Newsjournal of the Society for Industrial and Applied Mathematics sinews.siam.org

siam news

Volume 57/ Issue 2
March 2024

The Turbulent Tale of Icy Clouds

By Matthew R. Francis

Clouds are one of the most important influencers of climate on Earth as well as other worlds that have atmospheres. They comprise a major component of the hydrologic cycle, bounce light back into outer space, and trap radiation that is emitted or reflected from the ground. On Earth, clouds are made of water in the form of droplets and ice crystals; these components sometimes exist separately and sometimes form a mixture, depending on the elevation and atmospheric conditions.

Despite the importance of clouds and humanity's millennia-long interest in the sky, accurate mathematical descriptions of clouds remain elusive due to their internal complexity and complicated interactions with the atmosphere. Ice presents a particular challenge, as the shape and size of the grains raise difficulties in theory as well as experiment.

During the Ed Lorenz Lecture[1] at the 2023 Fall Meeting[2] of the American Geophysical Union (AGU)—which took place in San Francisco, Calif., in December 2023—physicist Alain Pumir of École Normale Supérieure de Lyon described new theoretical and experimental approaches that explain the self-organization of ice and other particles under turbulence. "All of the processes [within clouds] that you can think of are affected by turbulence," Pumir said, noting that cloud modeling requires an understanding of collective atmospheric phenomena. "When you have many droplets, you care about collective effects. That's also the case for little ice crystals."

The way in which light coherently reflects and refracts from atmospheric ice—e.g., as sun dogs, light pillars (see Figure 1), lunar haloes, and other beautiful phenomena—indicates that the microscopic crystals align with each other under certain conditions. Like snowflakes, these ice crystals exhibit hexagonal symmetry down to the micron scale, while the clouds that contain them can stretch for many kilometers both horizontally and vertically. Modeling efforts must therefore treat the relevant physical properties while identifying any superfluous aspects. "The worst greenhouse gas is water [vapor]," Pumir said, adding that the largest uncertainties in climate change models come from clouds, which contribute to reflection and radiation. "How [ice crystals] align or don't align has consequences in terms of reflection, either on Earth or from above."

And it's not just ice. In fact, many of Pumir's collaborators model nonspherical atmospheric particles like volcanic ash or microplastics, which also play a role in cloud formation when water or ice collects around them as nuclei. Some scientists have even suggested seeding clouds with small particles to cool the atmosphere; in this context, Pumir's research could lend essential insight as to whether this type of geoengineering project is even feasible.

Dropping Coins, Scientifically

As so often happens in science, Pumir's first attempts to solve the problem were unsuccessful. "We did it completely wrong," he cheerfully admitted, adding

See **Icy Clouds** *on page 2*

1 https://www.agu.org/honors/lorenz
2 https://www.agu.org/fall-meeting

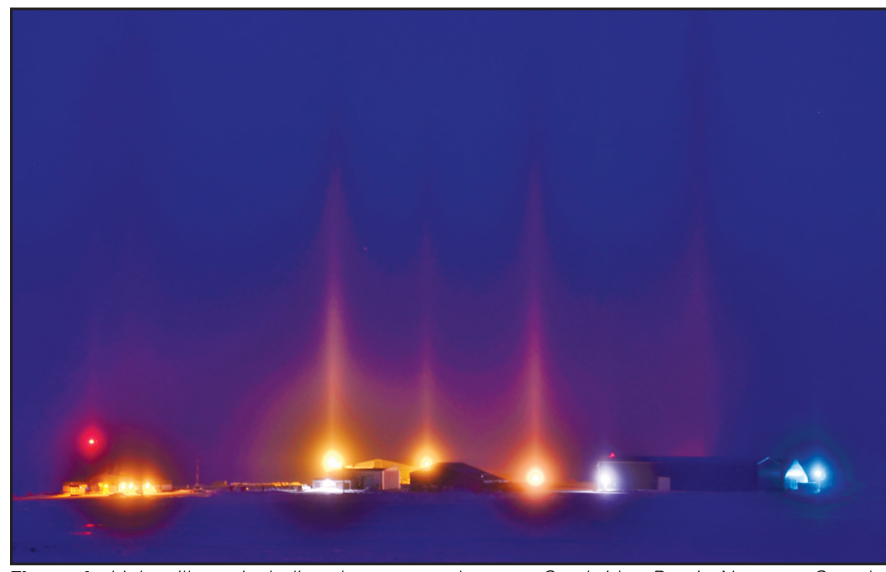

Figure 1. *Light pillars—including these examples over Cambridge Bay in Nunavut, Canada—form when light shines through atmospheric ice crystals. The crystals align as they fall through turbulent air, which allows for coherent light scattering. Figure courtesy of Eric Van Lochem and Flickr under the Attribution-ShareAlike (CC BY-SA 2.0) license.*

Fusing Artificial Intelligence and Optimization with Trustworthy Optimization Proxies

By Pascal Van Hentenryck

Recent years have seen significant interest in the fusion of machine learning (ML) and optimization for a variety of engineering applications [2, 11]. Optimization technologies are widely successful in industry; they help to dispatch power grids, route transportation and logistics systems, plan and operate supply chains, and schedule manufacturing systems. However, these technologies still face computational challenges in certain applications. For instance, real-time constraints may prevent the production of solutions, or planners and operators in the loop might require fast interactions with the underlying decision support systems.

Fortunately, engineering applications typically operate on physical infrastructures that change relatively slowly. As a result, optimization technologies must repeatedly solve the same core optimization problem on instances that are somewhat similar. These considerations have inspired the idea that ML could learn such *parametric optimization problems* and replace optimization altogether. Consider the parametric optimization problem

$$\min_{\mathbf{y}} f_{\mathbf{x}}(\mathbf{y}) \text{ subject to} \qquad (1)$$
$$\mathbf{h}_{\mathbf{x}}(\mathbf{y}) = 0 \text{ and } \mathbf{g}_{\mathbf{x}}(\mathbf{y}) \geq 0,$$

where $\mathbf{x}$ represents instance parameters that determine the objective function $f_{\mathbf{x}}$ and the constraints $\mathbf{h}_{\mathbf{x}}$ and $\mathbf{g}_{\mathbf{x}}$. We may view this optimization as mapping from an input $\mathbf{x}$ to an output $\mathbf{y}$ that represents its optimal solution (or a selected optimal solution). In response, we can train a ML model—such as a deep neural network—to approximate this mapping via *empirical risk minimization under constraints*. Unfortunately, such an approximation is typically unsatisfactory for engineering tasks; because they are regressions, the ML predictions are unlikely to satisfy the problem constraints and may have significant consequences when optimization models assist with the operation of physical infrastructures.

Optimization Proxies

Optimization proxies seek to overcome this difficulty by combining a predictive component (typically a deep neural network) that produces an approximation $\tilde{\mathbf{p}}$ with a *repair layer* that transforms $\tilde{\mathbf{p}}$ into a feasible solution $\bar{\mathbf{p}}$ (see Figure 1). In a first approximation, the repair layer acts as a projection of $\tilde{\mathbf{p}}$ into the feasible space of the optimization problem. In practice, however, it is often preferable to design dedicated repair layers that ensure fast training and inference times.

Optimization proxies have the potential to transform various classes of applications through orders-of-magnitude improvements in efficiency. For example, consider the real-time risk assessment framework in Figure 2 (on page 4), which runs a collection of Monte Carlo scenarios. Each scenario necessitates 288 optimizations, which equates to one every five minutes over a 24-hour period. This process takes about 15 minutes. But when we replace each optimization with its proxy, we can evaluate

See **Optimization Proxies** *on page 4*

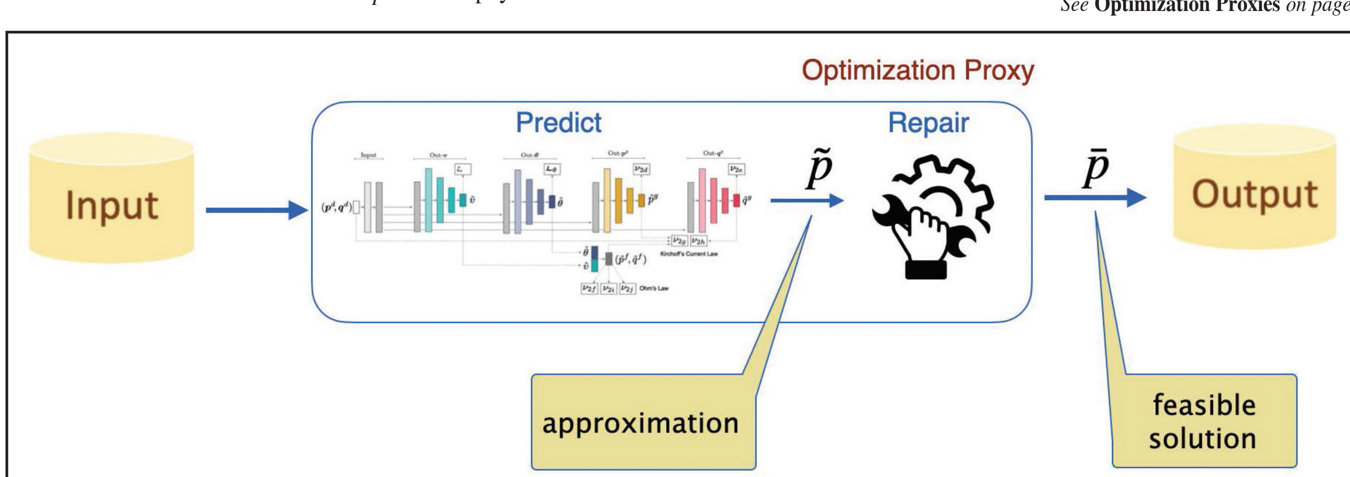

Figure 1. *A high-level outline of the architecture of optimization proxies. Figure courtesy of the author.*

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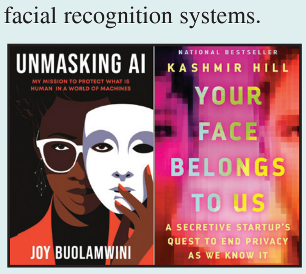
Volume 57/ Issue 2/ March 2024



Icy Clouds

Continued from page 1

that other cloud formation experts were happy to explain the shortcomings of his approach. Armed with new ideas and a broader web of collaborators, Pumir examined previous progress in related areas, such as the settling of volcanic ash. Water droplets and particles—whether liquid water, ice, or another form—fall under gravity but are simultaneously buoyed by air. Ice in particular forms hexagonal crystals that are wider than they are thick, which means that a physics-based treatment must consider orientation as well as motion in three dimensions.

In the absence of air resistance, a nonrotating object simply falls without tumbling. But a fall through any fluid dramatically changes the situation, spinning the crystal and ultimately yielding two preferred orientations: edge down or face down. The latter configuration is more stable, analogous to a dropped coin in a swimming pool. However, Pumir pointed out the immediate breakdown of this analogy: the density of liquid water is close to that of ice crystals, whereas ice is significantly denser than air $(\rho_{\text{ice}}/\rho_{\text{air}} \sim 1000)$.

To mitigate these problems, Pumir's collaborators—led by Gholamhossein Bagheri at the Max Planck Institute for Dynamics and Self-Organization—constructed a device that they call the *Göttingen turret*. This apparatus injects thin, 3D-printed plastic disks into a chamber of air in a highly controlled manner. The team tracked the disks' settling process with two pairs of state-of-the-art, high-speed cameras. These "million-dollar babies," as Pumir put it, operate at 2,932 frames per second and were situated at the top and bottom of the chamber — locations that allowed them to reconstruct the disks' tumbling motions (see Figure 2).

