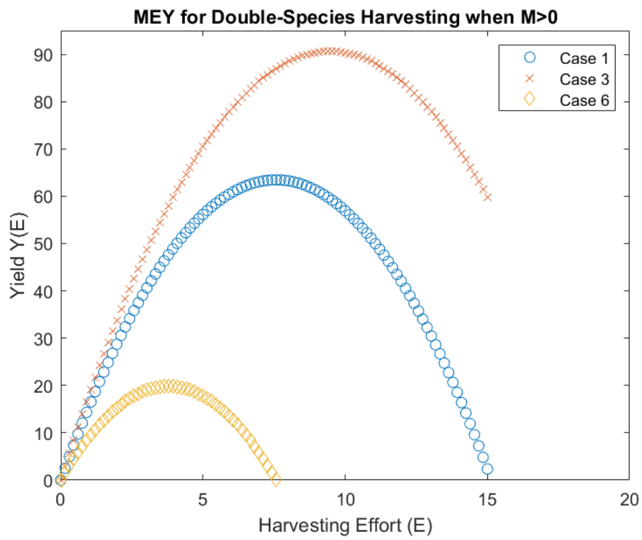

Figure 5: MEY for double-species harvesting in a system with strong interspecific competition and predation

Like in the single-species case, we see that the harvesting efforts E^* and $\bar{E}$ guarantee that the MEY is achieved before the MSY, that is, $\bar{E} < E^*$. Moreover, the economic yield is at least two times higher than the sustainability yield. Finally, when looking at both harvesting efforts E^* and $\bar{E}$, we see that about half the effort is required to harvest the two prey species (Case 6) than harvesting the predator and one of the prey species (Cases 1 or 3).

Double-species $M < 0$	$(E^*, \text{MSY}(E^*))$	$\left(\bar{E}, \text{MEY}(\bar{E})\right)$
Case 1 $(E_1, E_2, E_3) = (0, 1, 1)$	$(3.9212, 35.0494)$	$(3.7749, 97.4522)$
Case 3 $(E_1, E_2, E_3) = (1, 0, 1)$	$(3.6445, 32.3533)$	$(3.5077, 89.9076)$
Case 6 $(E_1, E_2, E_3) = (1, 1, 0)$	$(4.4860, 39.9602)$	$(4.3181, 111.0766)$

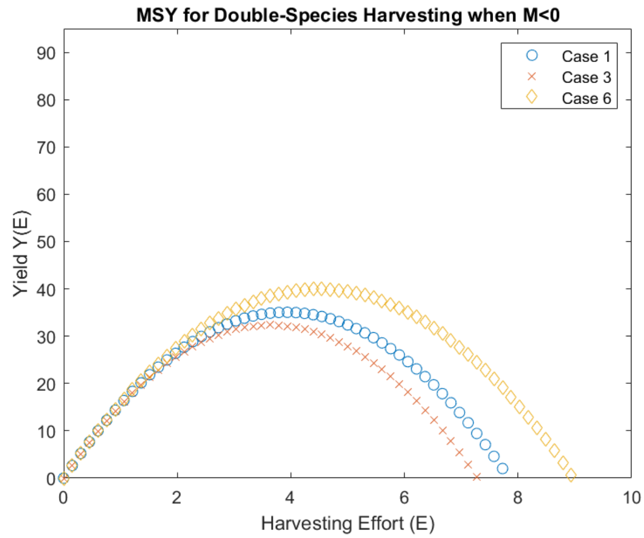

Figure 6: MSY for double-species harvesting in a system with weak interspecific competition and predation

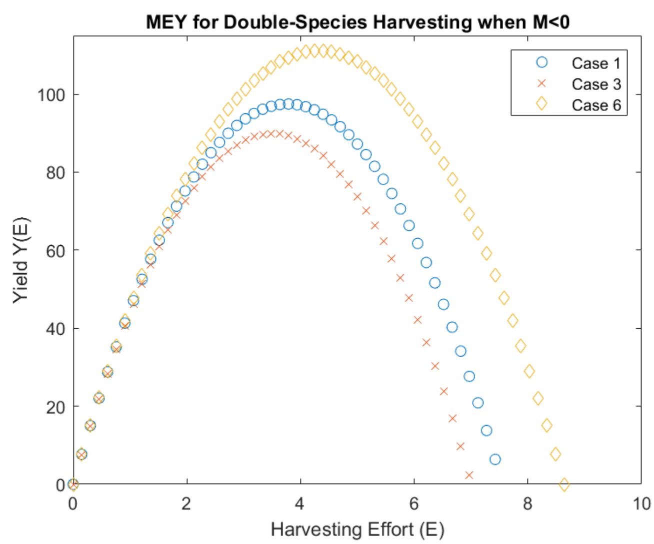

Figure 7: MEY for double-species harvesting in a system with weak interspecific competition and predation

The results above for double-species harvesting in case $M < 0$ further confirms the observation that $\bar{E} < E^*$ and that the yield amounts satisfy MEY($\bar{E}$) > MSY(E^*).

