SOCIETY for INDUSTRIAL and APPLIED MATHEMATICS

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Applied Mathematics Research in Space National Security
Submitted by the Society for Industrial and Applied Mathematics

The Society for Industrial and Applied Mathematics (SIAM) offers the following white paper on priority areas of applied mathematics and computational science research to enable defense technologies that meet the needs of the new United States Space Force (USSF). SIAM has over 14,000 members, including applied mathematicians, computational scientists, numerical analysts, data scientists, etc. They work in industrial and service organizations, academia, and government agencies and laboratories all over the world. SIAM stands ready to serve as a resource to USSF and the Air Force Office of Scientific Research as it continues to define a basic research agenda for space national security.

Historically, space-oriented research activities have focused on applied efforts, but the creation of USSF offers an opportunity to consider the full range of research and development needed to meet mission needs. Basic research will be a critical component of this portfolio to ensure a dynamic ecosystem of new technologies and capabilities that will enable USSF to stay at the cutting edge, keeping the U.S. ahead of our adversaries. Advances in applied mathematics and computational science are critical to address USSF challenges including for design and control of space systems, imaging and understanding the space environment, and solving complex challenges such as space debris. Developing and testing of hypersonics, autonomous systems, advanced image analysis, human-machine teams, automated sensor tasking, data fusion, and missile defense systems is expensive and arduous, and all require superior mathematical capabilities to design and reduce technological and financial risk. Decision makers, planners, and analysts also benefit from machine intelligence enabled by fundamental research in applied mathematics that increases confidence and understanding in the results, and improves response time and accuracy in models and assessments. Applied mathematics and computational science provide the foundation for the U.S. to maintain its technological edge in space by providing the basic underpinning of many technologies that enhance our capabilities.

Design, Control, Optimization, and Guidance
Space technologies are difficult to design and test therefore modeling, simulation, optimization, uncertainty quantification, and control theory are all critical aspects to successfully deploy new technologies. Currently, physical testing for space technologies, including hypersonics, is limited and expensive. Due to the high speeds involved, wind tunnels are constrained in their ability to test hypersonics, whereas flight testing requires specialized programs and hardware. For example, the X-51A program gathered approximately 90 seconds of scramjet propulsion flight test data at a cost in the billions of dollars. Also, the microgravity space environment cannot be produced on Earth, nor can frictionless orbital mechanics be reproduced, so new space technologies must be transported into orbit for testing which is extremely costly and limits the amount of data gathered and types of testing that can be conducted. Applied mathematics can help reduce these costs for development and testing of prototypes by enabling computational modeling and simulation to effectively test new technologies.

Simulations allow for low-cost testing prior to hardware development in many areas, including for additive manufacturing, new materials, radiation hardening, among other areas. Design of Experiments (DoE) techniques can help reduce the number of tests required for complex systems with large, high-dimensional parameter spaces to gain thorough assessment of system performance. Simulation also provides a relatively inexpensive test bed in which software upgrades can be vetted before integration into tactical systems; this supports algorithm development for, e.g., detection/classification, and control

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software for autonomous systems. To effectively realize these cost savings, and enable optimal designs, USSF needs to invest in fundamental research for applied mathematics and computational science in order to be able to apply them to their technology development programs. For example, improved multi-scale, multi-physics models would allow for design of transformational technologies such as hypersonics and better understanding of how these technologies interact in complex environments. To achieve optimal design requires the introduction of high-fidelity, computationally tractable, physics-based models that include better understanding of their uncertainty. Further, in order to develop multidisciplinary design optimization for hypersonic flight vehicles and other complex technologies, DOD needs to invest in fundamental research to enable the development of accurate models for entire vehicle characteristics, its subsystems and components, and their interactions and associated uncertainties to develop multidisciplinary design optimization for hypersonic flight vehicles and other complex technologies. It is critical that DOD invest in developing advanced modeling and simulations capabilities to enable low-cost efficient development timelines that will create a competitive advantage for the United States in these transformational areas.

Hypersonic missiles have the opportunity to revolutionize modern day warfare and the US development is noticeably behind our adversaries. The ability to predict critical aerodynamic and thermal characteristics for maneuverable hypersonic vehicles is a central challenge for design and control of hypersonic systems. To develop practical guidance and control laws one must design an extremely robust and tightly constrained control system models. This process requires the development of computational methods that produces faster and more accurate high-fidelity simulations as well as reduced order models for real time control. In addition, hypersonic vehicles are surrounded by shock waves and possibly plasma, which complicate the functioning of sensors and communication systems. Hypersonics also travel at such high speeds that physical testing is limited and not scalable, which means that simulation and modeling are required. USSF needs to be investing in this early-stage model development to enable the efficient and cost-effective development of hypersonics. Successful defense against hypersonic weapons will require new algorithms for sensing, detection, and networking for secure, robust, and fast communication amongst distributed sensors. Rapid, automated, physics-informed, data analysis will be required to fuse information from different sensing systems as well as from many locations to aid decision making.

Space Domain Awareness

Space activity is increasing, creating a more congested environment, and intensifying the need for accurate and timely tracking and predicting of future locations for a growing number (currently greated than 27,000) of space objects. This national security need is elevated by the recent Anti-satellite Weapon Test (ASAT), which both created a debris field increasing the number of items needing to be tracked and highlighted the potential for a hostile attack on our in-space infrastructure. As space and low Earth orbit become increasingly crowded, novel algorithms are needed to image and track fast moving and rotating objects.

In the event of an attack or potential collision, evasive maneuvering and self-defense tactics are needed. A first step in developing countermeasures is improving models for orbital maneuvers, optimal control algorithms, and software to characterize likely threats to space assets and models to analyze potential defensive maneuvers. Given recent advances in modeling, computational control and optimization methods, this area is ripe for investment. Applied mathematics can support optimization of tracking and controlling platforms in real-time to understand, mitigate and respond to any potential threats.

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Mathematical efforts are underway to use ground-based telescopes and radar to form images of satellites. This involves dealing with electromagnetic wave propagation through a turbulent atmosphere and includes techniques like adaptive optics and some newer passive radar sensing techniques. This affects situational awareness, which provides critical information for decision-making and tactics. It is also an important component of defense against missiles and other threats. Additionally, there are efforts to use receivers, either ground-based or spaceborne, to determine the location and emissions of other satellites that are emitting electromagnetic energy. Techniques of algebraic geometry, including numerical algebraic geometry software such as Bertini, may provide solutions. This has implications for both situational awareness (threat detection) and communications. The aggressive speed and maneuvering capabilities of space-based weapons require sensor networks with robust, high-speed communication capability. Furthermore, mathematical data analysis and computational science are essential components of the ability to protect our assets from collisions with orbiting debris and other objects through early detection of potential threats.

Data, Artificial Intelligence, and Decision-Making

Space-based technologies, imaging analysis, and other critical space-related tools increasingly rely on artificial intelligence (AI) to enable autonomy, better decision making, and deeper analytical capabilities. Advances in applied mathematics and computational science continue to aid improvements in artificial intelligence, machine learning, and data science; more improvements are needed to enhance the ability of these technologies to better address complex challenges and ensure reliability, usability, transparency, and explainability. Machine learning and AI applications include situational awareness, (i.e., sensing of the environment), performance prediction, autonomous vehicle navigation, data analysis, and information fusion. Research funding should enable partnerships to apply AI and machine learning research to USSF application needs.