Larger-scale simulations with multiple particles that fall in tandem are currently beyond the capability of the simple Göttingen turret. However, Bagheri's group has used hexagons and modified snowflake-like shapes to perform follow-up experiments[3] for better comparison with real-world ice crystals.

Turbulence Brings Us Together

Meanwhile, a full theoretical treatment of icy clouds requires an acknowledgment of the translational and rotational inertia of crystals alongside atmospheric fluid dynamics — i.e., how the crystals fall and tumble through turbulent air. At the same time, we intuitively (and mathematically) know that too much turbulence prevents the crystals from aligning and hence disrupts coherent light scattering. As such, the goal is to find the appropriate balance.

"The temptation was to say, 'It's a small particle [and] the Reynolds number (Re) is small,'" Pumir said, referring to the physical parameter that measures a fluid's smoothness and viscosity. In nonturbulent laminar flow, Re is much smaller than 1, while turbulence dominates a fluid at $\text{Re} \sim 1000$. Pumir and his colleagues focused on a middle regime—$\text{Re} \sim 10$—where turbulence is present but not dominant. "The Re is small but not *that* small, and that makes a world of difference," he said.

[3] https://www.ds.mpg.de/3865403/Bagheri_Mohsen

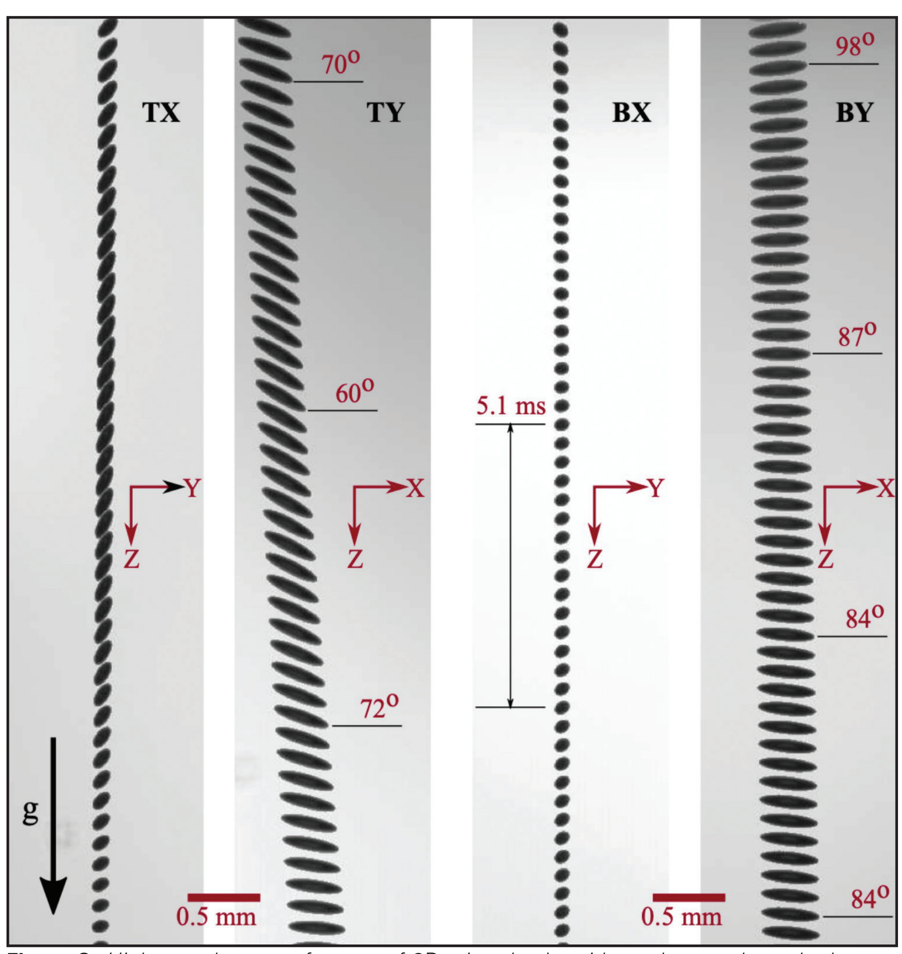


Figure 2. *High-speed camera footage of 3D-printed spheroids as they settle to the bottom of an air chamber. The two left panels depict the tumbling motion at the beginning of the fall from different viewpoints, and the two right panels show the spheroids at the end of the fall. Figure courtesy of [1].*

Accounting for all of these factors, the theorists built the following model based on Newtonian physics:

$$m\frac{d\mathbf{v}}{dt} = m\mathbf{g} + \mathbf{F}_{\text{h}}$$

$$\frac{d\mathbf{n}}{dt} = \boldsymbol{\omega} \times \mathbf{n}$$

$$\frac{d}{dt}(\mathbb{J}(\mathbf{n}) \cdot \boldsymbol{\omega}) = \mathbf{T}_{\text{h}}.$$

Here, $\mathbf{v}$ is the particle velocity, $\mathbf{n}$ is the vector that is normal to the flat face of the crystal, $\boldsymbol{\omega}$ is the angular velocity, and $\mathbf{g}$ is the gravity vector. $\mathbb{J}$ is the inertia tensor for a flat ellipsoid, which matches the experimental configuration and simplifies the math. The researchers treated the hydrodynamic force $\mathbf{F}_{\text{h}}$ and torque $\mathbf{T}_{\text{h}}$ perturbatively:

$$\mathbf{F}_{\text{h}} = \mathbf{F}_{\text{h}}^{(0)} + C_F \mathbf{F}_{\text{h}}^{(1)}$$

$$\mathbf{T}_{\text{h}} = \mathbf{T}_{\text{h}}^{(0)} + C_T \mathbf{T}_{\text{h}}^{(1)},$$

where the zero-order terms represent objects that are falling with air resistance but with a negligible Re. The correction terms with coefficients $\{C_F, C_T\}$ are a combination of empirical and theoretical analyses that account for small but finite turbulence [1].

In a vacuum, a coin-like shape will spin freely if it has an initial rotation. The addition of fluid resistance results in two energetically optimal orientations: unstable equilibrium when the ellipsoid is edge-down ($\mathbf{n}$ is perpendicular to $\mathbf{g}$), and stable equilibrium when the flat side is facing down. This theoretical result agrees with the Göttingen turret experiment and explains why ice crystals align to produce light pillars — at least, when turbulence does not dominate the system.

Clouds From Both Sides Now

Ed Lorenz—for whom the AGU lecture is named—is best known for work that proved that even simple atmospheric models lead to unpredictable outcomes, thus demonstrating that large-scale weather control is likely impossible. Lorenz and like-minded researchers helped to revolutionize modern interest in chaos and nonlinear phenomena — including turbulence, which is intrinsically a multiscale phenomenon. Turbulence affects the overall shape of clouds, all the way down to the microscopic length scales where individual droplets and ice crystals exist. "You can't just look at one part," Pumir said. "You have to do the whole problem."

Pumir noted that the system becomes a competition between turbulence and normal settling. Stronger turbulence makes particles tumble, but theoretical analysis revealed that a moderate amount of turbulence brings crystals together — possibly facilitating larger aggregations called graupels and sometimes leading to precipitation. However, the model found that the presence of larger particles, lower turbulence, or a combination of both typically causes the coin-like object to settle with its broad face downward.

As demonstrated by experiment and theory, understanding clouds requires complicated physics: the inertia of ice particles, turbulence in the air, and collisions between crystals. The results may be messy, but as with light pillars, sometimes disorder is necessary to produce beauty.

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[1] Bhowmick, T., Seesing, J., Gustavsson, K., Guettler, J., Wang, Y., Pumir, A., … Bagheri, G. (2024). Inertia induces strong orientation fluctuations of nonspherical atmospheric particles. *Phys. Rev. Lett.*, *132*(3), 034101.

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Five Key Concepts That Shaped Iterative Solution Methods for Linear Systems

By Yousef Saad

The advent of electronic computers in the mid-20th century played a pivotal role in defining a new era for numerical linear algebra. George Forsythe's remarkable 1953 article—enigmatically titled "Solving Linear Algebraic Equations Can Be Interesting"—serves as a testament to these origins [2]; his writing displays amazing vision and addresses key topics in the nascent field, several of which would become foundational. The article introduces the concept of the condition number, considers the impact of finite precision arithmetic, and contemplates the idea of acceleration. It also discusses iterative methods, including the conjugate gradient (CG) algorithm — the "newest process on the roster" that had been discovered a few years earlier [5]. However, hints of tension are evident in the early parts of the paper. Forsythe seems somewhat apologetic about the topic, warning that "The subject of this talk is mathematically a lowly one." A footnote also informs readers that the original title was "Solving Linear Equations Is Not Trivial." It is important to recognize that during the 1950s, numerical linear algebra (NLA) was just beginning to establish its presence and had not yet gained widespread acceptance as a legitimate field.

We can trace the evolution of iterative methods from Carl Friedrich Gauss' era to the present day by examining a few pivotal concepts. Five such "big ideas" constitute the foundational elements of these methods.

First Big Idea: Relaxation

In the early 19th century, Gauss and Adrien-Marie Legendre each invented the method of least squares (LS) within a few years of each other, and both noted that an *ordinary elimination*—i.e., a direct method—can yield the solution of the resulting system [4]. In an 1823 letter to his former doctoral student Christian Ludwig Gerling, Gauss introduced a method of *indirect elimination* to solve these systems. We write the system in question as

$$Ax=b \quad \text{or} \quad a_i^T x=\beta_i \quad \text{for} \quad i=1,2,\cdots,n, \tag{1}$$

where a_i^T is the ith row of A and $b=[\beta_1,\beta_2,\cdots,\beta_n]^T$. The residual vector is $r=b-Ax,=[\rho_1,\rho_2,\cdots,\rho_n]^T$. Gauss started with an initial approximation to the solution and then updated its components one by one to zero out the largest residual component at each time. If $|\rho_i|=\max_{j=1:n}|\rho_j|$, he would thus modify the ith component of the current x to $x_i^{(\text{new})}:=x_i+\delta_i$ so that $\rho_i^{(\text{new})}=0$. The condition $a_i^T(x+\delta e_i)-\beta_i=0$ yields $\delta_i=\frac{\rho_i}{a_{ii}}$. This process repeats until every ρ_i is sufficiently small.

In addition to highlighting its simplicity, Gauss emphasized another significant advantage of the method: users could calculate updates with only a few digits of accuracy, which was particularly appealing since calculations were done by hand at the time. Clearly excited by this new method, Gauss concluded his letter by saying, "I recommend this method to you for imitation. You will hardly ever again eliminate directly, at least not when you have more than two unknowns."

Extensions of this basic approach or techniques that were similar in spirit dominated the iterative method scene for decades, and the dawn of electronic computers in the 1950s brought about a renewed interest in relaxation methods. These methods—aided by contributions from giants like David Young, Stanley Frankel, and Richard Varga—took center stage until approximately the 1970s.

Second Big Idea: Projection

Projection processes allow us to extract an approximate solution to a problem from a subspace. When we apply these processes to linear systems of equations, we assume the knowledge of some initial guess x_0 to the solution and two subspaces K and L (both of dimension m). We use these assumptions to formulate the following projected problem:

$$\begin{aligned} &\text{Find} \quad \tilde{x}=x_0+\delta,\ \delta\in K \\ &\text{such that} \quad b-A\tilde{x}\perp L. \end{aligned} \tag{2}$$

With m degrees of freedom (K) and m constraints (L), (2) results in an $m\times m$ linear system that, under mild conditions, is nonsingular. We can translate it into matrix form by exploiting bases $V=[v_1,\ldots,v_m]$ for K and $W=[w_1,\ldots,w_m]$ for L. The approximate solution then becomes $\tilde{x}=x_0+\delta\equiv x_0+Vy$, where $y\in\mathbb{R}^m$. The orthogonality constraints yield

$$W^T(r_0-AVy)=0\rightarrow\tilde{x}= x_0+V[W^TAV]^{-1}W^Tr_0.$$

This projection process has important *optimality properties* in two particular cases.