Unlike the case $M > 0$, we see that the highest effort is needed to harvest both prey species (Case 6) than harvesting the predator and one of the prey species; this is because of the choice of parameters between systems having positive M and negative M. However, this effort 4.3181 for MEY and 4.4860 for MSY are not much higher than the efforts to harvest at least one of the prey species. Since the MEY when harvesting the two preys (Case 6) yields the highest yield amount at an effort that is roughly the same as the other yields, the numerical simulations suggest that a prey-oriented harvesting approach is recommended when harvesting double-species in a system with weak inter-specific competition between prey species and weak predation effects.

Combined species harvesting

All-species $M < 0$	$(E^*, \mathrm{MSY}(E^*))$	$\left(\bar{E}, \mathrm{MEY}(\bar{E})\right)$
Case 4 $(E_1, E_2, E_3) = (1, 1, 1)$	$(3.6302, 48.5060)$	$(3.4944, 134.8312)$
All-species $M > 0$	**$(E^*, \mathrm{MSY}(E^*))$**	**$\left(\bar{E}, \mathrm{MEY}(\bar{E})\right)$**
Case 4 $(E_1, E_2, E_3) = (1, 1, 1)$	$(3.3829, 14.7642)$	$(2.9953, 34.7255)$

Numerical simulation results when harvesting all three species confirm that $\bar{E} < E^*$. Moreover, whether M is negative or positive, yield values for MEY are more than two times higher than the yield values for MSY.

7 Conclusion and Recommendations

This paper considers a three-species fishery system, in which there are two prey species in competition, along with a predator that preys upon both prey species. We computed the coexistence equilibrium, expressing each component in terms of a common harvesting effort E. Then we considered the problem of maximizing the yield function and the profit function in terms of E.

For completeness, necessary conditions for the maximum sustainable yield and the maximum economic yield under all eight harvesting scenarios are investigated. Numerical simulations were conducted to confirm theoretical results. Due to the many possible ecological interactions in the given three-species system, we considered two cases in this project which ultimately can be measured by the sign of M in (4).

The authors are aware that numerical simulations in this paper are experimental and do not necessarily reflect actual and current data. We recommend that investigators from the fishing management communities apply our results when the economic parameters p_i, c_i, and q_i and the ecological parameters (growth, competition, predation) are known.

In all eight harvesting scenarios and whether $M < 0$ or $M > 0$, we see that $\bar{E}$ (the harvesting effort required to realize MEY) is always less than the corresponding E^* (the harvesting effort required to realize MSY). Ultimately, this paper has theoretically validated the desirable result that the harvesting effort required to achieve MEY is less than the harvesting effort required to achieve MSY. In other words, increasing the effort beyond $\bar{E}$ may not provide more economic benefits and may even cause sustainability problems.

Whether M is positive or negative, results show that for single-species harvesting, a predator-oriented harvesting approach is recommended as it will provide the largest yield at the MEY level. For double-species harvesting in a system with $M < 0$ (weak interspecific competition and weak predation), a prey-oriented harvesting approach is recommended. For double-species harvesting in a system with $M > 0$ (strong interspecific competition and strong predation), a predator-oriented harvesting approach is recommended.

This paper considers a careful MSY and MEY investigations in all eight harvesting scenarios in a multi-species system, in particular, when there are two competing prey species and one predator. Results show that ecological interactions between the species may give rise to different harvesting recommendations.

Finally, a possible direct extension of this project is on investigating MSY and MEY in the case that the harvesting efforts E_1, E_2, E_3 are not the same. In this case, the yield function becomes a function of three variables E_1, E_2, E_3; ecologically, such an assumption may be necessary when investigating a system where one of the species is invasive and hence, the harvesting effort on such species is required to be bigger than the harvesting effort for the native species.

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