Orthogonal Projection (OP) Methods: When $K=L$ and matrix A is symmetric positive definite (SPD), we can show that $\tilde{x}$ minimizes $(A(x-x_*),(x-x_*))\equiv\|x-x_*\|_A^2$ over all vectors x in the affine space x_0+K, where x_* is the exact solution.

A representative of the OP class of methods is the well-known *steepest descent algorithm* for SPD matrices. This algorithm corresponds to the application of a projection step with one-dimensional subspaces $L=K=\text{Span}\{r\}$. We can describe the iteration as $\tilde{x}=x+\alpha r$, where $\alpha:=(r,r)/(Ar,r)$.

Minimal Residual (MR) Methods: When $L=AK$, $\tilde{x}$ minimizes the Euclidean norm of the residual over the affine space x_0+K. A representative of the MR class is the *minimal residual iteration*, which corresponds to taking $K=\text{Span}\{r\}$ and $L=AK$. The iteration thus becomes $\tilde{x}=x+\alpha r$, where $\alpha:=(r,Ar)/(Ar,Ar)$. It does not break down if A is nonsingular and will converge if $A+A^T$ is positive definite.

Third Big Idea: Polynomial Acceleration

Consider an iterative scheme that takes the form $x_{k+1}=x_k+\alpha_k r_k$. Steepest descent and the MR iteration are both of this form, where α_k is calculated in a greedy, shortsighted way. Polynomial iteration aims to calculate the α_is by taking a more global view. From $r_{k+1}=b-A(x_k+\alpha_k r_k)$, we obtain $r_{k+1}=r_k-\alpha_k Ar_k=(I-\alpha_k A)r_k$. This leads to the relation

See **Linear Systems** *on page 7*

The Operator is the Model

By Igor Mezić

Modeling of physical processes is the art of creating mathematical expressions that have utility for prediction and control. Historically, such models—like Isaac Newton's dynamical models of gravity—relied on sparse observations. The late 20th and early 21st centuries have witnessed a revolutionary increase in the availability of data for modeling purposes. Indeed, we are in the midst of the *sensing revolution*, where the word "sensing" is used in the broadest meaning of data acquisition. This proliferation of data has caused a paradigm shift in modeling. Researchers now often use machine learning (ML) models (under the umbrella of artificial intelligence) to analyze and make sense of data, as evidenced by the explosive popularity of large language models that rely on deep neural network technology. Because these models are typically vastly overparametrized (i.e., the number of weights is massive, in the billions or even trillions), an individual weight does not mean much. To guarantee efficient human-machine correspondence, we must extract *human-interpretable models* through which we can make our own sense of the data.

Koopman operator theory (KOT) has recently emerged as the primary candidate for this task. Its key paradigm is that *the operator is the model* [9, 10]. Namely, KOT assumes the existence of a linear operator U—essentially an infinite-dimensional matrix—such that *any* observation f of system dynamics U enables the prediction of the time evolution to the next observation f^+ via the equation

$$f^+=Uf, \tag{1}$$

where f is a function on some underlying state space M. Modelers must then *find a finite number of observables that are useful for prediction and control*. Rather than ask, "If position and momentum are observable, what equations describe their dynamical evolution?", we instead inquire, "Given the available data, what observables parsimoniously describe their dynamics?". This change of setting—from dynamics on the state space to dynamics on the space of observables $\mathcal{O}$—inspired a new modeling architecture that takes $\mathcal{O}$, rather than the state or phase space, as its template.

The resulting approach finds use in a variety of applied contexts, such as fluid dynamics, autonomy, power grid dynamics, neuroscience, and soft robotics. The theory relies on a beautiful combination of operator-theoretic methods, geometric dynamical systems, and ML techniques.

History

Driven by the success of the operator-based framework in quantum theory, Bernard Koopman made a proposal in the 1930s to treat classical mechanics in a similar way; he suggested using the spectral properties of the composition operator that is associated with dynamical system evolution [5]. But it was not until the 1990s and 2000s that researchers realized the potential for wider applications of the Koopman operator-theoretic approach [10]. In the past decade, the trend of applications has continued. Earlier work emphasized the utilization of Koopman theory to find finite-dimensional models from data [9]. These models exist on invariant subspaces of the operator that are spanned by eigenfunctions. Finding an eigenfunction ϕ of the operator that is associated with a discrete-time, possibly nonlinear process yields a reduced model of the process whose dynamics are governed by $\phi^+=\lambda\phi$; as such, the result is a potentially reduced order but linear model of the dynamics. The level sets of the eigenfunction on the original state space yield geometrically important objects like invariant sets, isochrons, and isostables [8]. This outcome led to a realization that geometrical properties can be *learned* from data via the computation of spectral objects, thus initiating a strong connection between ML and dynamical systems communities that continues to grow. The key notion that drives these developments is the idea of representing a dynamical system as a linear operator on a typically infinite-dimensional space of functions.

However, it is interesting to invert this question and start from the operator, rather than the state-space model; U is the property of the system, but does it have a finite-dimensional (linear or nonlinear) representation? We formalized the concept of *dynamical system representation* in 2021 [10]. Instead of starting with the model and constructing the operator, we construct the finite-dimensional linear or nonlinear model *from* the operator. Doing so facilitates the study of systems with *a priori unknown physics*, like those in soft robotics [2, 4].

Operator Representations

The modeling exercise usually begins with a catalogue of available observations in vector $\mathbf{f}=(f_1,\ldots,f_n)$. We can organize n different streams of data into the $n\times m$ matrix $[\mathbf{f}(1),\ldots,\mathbf{f}(m)]$, where m is the number of "snapshots" of observations (in ML parlance, the observables are "features"). For simplicity, we assume that these snapshots are taken at regular time intervals and sequentially organized into columns. Assuming that the dynamics are evolving on some underlying state space M (that we might not know) according to a potentially unknown mapping $\mathbf{x}(k+1)=T(\mathbf{x})$, the Koopman (composition) operator U is defined by $U\mathbf{f}=\mathbf{f}\circ T$. Then, $\mathbf{f}(k+1)=U\mathbf{f}(k)$. Is there an $n\times n$ matrix A such that $U\mathbf{f}=A\mathbf{f}$? This is indeed the case when $\mathbf{f}$ is within the span of n (generalized) eigenfunctions of U [10]. Methodologies to find eigenfunctions include spectral methods [9] and (extended) dynamic mode decomposition [6, 13]. We could also ask a more general question: Is there a map $\mathbf{F}:\mathbb{C}^n\rightarrow\mathbb{C}^n$ and observables (functions) $\mathbf{g}:X\rightarrow\mathbb{C}^d$ such that

See **Operator is the Model** *on page 5*

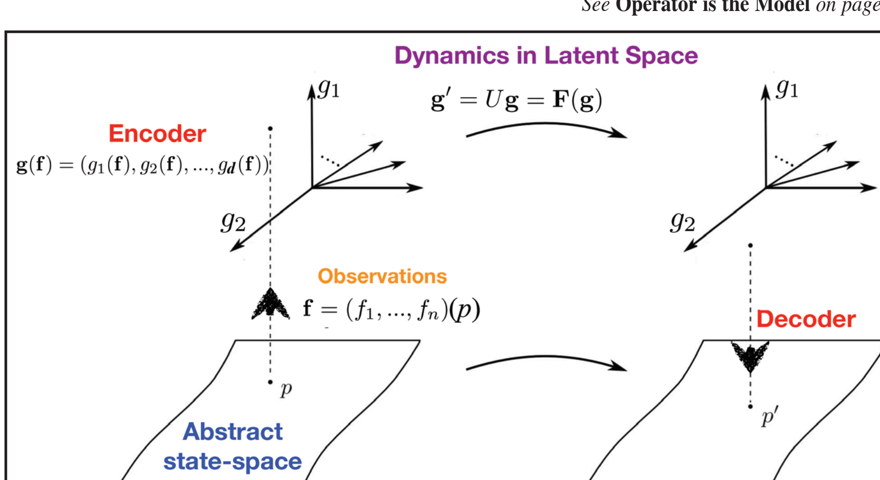

Figure 1. *The Koopman-operator-based modeling architecture. Figure courtesy of the author.*

Optimization Proxies

Continued from page 1

every scenario in roughly five seconds [3]. Other possible applications include dashboards for tactical planning and operations in areas such as energy distribution, supply chains, and manufacturing. It may also be possible to deploy models with much higher fidelity, which would shift most of the computational burden offline during training. Here, I describe some of the science and engineering concepts that underly optimization proxies and highlight research in trustworthy optimization proxies that guarantee feasibility and quality.

Differentiable Programming

The repair layers of optimization proxies rely on *differential programming*: the use of dynamic computational graphs that can be automatically and transparently differentiated. Consider a simplified version of the economic dispatch that is utilized by the Independent System Operators[1] (ISOs) in the U.S.:

$$\min_{\mathrm{p,r},\xi_{\mathrm{th}}} \quad c(\mathrm{p}) + M_{\mathrm{th}} \|\xi_{\mathrm{th}}\|_1 \tag{2a}$$

$$\text{s.t.} \quad \mathbf{e}^\top \mathbf{p} = \mathbf{e}^\top \mathbf{d}, \tag{2b}$$

$$\mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}}, \tag{2c}$$

$$\underline{\mathbf{f}} - \xi_{\mathrm{th}} \le \Phi(\mathbf{p} - \mathbf{d}) \le \bar{\mathbf{f}} + \xi_{\mathrm{th}}, \tag{2d}$$

$$\xi_{\mathrm{th}} \ge \mathbf{0}. \tag{2e}$$

ISOs solve this fundamental optimization problem to balance generation and demand in electricity grids while also accounting for reserve constraints and thermal limits. Constraint (2b) captures the global power balance, (2c) enforces minimum and maximum limits on each generator's active power, and (2d) uses a power transfer distribution factor to express the thermal constraints on every branch. The thermal constraints in U.S. electricity markets are traditionally soft; they can be violated, but doing so incurs a high cost [5, 6]. Since it is a regression, the approximation $\tilde{\mathbf{p}}$ in the proxy does not satisfy the power balance constraint. However, the repair layer can utilize control systems concepts to scale the generators proportionally and obtain a feasible solution $\bar{\mathbf{p}}$:

$$\bar{\mathbf{p}} = \begin{cases} (1-\zeta^\uparrow)\tilde{\mathbf{p}} + \zeta^\uparrow \bar{\mathbf{g}} & \text{if } \mathbf{1}^\top \tilde{\mathbf{p}} < \mathbf{1}^\top \mathbf{d}, \\ (1-\zeta^\downarrow)\tilde{\mathbf{p}} + \zeta^\downarrow \underline{\mathbf{g}} & \text{otherwise,} \end{cases} \tag{3}$$

where $\zeta^\uparrow$ and $\zeta^\downarrow$ are defined as

$$\zeta^\uparrow = \frac{\mathbf{1}^\top \mathbf{d} - \mathbf{1}^\top \tilde{\mathbf{p}}}{\mathbf{1}^\top \bar{\mathbf{g}} - \mathbf{1}^\top \tilde{\mathbf{p}}}, \qquad \zeta^\downarrow = \frac{\mathbf{1}^\top \tilde{\mathbf{p}} - \mathbf{1}^\top \mathbf{d}}{\mathbf{1}^\top \tilde{\mathbf{p}} - \mathbf{1}^\top \underline{\mathbf{g}}}. \tag{4}$$

This layer is differentiable almost everywhere, which means that we can naturally integrate it into the training process of the ML model. This type of overall architecture guarantees feasibility during training and inference and can generate near-optimal feasibility for economic dispatch problems in mere milliseconds [3].

Self-supervised Optimization Proxies

One appealing feature of optimization proxies is the possibility of *self-supervised learning*: training proxies without labeled data [4, 8]. For instance, self-supervised learning is possible for the aforementioned economic dispatch if we use the original objective function as the loss function to train the optimization proxy P_θ, i.e., $\mathcal{L}(\mathbf{y}|\theta) = f_{\mathbf{x}}(\mathbf{y})$. Stochastic gradient descent could then learn the parameters θ. Self-supervised learning thus removes the need to solve the optimization problems that are specified by the dataset $\mathcal{D}$.

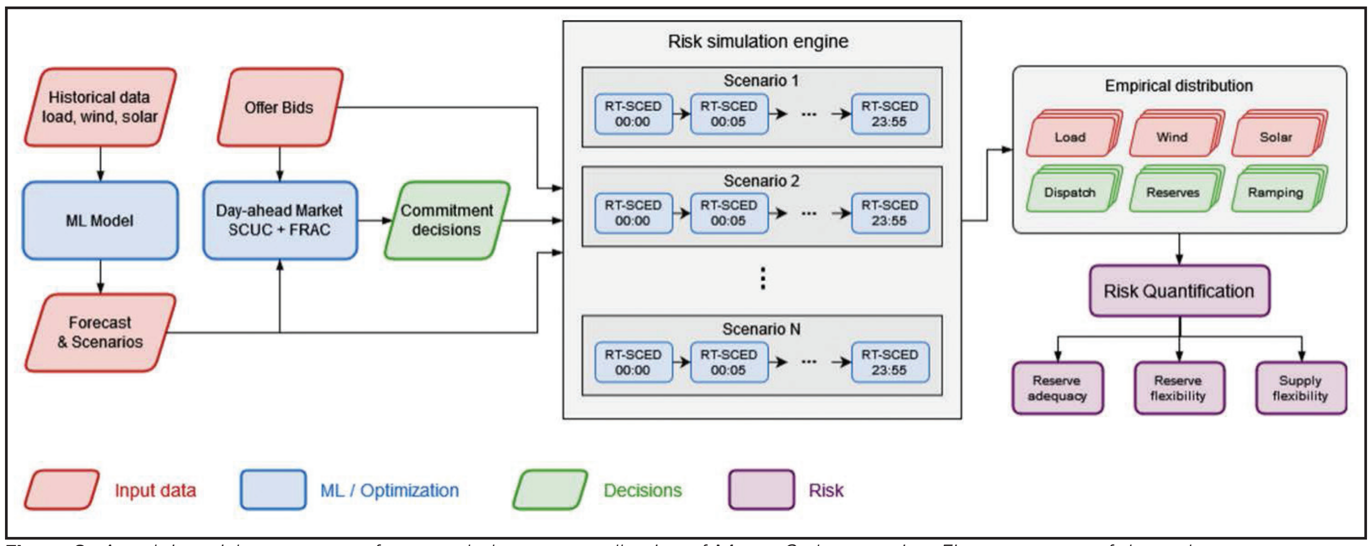

Figure 2. *A real-time risk assessment framework that runs a collection of Monte Carlo scenarios. Figure courtesy of the author.*

Dual Feasible Solutions

An ideal outcome in optimization practice is a pair of primal and dual solutions with a small duality gap. Another attractive feature of optimization proxies is their ability to find these dual feasible solutions for many convex optimization problems that arise in engineering. Consider the following linear program and its dual:

$$\begin{array}{llll} \min & c^T\mathbf{x} & \min & b^T\mathbf{y} \\ s.t. & A\mathbf{x} \ge b & s.t. & \mathbf{y}A - \lambda + \gamma = c \\ & l \le \mathbf{x} \le u & & \mathbf{y}, \lambda, \gamma \ge 0. \end{array}$$

The formulation of the primal optimization reflects the fact that decision variables have lower and upper bounds in many engineering problems. As a result, it is relatively easy to restore feasibility in the dual space. Indeed, the optimization proxy for the dual optimization can first predict $\mathbf{y}$ and then use λ and γ to determine a dual feasible solution. One study applied the same idea to the second-order cone relaxation of the alternating current power flow equations, wherein the reconstruction utilizes properties of the optimal solutions [9]. Experimental results confirm that the ensuing proxies can find near-optimal dual feasible solutions.

Primal-dual Learning

Optimization proxies also offer a unique opportunity for researchers to adapt traditional optimization algorithms to the learning context. Consider the constrained optimization problem

$$\min_{\mathbf{y}} f_{\mathbf{x}}(\mathbf{y}) \;\text{ subject to }\; \mathbf{h}_{\mathbf{x}}(\mathbf{y}) = 0, \tag{5}$$

where $\mathbf{x}$ represents instance parameters that determine the objective function $f_{\mathbf{x}}$ and equality constraint $\mathbf{h}_{\mathbf{x}}$.

Primal-dual learning (PDL) [8] jointly learns two neural networks: (i) a primal neural network P_θ that learns the input/output mapping of the unconstrained optimizations of the *augmented Lagrangian method* (ALM), and (ii) a dual network $\boldsymbol{\lambda}$ that learns the dual updates. At each iteration, the *primal learning* step updates the parameters θ of the primal network while keeping the dual network D_ϕ fixed. After primal learning is complete, PDL applies a *dual learning* step that updates the parameters ϕ of the dual network D_ϕ. Training the primal network involves the loss function

$$\mathcal{L}_{\mathrm{p}}(\mathbf{y}|\boldsymbol{\lambda},\rho) = f_{\mathbf{x}}(\mathbf{y}) + \boldsymbol{\lambda}^T\mathbf{h}_{\mathbf{x}}(\mathbf{y}) + \frac{\rho}{2}\mathbf{1}^\top \nu(\mathbf{h}_{\mathbf{x}}(\mathbf{y})),$$

which is the direct counterpart of the ALM unconstrained optimization; here, ρ is a penalty coefficient and $\boldsymbol{\lambda}$ represents the Lagrangian multiplier approximations. The dual learning training uses the loss function

$$\mathcal{L}_{\mathrm{d}}(\boldsymbol{\lambda}|\mathbf{y},\boldsymbol{\lambda}_k,\rho) = \|\boldsymbol{\lambda} - (\boldsymbol{\lambda}_k + \rho\mathbf{h}_{\mathbf{x}}(\mathbf{y}))\|,$$

which is the direct counterpart of the update rule for the Lagrangian multipliers of the ALM. These two steps—the training of the primal and dual networks—are iterated in sequence until convergence. Note that PDL is self-supervised and does not require labeled data. Researchers have applied PDL to preventive security-constrained optimal power flow problems with automatic primary response [7] — an application that even state-of-the-art optimization cannot solve. This work highlights optimization proxies' ability to deploy optimization models that would otherwise be too complex to meet real-time requirements.

Convex Optimization Proxies

Optimization models often appear as components in other types of optimization models, including decomposition techniques, bilevel optimization, and stochastic optimization. Can optimization proxies be similarly compositional? While we can encode a neural network with rectified linear unit (ReLU) activation functions as a mixed-integer program (MIP), the resulting non-convexities and computational challenges make this tactic undesirable. An intriguing alternative is the use of *convex neural networks* — or more precisely, *input-convex neural networks*. A neural network with ReLU activation functions computes a convex function if all of its weights are nonnegative. An input-convex neural network generalizes this idea by introducing a first layer whose weights are unconstrained and adding skip connections to all other "convex" layers. More precisely, the kth layer of an input-convex neural network takes the form

$$\mathbf{x}^k = h^k(\mathbf{x}^{k-1}) = \mathrm{ReLU}(W^k\mathbf{x}^{k-1} + H^k\mathbf{x}^0 + d^k). \tag{6}$$

Here, $\mathbf{x}^k$ and $\mathbf{x}^{k-1}$ denote the outputs of layer k and $k-1$, $\mathbf{x}^0$ signifies the input of the iterative convolutional neural network (ICNN), d^k is the bias vector, and W^k and H^k are weight matrices. Skip connections feed the ICNN input $\mathbf{x}^0$ to each layer. The coefficients of W^k are nonnegative, whereas H^k may take positive or negative values without affecting convexity [1]. Once it is trained, the input-convex neural network provides a function and gradient with respect to its inputs — thereby meeting the requirements of multiple applications. Input-convex neural networks can approximate the objective value of alternating current optimal power flow, its second-order cone relaxation, and its direct current approximation with high accuracy, which reveals many promising avenues for practical implementation [10].

Applications

Optimization proxies can find utility in a variety of applications, including energy systems, mobility engineering, transportation, and supply chains. In mobility settings, the use of optimization proxies within a model-predictive control framework can help relocate vehicles in an ideal manner. Reinforcement learning fine-trains these proxies to capture long-term effects, and the repair layers use linear programming (transportation) models that are solvable in polynomial time. In transportation and supply chain contexts, proxies approximate complex MIPs with poor linear relaxations, resulting in an order-of-magnitude reduction in MIP size for real-time solutions. And as evidenced by this article, researchers have studied proxies for energy systems in significant depth.

Pascal Van Hentenryck delivered an invited lecture[2] on this topic at the 10th International Congress on Industrial and Applied Mathematics,[3] which took place in Tokyo, Japan, last year.

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[1] https://www.ferc.gov/power-sales-and-markets/rtos-and-isos

[2] https://iciam2023.org/3488#Hentenryck
[3] https://iciam2023.org

Operator is the Model

Continued from page 3

$$U\mathbf{g} = \mathbf{F}(\mathbf{g}), \qquad (2)$$

where X is some latent space and $\mathbf{g} = \mathbf{g}(\mathbf{f})$? Typically, $d \geq n$. A simple example of finding a nonlinear representation of the Koopman operator is available in [11].

Figure 1 (on page 3) provides a graphical representation of the modeling process architecture. If we take our original observables $\mathbf{f}$ and set $\mathbf{g} = \mathbf{f}$, it may be impossible to find an $\mathbf{F}$ that satisfies (2). In this case, the observations do not provide us with a "closure" — i.e., we cannot uniquely predict the next state of observations based on the current state, but a more sophisticated "embedding" $\mathbf{g}$ might do the job. Interestingly, the architecture is similar to the transformer architecture of large language models [11].

The problem of finding $(\mathbf{F}, \mathbf{g})$ in (2) is called the *representation eigenproblem* [10]. A rigorous result exposes how the nature of the representation depends on the spectrum of the Koopman operator; finite linear representations are possible if the operator has a discrete spectrum, while finite nonlinear representations—which pertain to infinite dimensional invariant subspaces—are needed when the spectrum of the operator is continuous [10].

One approach to solving the representation problem utilizes the standard neural network architecture to minimize

$$(\beta^*, \gamma^*) = \min_{\beta,\gamma} ||\mathbf{g}_\beta(k+1) - \mathbf{F}_\gamma(\mathbf{g}_\beta(k))||, \qquad (3)$$

where some or all of the components g_j, F_k are parametrized by neural networks with weights γ, β. This approach can be used in combination with the concept of *parenting* in learning, in that domain experts can suggest some key observables. Experts in classical dynamics would likely propose $\sin\theta$ as a good observable for learning rigid pendulum dynamics, for instance, but would presumably use neural networks or time-delay observables to learn appropriate observables for a soft pendulum whose physical laws are hard to derive [2, 4]. This tactic enables a mixture of human-prescribed and machine-learned observables.

Extensions and Relationships to Other Machine Learning Methods

Koopman-based ML of dynamical models is particularly suitable for an extension to control systems [8]. Another applicable extension is to ML for general nonlinear maps between different spaces; random dynamical systems have also been treated in the framework of the stochastic Koopman operator [9, 10].

Researchers have drawn multiple connections between "pure" Koopman operator-based methodologies and other ML techniques. The version of the framework that has a predefined set of observables is conceptually equivalent to the kernel methods that are popular in ML. One can view the class of autoregressive integrated moving average (ARIMA) models as a subset of Koopman-based methods, and deep learning can help identify effective observables and make connections to transformer architectures that are common in large language models.

Furthermore, a well-known methodology of reinforcement learning (RL) has connections to KOT modeling. However, there is a fundamental difference in the approaches to optimal control with KOT versus RL; specifically, the exploration strategy in RL can lead to dangerous scenarios. In the KOT approach, the model is formed first to ensure the execution of only *safe scenarios*. We then specify a cost function to enable optimization of the task while simultaneously preserving safety. KOT methodologies also typically require orders-of-magnitude fewer executions of learning tasks than RL.

Because of their explicit treatment of the time dimension, Koopman operator models are well suited to handle causal inference [12]. For example, we can use Koopman control models to answer counterfactual questions such as "What if I had acted differently?" In fact, generative Koopman control models help to overcome obstacles in the development of autonomous systems that exhibit human-level intelligence — robustness, adaptability, explainability (interpretability), and cause-and-effect relationships. The methodology has now penetrated most dynamics-heavy fields and inspired recent advances in soft robotics [2, 4] and game modeling [1]; Koopman operators even furthered the study of neural network training [3, 7]. These successes are due to the effectiveness of developed ML algorithms as well as the depth of the underlying theory that enhances interpretability (which is prevalent in applied mathematics but absent from some ML approaches).

Despite the aforementioned progress, there is still much to do. The current decade promises to be an exciting one for this growing set of data-driven artificial intelligence methodologies that will boost the discovery of models of dynamical processes.

An expanded version of this article is available online [11].

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Igor Mezić is a professor of mechanical engineering at the University of California, Santa Barbara. His work encompasses the operator theoretic approach to dynamical systems and incorporates elements of geometric dynamical systems theory and machine learning. Mezić applies such mathematical methods in various contexts.

Thomas Yizhao Hou Won William Benter Prize in Applied Mathematics 2024

Professor Thomas Yizhao Hou

Professor Thomas Yizhao Hou, Charles Lee Powell Professor of Applied and Computational Mathematics, California Institute of Technology, US, won the William Benter Prize in Applied Mathematics 2024.

Professor Hou, an outstanding applied mathematician with exceptional strengths in both numerics and analysis, has made pioneering and ground-breaking contributions in several areas of applied mathematics. For fluid interface problems, Professor Hou and collaborators developed the Small Scale Decomposition method which has many applications ranging from fluid dynamics to materials science and biology. The first level set method to study incompressible multiphase flows was developed by Chang, Hou, Merriman and Osher in 1996. The work has generated a significant impact in the computational fluid dynamics community.

The Multiscale Finite Element Method (MsFEM) developed by Hou and Wu in 1997 has generated a considerable impact in both the applied math and engineering communities. Some major oil companies have adopted a version of MsFEM in their next generation flow simulators. The Generalized Multiscale Finite Element Method (GMsFEM) developed by Efendiev, Galvis and Hou is another remarkable contribution. The GMsFEM has been used to derive macroscopic equations for a variety of applications and has found many applications in geoscience and materials science. It also provides a rigorous justification for the widely used multicontinuum theories in the engineering community.

Whether the 3D incompressible Euler equations can develop a finite time singularity from smooth initial data is considered as one of the most challenging problems. Professor Hou and collaborators established a localized non-blowup criterion for 3D Euler equations, discovered and analyzed the surprising stabilizing effect of advection, and proved the existence of globally smooth solutions for the 3D Navier-Stokes equations with large smooth initial data of finite energy. In 2014, Lou and Hou discovered a new blowup scenario for the 3D axisymmetric Euler equations with boundary. They designed an extremely effective adaptive mesh strategy to achieve a remarkable level of resolution, and obtained strong numerical evidence of finite time singularity. Recently, Professor Hou and his former PhD student, Jiajie Chen, made a major breakthrough by providing a rigorous computer-assisted proof of the Hou-Luo blowup scenario. Their method is very powerful and it can be potentially used to study self-similar blowup of other nonlinear PDEs. Very recently, Professor Hou made another important breakthrough by discovering a new class of potentially singular solutions of the axisymmetric Navier-Stokes equations.

For his outstanding contributions in applied mathematics, Professor Hou has received many honors and awards. He was an ICM invited speaker in 1998 and a plenary speaker of ICIAM in 2003. He was elected to Fellow of American Academy of Arts and Sciences in 2011, an inaugural SIAM and AMS Fellow. He also co-founded the highly influential SIAM interdisciplinary Journal on Multiscale Modelling and Simulation Journal in 2002.

The William Benter Prize will be presented during the opening ceremony for the International Conference on Applied Mathematics (ICAM 2024), which is co-organized by the Liu Bie Ju Centre for Mathematical Sciences (LBJ) and the Department of Mathematics of City University of Hong Kong.

The William Benter Prize in Applied Mathematics was set up by LBJ in honour of Mr William Benter for his dedication and generous support to the enhancement of the University's strengths in mathematics. The prize recognizes outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance and engineering applications. The cash prize of US$100,000 is given once every two years.

– City University of Hong Kong

Pursuing Computational Goals as an LLF-SIAM Undergraduate Fellow

By Everett Grethel

Computational science is seemingly emerging as a crucial subfield in nearly every scientific domain. A large number of researchers—from biologists to physicists—are beginning to describe themselves as "computational." Scientists with experimental or theoretical backgrounds may therefore wonder if they have the requisite skill sets to pursue these new lines of inquiry. Several years ago, I embarked on the path towards becoming a computational scientist and resolved to work at the intersection of machine learning and physical science.

Last summer, I was afforded the opportunity to intern at Lawrence Livermore National Laboratory[1] (LLNL) as the 2023 inaugural LLF-SIAM Undergraduate Fellow.[2] SIAM partnered with the Livermore Lab Foundation[3] (LLF) to support an undergraduate student internship at LLNL's world-class research facility that offers one lucky participant the chance to advance their applied mathematics knowledge and address relevant scientific and technological challenges. Bruce Hendrickson, Principal Associate Director for Computing at LLNL, made the initial connection between LLF and SIAM. Here, I will describe both my transition to computational research and the Fellowship's role in bringing clarity to my path.

Last year, SIAM partnered with the Livermore Lab Foundation (LLF) to support an undergraduate student internship at Lawrence Livermore National Laboratory. Everett Grethel of the University of Connecticut was the 2023 inaugural LLF-SIAM Undergraduate Fellow. Photo courtesy of LLF.

In my experience, students or professionals who are considering computational work frequently have a background in the natural sciences. Often, they seek to develop a sense of intuition for incorporating computational and data science into their respective fields. Experience in the natural sciences does allow for a relatively seamless transition to computation because of the major overlap in prerequisite knowledge between the two areas. In contrast, computer scientists who wish to apply their expertise to other scientific fields may encounter more difficulties due to their lack of familiarity with the targeted domain.

I began my journey with no background whatsoever in science, technology, engineering, and mathematics (STEM), having switched my major from fine arts to computer science (I recently graduated with bachelor's degrees in both computer science and digital media and design from the University of Connecticut). Three key aspects contributed to my successful transition. First, I learned how to study in a manner that worked for me — a crucial part of finding success as a STEM student. Second, I recognized math's relevance to my field of choice. Concepts in probability, multivariable calculus, and linear algebra take on new meaning when they are contextualized in a neural network. Third, research experiences helped me appreciate the symbiotic relationship between computer science and the natural sciences. By abstracting the physical world's phenomena into data, I realized that I can use techniques such as machine learning to extract new information; conversely, physical phenomena can inspire novel algorithms in areas like genetic programming or neural networks.

My newfound understanding and appreciation for STEM led me to wonder how I could apply computer science to real-world problems in other scientific domains. While university research certainly provided a foundation, the LLF-SIAM Undergraduate Fellowship ultimately offered a broader view of this enterprise. The Fellowship targets students who are interested in applied mathematics, computational science, and/or data science, though LLF also sponsors additional Fellowships[4] in other areas of STEM. Students are paired with an LLNL staff mentor whose research aligns with their interests and skills; they take full ownership of their assigned project components and often collaborate with multiple

See **Undergraduate Fellow** on page 7

1 https://www.llnl.gov
2 https://livermorelabfoundation.org/2023/10/17/llf-siam-undergraduate-fellowship
3 https://livermorelabfoundation.org
4 https://livermorelabfoundation.org/stem

The Perils of Automated Facial Recognition

Unmasking AI: My Mission to Protect What is Human in a World of Machines. *By Joy Buolamwini. Random House, New York, NY, October 2023. 336 pages, $28.99.*

Your Face Belongs to Us: A Secretive Startup's Quest to End Privacy as We Know It. *By Kashmir Hill. Random House, New York, NY, September 2023. 352 pages, $28.99.*

BOOK REVIEW
By Ernest Davis

Automated facial recognition is one of the most widely deployed and technically successful forms of artificial intelligence (AI). AI systems can match faces with roughly the same level of accuracy as humans themselves. These technologies can find photos on the internet from a decades-old party that even the subject has never seen. They are able to match low-quality images and photos where the individual in question is inconspicuously in the background, part of a large group, wearing a mask, or sporting a completely different hairstyle at a much younger age.

Facial recognition is also one of the most problematic AI technologies, with very serious implications for personal privacy and inequality. The toxic combination of power, ubiquity, invasiveness, and bias has brought forth a uniquely troubling situation. Two important recent books—*Unmasking AI: My Mission to Protect What is Human in a World of Machines* by computer scientist Joy Buolamwini, and *Your Face Belongs to Us: A Secretive Startup's Quest to End Privacy as We Know It* by *New York Times* reporter Kashmir Hill—raise serious concerns about the impact of facial recognition systems and the difficulty of controlling them.

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Joy Buolamwini is best known for exposing the fact that many common facial recognition systems are much less accurate for women and people with dark skin than for white males. *Unmasking AI* is simultaneously an autobiography, an explanation of her scientific work, and a statement of principles that should guide AI development.

While conducting undergraduate research with a robot that had a camera that utilized facial recognition technology, Buolamwini noticed that the robot was often unable to see her face — despite its ability to identify her white classmates. She later encountered the same issue in graduate school; though she was using more advanced facial detection software, it did not process her face until she donned a white Halloween mask. Buolamwini examined this problem systematically as part of her doctoral research and found that all types of facial recognition systems had substantially higher failure rates for women, people with dark complexions, and especially dark-complexioned women.

Buolamwini has since developed industry-wide techniques to ameliorate these biases. She currently researches the detection and correction of biases in AI systems and explores the deployment of AI in ways that promote societal justice and equity. Buolamwini's career has been marked by meteoric success, including a doctoral degree from the MIT Media Lab at the Massachusetts Institute of Technology, various TED talks, testimony to U.S. Congress, and a group meeting with U.S. President Joe Biden — all by the age of 33. Sadly, it has also been punctuated by the kinds of slights and insults that Black women in technology encounter all too often: participants at conferences who assume she is staff, security guards who block her entrance to events where she is presenting, and so on.

One particularly interesting aspect of *Unmasking AI* is Buolamwini's struggle with the ethical issues that arose in her own research. Quantitative documentation of a vision program's bias against women and people who are Black requires a benchmark collection of facial photographs that are tagged with their race and gender. Existing benchmarks' biases towards white male faces rendered them unusable. In order to create an unbiased, high-quality assembly of benchmark photos, Buolamwini decided to personally collect the images and tag them herself. She assigned each face a numerical measure of skin color and gender, acknowledging that neither measure is fully objective and both are sometimes difficult to judge from a photograph.

Image collection presented its own set of complications. Buolamwini used photographs of global parliamentarians from official websites to avoid copyright issues and ensure that the subjects had consented to publication. Nevertheless, some concerns remained. For instance, there was no reason to suppose that, when agreeing to circulate their likenesses, the image subjects had also intended to give permission for the use of their photos in this context. In addition, a collection of parliamentarians is obviously not a representative sample in areas like age, social status, and image quality.

Finally, Buolamwini wondered about the overall impact of her own work; to what extent would it make the world better, fairer, and more equitable, and to what extent was it merely improving the technology that served as a tool for surveillance capitalism?

Although Buolamwini's research—and that of the scientists who have followed in her footsteps—has led to significant reductions in gender and racial bias in facial recognition programs, the problem still persists. A recent *New Yorker* article discussed the wrongful arrest of Alonzo Sawyer in 2022 based purely on a match by AI software, despite a wealth of contrary evidence [3]. Racial and gender bias also infects other kinds of AI software, such as image generation programs. For example, when the authors of a *Washington Post* article entered the prompt "attractive people" into the popular Stable Diffusion model, it produced images of young, light-skinned individuals [4].

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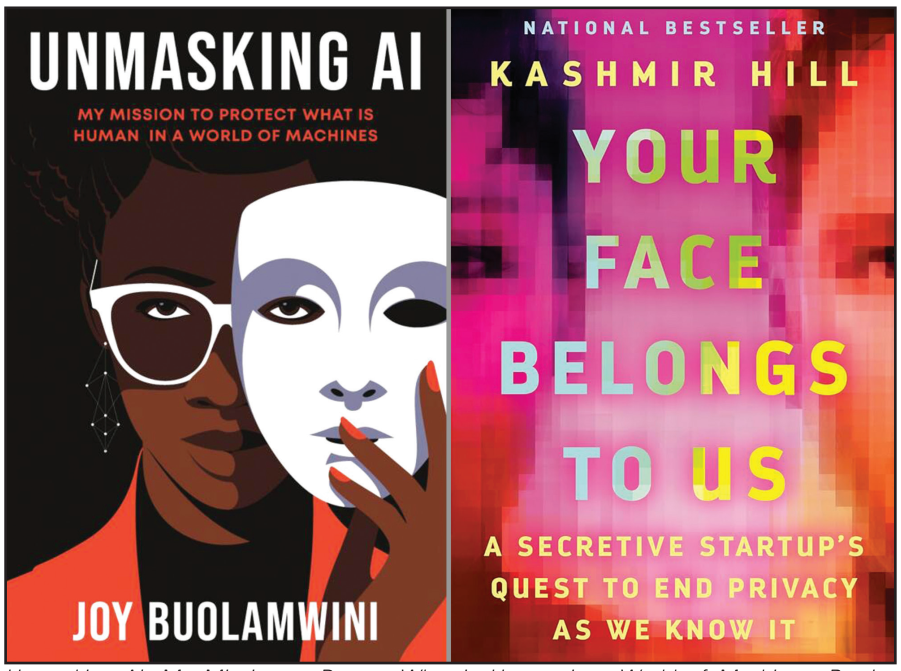
On January 18, 2020, the front page of *The New York Times* featured an extraordinary story about a completely unknown company called Clearview AI [2]. Clearview had downloaded billions of photos from the web and built an app that matched an input image against its collection with startling scope and accuracy. When Kashmir Hill, the author of the article, submitted her own photograph, the app returned "numerous results, dating back a decade, including photos of myself that I had never seen before." The Clearview app creators sold their product—without any public notification, scrutiny, or independent evaluation of its error rate or biases—to more than 600 different law enforcement agencies and a handful of companies. The police departments that purchased it were very enthusiastic and had already used the technology



Unmasking AI: My Mission to Protect What is Human in a World of Machines. By Joy Buolamwini. *Your Face Belongs to Us: A Secretive Startup's Quest to End Privacy as We Know It.* By Kashmir Hill. Images courtesy of Random House.

See **Facial Recognition** on page 8

Linear Systems

Continued from page 3

$$r_{k+1} = (I-\alpha_k A)(I-\alpha_{k-1}A)\cdots (I-\alpha_0 A)r_0 \equiv p_{k+1}(A)r_0, \tag{3}$$

where $p_{k+1}(t) = (1-\alpha_k t)(1-\alpha_{k-1}t)\cdots(1-\alpha_0 t)$ is a polynomial of degree $k+1$. Note that $p_{k+1}(0)=1$.

Lewis Fry Richardson was the first person to formulate the problem of selecting the α_is with a goal of minimizing the error $u_{k+1} \equiv A^{-1} r_{k+1}$. Assume that A is SPD with eigenvalues in an interval $[\alpha, \beta]$ where $\alpha > 0$, and let $\mathbb{P}_{k+1,0}$ be the set of polynomials p of degree $k+1$ such that $p(0)=1$. To minimize the maximum deviation of $p_{k+1}(t)$ from 0 in the interval, we must find a polynomial $p \in \mathbb{P}_{k+1,0}$ for which $\max_{t\in[\alpha,\beta]} |p(t)|$ is minimal. The solution is known in terms of $C_{k+1}(t)$, i.e., the Chebyshev polynomial of the first kind of degree $k+1$:

$$T_{k+1}(t) \equiv \frac{1}{\sigma_{k+1}} C_{k+1}\left(\frac{\beta+\alpha-2t}{\beta-\alpha}\right) \quad \text{with} \quad \sigma_{k+1} \equiv C_{k+1}\left(\frac{\beta+\alpha}{\beta-\alpha}\right). \tag{4}$$

In the context of (3), we see that the best α_ks are the inverses of the roots of T_{k+1}. Richardson was seemingly unaware of Chebyshev polynomials, as he selected the roots $1/\alpha_i$ by spreading them in an *ad hoc* fashion within $[\alpha, \beta]$.

It took more than four decades for ideas based on Chebyshev polynomials to emerge. Young, Cornelius Lanczos, and George Shortley were among the first researchers to invoke the concept, though their methods did not yield numerically reliable algorithms because they did not fully exploit the three-term recurrence of Chebyshev polynomials. While Lanczos did employ the three-term recurrence, his approach was a preprocessing scheme that was not quite related to Chebyshev acceleration. In fact, it seems that John von Neumann actually described the first acceleration scheme based on Chebyshev polynomials that exploits the three-term recurrence [1]. The article published in 1959 but von Neumann died in early 1957, so he must have developed the method in 1956 or earlier. In 1961, Varga and Gene Golub published a very similar technique—called the *semi-iterative method*—that acknowledged von Neumann's prior work in a footnote [3]. We can easily derive the Chebyshev acceleration algorithm from (4) and the three-term recurrence of the C_ks [1, 3, 8].

Fourth Big Idea: Krylov Subspace Methods

Krylov subspace methods for the solution of (1) are projection methods on the Krylov subspace $K_m(A,b) = \mathrm{Span}\{b, Ab, \cdots, A^{m-1}b\}$. Here, we simplify the notation by assuming that the initial guess is $x_0 \equiv 0$. We now return to the two aforementioned cases.

OP Case: When A is SPD, an OP approach yields an approximate solution that minimizes the scalar function $f(x) = \frac{1}{2}x^T Ax - b^T x$ over all x vectors in $K_m(A,b)$. Magnus Hestenes and Eduard Stiefel's implementation of this method resulted in the *conjugate gradient algorithm* [5]. The process invokes purely geometric arguments; Hestenes and Stiefel's insights from the two-dimensional case and their knowledge of conics inspired them to exploit *conjugate directions*.

Lanczos developed a similar method from a completely different viewpoint. He utilized an MR approach and relied on what is now called a Lanczos basis to implement it. Initially, critics considered the CG algorithm to be an unstable direct solution method. It therefore laid dormant until the mid-1970s, when it resurfaced with force.

MR Case: Several projection techniques on the subspace $K_m(A,b)$ emerged in the late 1970s, with the objective of minimizing the residual norm over the subspace. An implementation with an orthonormal basis $K_m(A,b)$ leads to the generalized minimal residual (GMRES) method; other implementations include generalized CG-LS, Orthomin, Orthodir, and the generalized conjugate residual. A considerable volume of work has followed these beginnings of Krylov methods for nonsymmetric linear systems [7].

Fifth Big Idea: Preconditioning

Krylov subspace methods can achieve fast convergence when matrices have spectra that are clustered around 1 and are not highly nonnormal; resorting to preconditioning can exploit these properties. For example, we can use a Krylov subspace method to solve a system like $M^{-1}Ax = M^{-1}b$ (instead of the original system), where the preconditioning matrix M is close to A in some sense. However, systems such as $Mx = f$ are inexpensive to solve. A common approach is the incomplete lower-upper (LU) factorization $M = LU$, which stems from an approximate LU factorization of A. The combination of Krylov methods and various forms of preconditioning gives rise to one of the most important and effective present-day iterative methods.

Looking Forward

The new wave in NLA embraces *randomness* and *statistical analysis*; in this context, standard "optimal" methods (such as CG and GMRES) are not as useful and those that exploit short-term recurrences lose their appeal. The current buoyant activity in randomized NLA is somewhat reminiscent of the golden era of iterative methods three decades ago. It remains to be seen whether this ongoing transformation will last. Nevertheless, the basic tools from the past still constitute key ingredients for future methods.

What will be the next "big idea" in NLA? Big ideas typically result from a pressing need to solve well-defined problems (e.g., the "flutter problem" in the 1940s) and the drive of bright, motivated researchers with exceptional knowledge and vision [6]. It is evident that machine learning and data-driven approaches have become the primary catalysts in research. As for motivating bright researchers, it is important to actively share our work to capture readers' interests and potentially spark inspiration for the next prominent star. The significance of thoughtfully conveying ideas and disseminating software and other artifacts cannot be overstated, but the fast-paced and competitive nature of our discipline often leaves little time for such initiatives. Nonetheless, the gratification of witnessing their impact validates these efforts.

Yousef Saad received the 2023 John von Neumann Prize[1] *and delivered the associated lecture*[2] *at the 10th International Congress on Industrial and Applied Mathematics,*[3] *which took place in Tokyo, Japan, last year.*

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Yousef Saad is a College of Science and Engineering Distinguished Professor in the Department of Computer Science and Engineering at the University of Minnesota. He joined the faculty of Minnesota in 1990.

From left to right: Magnus Hestenes, Eduard Stiefel, and Cornelius Lanczos — three major contributors to Krylov subspace methods. Image of Hestenes courtesy of Konrad Jacobs and Wikimedia under the Creative Commons Attribution-ShareAlike 2.0 (CC BY-SA 2.0) Germany license; image of Stiefel courtesy of ETH-Bibliothek Zürich, Bildarchiv and Wikimedia under the Creative Commons Attribution-ShareAlike 3.0 (CC BY-SA 3.0) Unported license; and image of Lanczos courtesy of Ida Rhodes and SIAM.

1 https://go.siam.org/gPv44z
2 https://iciam2023.org/3488#Saad
3 https://iciam2023.org

Undergraduate Fellow

Continued from page 6

team members. By interning at a national laboratory, Fellows receive a paid, full-time, 10- to 12-week opportunity to work on major problems with the resources and support of a high-level research center. LLF also hosts a Fellows Week, during which the recipients of various LLF Fellowships gather to connect with each other and foster career-related skills.

I applied for the LLF-SIAM Undergraduate Fellowship to acquire additional research experience and gain a better understanding of what a career in computational science might look like for me. Additionally, I hoped to encounter researchers whose work blended computation with applications to particular scientific domains. Though I was officially a computing intern, I was paired with Diego Oyarzun Dinamarca and Aldair Gongora: staff scientists in mechanical and materials engineering. My assignment involved uncertainty quantification for heat transfer simulations within an overarching project about *autonomous experimentation,*[5] which combines robotics and real-world experiments with machine learning methods like Bayesian optimization. The tasks of performing experiments and making adjustments are automated via robotics and machine learning respectively — an approach that accelerates the iterative process of experimentation to reach a particular goal.

My project sought to reduce the quantity of required experiments for certain heat transfer problems. I spent half of my time at a desk working with programmers, and the other half in a wet lab amongst chemists and engineers. This dual environment allowed me to interact with researchers and interns who had a diverse array of expertise. Some staff members were kind enough to have lunch with me and answer my questions about their work; our conversations offered insight into possible areas of focus for my career. I had also been concerned about the feasibility of working in both computer and physical science, but I met a number of individuals who successfully bridged this gap and came from a variety of backgrounds. Some of these researchers were computer scientists who were pursuing physical science projects and possessed only minimal domain knowledge, while others were physical scientists who had taken up a computational role while still focusing on their domain. Despite these differences, I realized that everyone and everything had its own niche; each approach had its own strengths and weaknesses that were better suited for certain projects and team structures than others. As I continue on my own career journey, I will need to figure out what niche I wish to fill.

My time as an LLF-SIAM Undergraduate Fellow provided clarity and answers to some of my lingering career questions. There is no one correct path that intersects computer science and the natural sciences. Instead, a spectrum of approaches—each with requirements that depend on one's specific background—offer different types of opportunities. The mentorship and knowledge that I gained from the Fellowship were invaluable, and I hope that students with similar aspirations in the SIAM community and beyond will have equally positive experiences in the years to come.

SIAM looks forward to partnering with LLF again in 2024 to sponsor another LLF-SIAM Undergraduate Fellow, who will be selected in the coming months. If you have questions about philanthropically supporting this partnership or other undergraduate programs at SIAM, please contact Abby Addy, Director of Development and Corporate Relations, at ***aaddy@siam.org*** *or (267) 648-3529.*

Everett Grethel is an Academic Graduate Appointee at Lawrence Livermore National Laboratory. He aims to develop methods that handle the unique challenges of machine learning in materials science and other physical sciences.

5 https://go.siam.org/r53HZi

Hong Kong Polytechnic University SIAM Student Chapter Hosts Dialogue with World-leading Scholars

By Yixuan Zhang

In December 2023, the Hong Kong Polytechnic University (PolyU) SIAM Student Chapter[1] successfully organized an exciting event called "Dialogue with World-leading Scholars." The session was part of the three-day Workshop on Nonsmooth Optimization and Variational Analysis[2]—which took place at PolyU from December 4-6—and featured five distinguished panel members: R. Tyrrell Rockafellar (University of Washington), Boris Mordukhovich (Wayne State University), Yurii Nesterov (Université Catholique de Louvain), Kim-Chuan Toh (National University of Singapore), and Radu Ioan Boţ (University of Vienna).

These esteemed experts in the field of optimization discussed their early-career experiences and elaborated on their professional journeys. A question-and-answer format allowed for a more interactive and engaging experience between the students and speakers, the latter of whom also offered practical advice and encouraged attendees to continually learn new knowledge in a variety of mathematical fields. Audience questions covered a wide range of topics, from effective strategies for research and publishing to career development in both academia and industry. The panelists' insights provided students with a clearer roadmap for their future endeavors in the fields of applied mathematics, computational science, and data science.

The Workshop on Nonsmooth Optimization and Variational Analysis was sponsored by PolyU's Department of Applied Mathematics[3] (AMA); the organizing committee comprised academic staff at AMA who study optimization, including co-chairs Xiaojun Chen (faculty advisor of the PolyU SIAM Student Chapter and Chair Professor of Applied Mathematics) and Defeng Sun (Chair Professor of Applied Optimization and Operation Research). Throughout the first two days of the workshop, 16 scholars from around the world delivered enlightening talks that introduced cutting-edge progress in nonsmooth optimization and variational analysis. The third and final day commenced with a "Research Salon," during which the aforementioned panel members from the SIAM Student Chapter event addressed the positive and negative effects of artificial intelligence; the "Dialogue with World-leading Scholars" session immediately followed the salon.

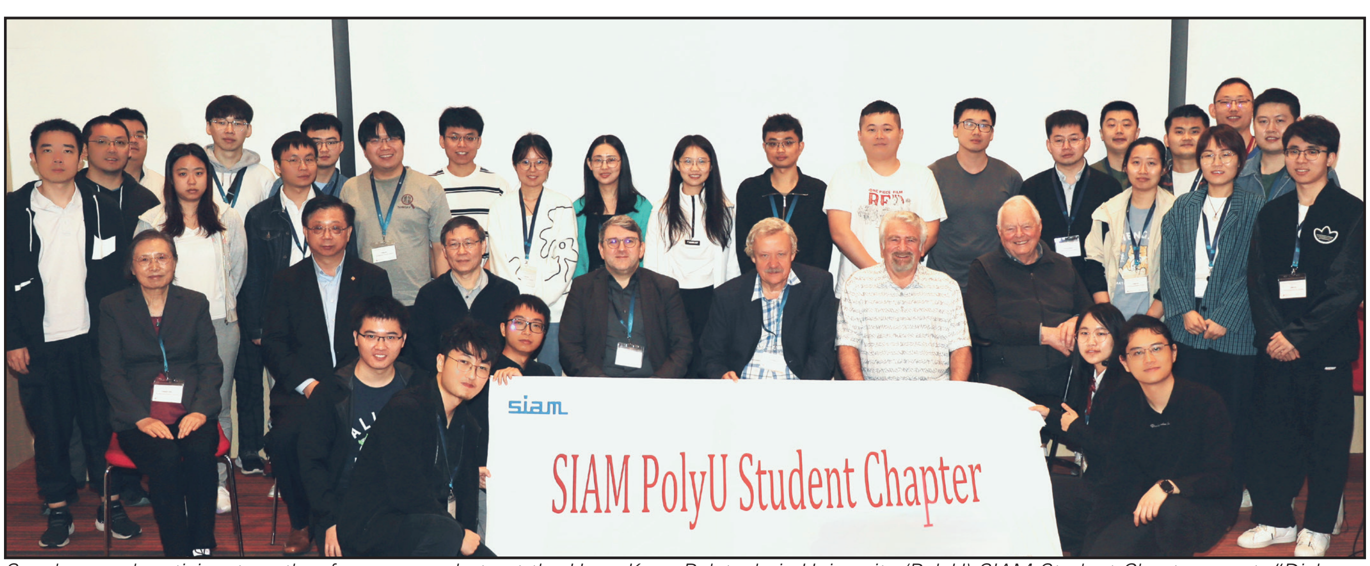
Members of the PolyU SIAM Student Chapter appreciated the opportunity to converse with optimization scholars, learn from their experiences, and glean inspiration and career direction. Student organizers and officers of the chapter—president Yixuan Zhang, vice presidents Cunxin Huang and Bei Sun, secretary Yuan Gao, treasurer Zexian Li, and webmaster Gaohang Chen—would like to thank all attendees, including their fellow chapter members and other participating students and researchers from Hong Kong and beyond. We are especially grateful to the panel members who generously contributed their time and knowledge, as well as for support from the organizing committee of the concurrent Workshop on Nonsmooth Optimization and Variational Analysis.

2023 marked the milestone 10th anniversary of the PolyU SIAM Student Chapter. Established in 2013, this group was the first SIAM student chapter to form in Hong Kong. The chapter generates student interest in applied mathematics, computational science, and data science by organizing various activities and occasions for the dozens of members to gather with contemporaries and faculty from relevant departments both within and outside of PolyU's campus. Recent activities include a variety of lectures, several discussion salons, and an online workshop during the COVID-19 pandemic. Some of these events were held jointly with SIAM student chapters at nearby sister institutions, such as the Chinese Academy of Sciences, the Chinese University of Hong Kong, and the City University of Hong Kong. We invite readers to stay tuned for more upcoming events!

Yixuan Zhang is a Ph.D. student in the Department of Applied Mathematics at the Hong Kong Polytechnic University (PolyU). She currently serves as president of the PolyU SIAM Student Chapter.

[1] https://www.polyu.edu.hk/ama/research-and-consultancy/siam-polyu-student-chapter
[2] https://events.polyu.edu.hk/nova/home
[3] https://www.polyu.edu.hk/en/ama

During the Hong Kong Polytechnic University (PolyU) SIAM Student Chapter's "Dialogue with World-leading Scholars," experts in optimization shared insights and advice based on their own career experiences. From left to right: panel members Kim-Chuan Toh (National University of Singapore), Radu Ioan Boţ (University of Vienna), Yurii Nesterov (Université Catholique de Louvain), Boris Mordukhovich (Wayne State University), and R. Tyrrell Rockafellar (University of Washington), as well as moderator Yixuan Zhang (president of the PolyU SIAM Student Chapter). Photo courtesy of Xin Qu.



Speakers and participants gather for a group photo at the Hong Kong Polytechnic University (PolyU) SIAM Student Chapter event, "Dialogue with World-leading Scholars," which took place during the three-day Workshop on Nonsmooth Optimization and Variational Analysis at PolyU in December 2023. Photo courtesy of Eric Lam Kwok-lung and Peter Lo Charn-tong.

Facial Recognition

Continued from page 6

to identify perpetrators and victims in cases of murder, assault, sexual abuse, and theft. Hill's 2023 book, *Your Face Belongs to Us*, expands upon her original article, details her investigation, offers additional information about the history of Clearview and its founder Hoan Ton-That, and provides readers with updates to the narrative.

The prologue of the book, which recounts the first stages of Hill's investigation, is particularly fascinating. When Hill learned about the existence of Clearview, the organization was wrapped in secrecy — despite the fact that it was already aggressively promoting the app to police departments. The scant company website listed a nonexistent New York address. When Hill called or emailed police departments, they subsequently avoided all communication. She hired a private investigator who contacted Clearview while posing as a potential customer; when he tried to test their product with Hill's photo, they immediately severed the connection.

One particularly notable characteristic of Clearview is the comparatively fly-by-night way in which it came about. In recent years, impactful AI products have mostly originated in large corporate labs with huge teams of top-notch scientists, enormous budgets, and a plethora of computing resources. By contrast, Ton-That seemingly built, deployed, marketed, and maintained Clearview largely by himself (though he was joined by then-computational physicist Terence Liu for a few months). The product's code used a combination of open-source software and techniques from the published literature; there is no indication that its construction involved any particular technical innovation.

When I first read Hill's *New York Times* article, I assumed that it would mark the end of Clearview, much like how John Carreyou's exposé of Theranos in *The Wall Street Journal* ultimately led to that company's downfall [1]. In fact, the outcome was quite the contrary. The article's publicity brought Clearview even more customers, though it did generate a certain amount of legal trouble for the organization. The American Civil Liberties Union[1] (ACLU) filed a lawsuit against Clearview, but their argument was based on the narrow legal grounds that Clearview's use of biometric measurements was illegal; the ACLU agreed with Clearview's lawyer that simply scraping images from the web, matching them, and distributing them was protected free speech. The two groups eventually settled on the compromise that Clearview would not sell its app to private individuals or companies within the U.S., but it could continue selling to U.S. government agencies — including police departments.

[1] https://www.aclu.org

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What will the future bring? Buolamwini's book is not very hopeful in that regard, and Hill's text is downright depressing. Buolamwini and her colleagues at the Algorithmic Justice League[2] (an association that she founded and runs) have an admirable mission in trying to make AI a realistic tool for human prosperity, dignity, and equity, but they face formidable headwinds as many large corporations and nations continue to develop and deploy AI systems in a seemingly reckless manner. The outlook for privacy is even worse. Facial recognition systems like Clearview are powerful and easy to use; cameras are ubiquitously deployed by the police, carried by citizens in cell phones, and hidden in household devices; and the public's fascination with social media is apparently inexhaustible. We may soon arrive at a dystopia in which anyone can examine practically everything about someone's life and publish it to the world whenever they choose.

Of course, unlike with climate change or pandemics, society as a whole has complete collective agency over computer technology. Given the will, nothing would stop us from eliminating all facial recognition software from our lives; doing so would not even cost much. We are in charge, not the AIs. But we must jointly identify our most important values and figure out how to protect them. Doing so is not an easy task, and we may not have much time before the situation becomes relatively dire.

[2] https://www.ajl.org

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InsideSIAM

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SIAM Conference on Applied Linear Algebra
May 13–17, 2024
Paris, France
Sponsored by the SIAM Activity Group on Linear Algebra

SIAM Conference on Mathematical Aspects of Materials Science
May 19–23, 2024
Pittsburgh, Pennsylvania, U.S.
Sponsored by the SIAM Activity Group on Mathematical Aspects of Materials Science

SIAM Conference on Imaging Science
May 28–31, 2024
Atlanta, Georgia, U.S.
Sponsored by the SIAM Activity Group on Imaging Science

SIAM Conference on the Life Sciences
June 10–13, 2024
Portland, Oregon, U.S.
Sponsored by the SIAM Activity Group on Life Sciences

SIAM Conference on Mathematics of Planet Earth
June 10–12, 2024
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Sponsored by the SIAM Activity Group on Mathematics of Planet Earth

SIAM Conference on Nonlinear Waves and Coherent Structures
June 24–27, 2024
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2024 SIAM Annual Meeting
July 8–12, 2024
Online Component July 18–20, 2024
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SIAM Conference on Applied Mathematics Education
July 8–9, 2024
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SIAM Conference on Discrete Mathematics
July 8–11, 2024
Spokane, Washington, U.S.
Sponsored by the SIAM Activity Group on Discrete Mathematics

ICERM-SIAM Workshop on Empowering a Diverse Computational Mathematics Research Community
July 22–August 2, 2024
Providence, Rhode Island, U.S.

SIAM Conference on Mathematics of Data Science
October 21–25, 2024
Atlanta, Georgia, U.S.
Sponsored by the SIAM Activity Group on Data Science

ACM-SIAM Symposium on Discrete Algorithms
January 12–15, 2025
New Orleans, Louisiana, U.S.
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Student Chapter Funding for 2023–2024

SIAM awarded more than $60,000 to over 170 chapters for events and activities taking place throughout the 2023-2024 academic year. SIAM Student Chapters organized meetings or seminars, invited speakers from industry, labs, and academia, ran programming workshops, professional development, and other interesting activities. Below are a few exciting events our chapters held in 2023 or plan to do in spring 2024. For information about obtaining funding for your chapter, go to *www.siam.org/Students-Education/Student-Chapters/Chapter-Resources/Detail/student-chapter-funding*.

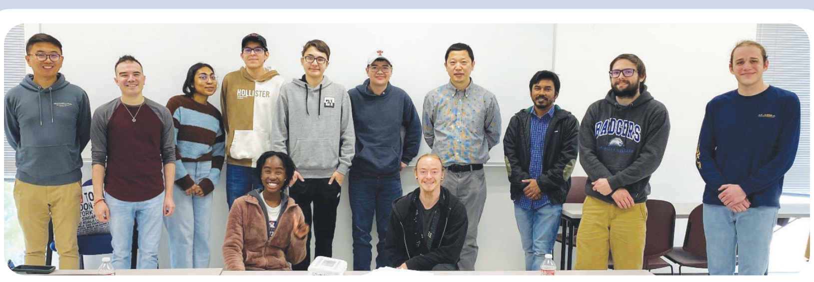
Students from the **Pontificia Universidad Catolica de Valparaiso** chapter celebrate International Day of Women in Mathematics.

Eastern Washington University chapter participated in a STEM outreach night at a local elementary school, with activities that promoted mathematics in a fun way.

The chapter at **Charles University** held a Christmas Chess Tournament, bringing together students and professors for a day of strategic battles and holiday cheer.

The chapter at **Duke University** will hold a Q&A graduate panel in 2024. Several current graduating students will share their experiences and advice on going to the next stage — how they got their next positions, both academic positions and industry/private-sector. In addition to answering questions about applying to jobs, graduates also share tips, FAQ, and experiences about all parts of their time in the duke math PhD program.

The **Florida Atlantic University** student chapter has a brand-new Reading Group this semester. The mission is to advance undergraduate and graduate student interdisciplinary collaborations across the STEM fields.



Students at **West Texas A&M University** participated in Math Bowl 2023, a math competition open to all WT students who are passionate about mathematics and who enjoy solving challenging problems.

Students from **Heidelberg University** took a chapter field trip to Zeiss SMT, a prominent company in the precision mechanics and optics industries. Zeiss Semiconductor Manufacturing Technologies (SMT) is Zeiss's business subgroup for the development and production of equipment for the semiconductor industry.

The **Sukkur IBA University** chapter hosts the Undergraduate Research Circle (URC) twice each month. Undergraduate student members of any institute meet, learn and share their current research or talk about some interesting math. URC welcomes all talks from conference practice and preparation to general interest. The discussion and talks are held in a friendly atmosphere with faculty and the visitors.

Jaypee University of Information Technology has a Distinguished Speaker Seminar Series which enables the members to gain exposure and wisdom directly from the best in academics and industry. Under this initiative renowned personalities who have achieved success in both academic and professional spheres share their wisdom and insights with the Chapter members.

The **Kyoto University** chapter put on a student poster session. The event aims to promote interdisciplinary exchange among students conducting research related to applied mathematics. Students from various fields such as physics, chemistry, biology, engineering, and information science, among others, are welcome to participate.

A two-month-long student modelling competition for students at the **University of Edinburgh** (UoE) and **Heriot-Watt University** (HWU). The aim is to call students to put their academic knowledge and experience into practice and tackle problems under real-life industrial applications. Groups of students will investigate problems and data contributed by industrial partners and try to solve them. Then, a jury would evaluate the quality and impact of the work. The groups of students with the best contributions would be asked to present their work to a panel.

The SIAM Gators at the **University of Florida** host the Applied Math Book Club (ABC). Each meeting students discuss papers and books that are of interest to the group. The aim is to increase understanding of concepts outside our areas of research.

SIAM welcomes its newest student chapters:
The Cooper Union
Queens College, CUNY
Tarleton State University
TU Munich
Oklahoma State University
Universidad de Costa Rica
Texas State University
Universidad de Rosario

The **University of Iowa** chapter organized weekly writing group sessions to encourage SIAM student chapter members and other graduate students to work on their research or assignments regularly. Students could work on their own research and encourage one another to continue.

The chapter at **University of Utah** hosted "Utah's Next Top Model" math modeling competition. This competition consists of a team of three or four undergraduates who will then create, analyze, and write a report on a mathematical model for an open-ended real-world problem over the span of a weekend. Graduate students in the department of mathematics judged the reports on merits such as practicality of the model and clarity of the report.

Nominate your students for free membership in 2024!

SIAM members (excluding student members) can nominate up to two students per year for free membership. Go to ***my.siam.org/forms/nominate.htm*** to make your nominations.

siam Fellow

The 2024 Class of SIAM Fellows will be announced March 30, 2024. Learn more at ***siam.org/Prizes-Recognition/Fellows-Program***.

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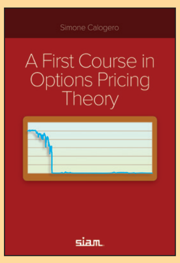
SIAM | BOOKS

Quality and Value in Mathematical Science Literature

Essential Reading in Financial Mathematics

A First Course in Options Pricing Theory

Simone Calogero

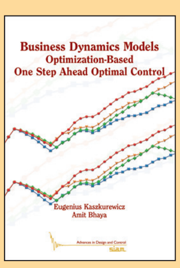


Options pricing theory utilizes a wide range of advanced mathematical concepts, making it appealing to mathematicians, and it is regularly applied at financial institutions, making it indispensable to practitioners. The emergence of artificial intelligence in the financial industry has led to further interest in mathematical finance. This book presents a self-contained introduction to options pricing theory and includes a complete discussion of the required concepts in finance and probability theory.

2023 / xii + 286 pages / Softcover / 978-1-61197-763-9
List $79.00 / SIAM Member $55.30 / OT192

Business Dynamics Models: Optimization-Based One Step Ahead Optimal Control

Eugenius Kaszkurewicz and Amit Bhaya



This book introduces optimal control methods, formulated as optimization problems, applied to business dynamics problems. It includes solutions that provide a rationale for the use of optimal control and guidelines for further investigation into more complex models, as well as formulations that can also be used in a so-called flight simulator mode to investigate different complex scenarios. The text offers a modern programming environment (Jupyter notebooks in JuMP/Julia).

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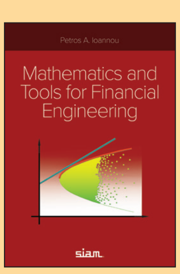
Mathematics and Tools for Financial Engineering

Petros A. Ioannou

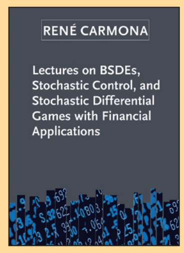


This book presents an overview of fundamental concepts in mathematics and how they are applied to basic financial engineering problems, with the goal of teaching students to use mathematics and engineering tools to understand and solve financial problems. Part I covers mathematical preliminaries and Part II addresses financial topics ranging from low- to high-risk investments.

2021 / xvi + 268 pages / Softcover / 978-1-611976-75-5
List $79.00 / SIAM Member $55.30 / OT176

Lectures on BSDEs, Stochastic Control, and Stochastic Differential Games with Financial Applications

René Carmona

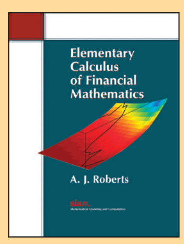


This book will be helpful to students who are interested in stochastic differential equations (forward, backward, forward-backward); the probabilistic approach to stochastic control (dynamic programming and the stochastic maximum principle); and mean field games and control of McKean–Vlasov dynamics.

2016 / x + 265 pages / Softcover / 978-1-611974-23-2
List Price $93.00 / SIAM Member Price $65.10 / FM01

Elementary Calculus of Financial Mathematics

A. J. Roberts



This book introduces the fascinating area of financial mathematics and its calculus in an accessible manner geared toward undergraduate students. Using little high-level mathematics, the author presents the basic methods for evaluating financial options and building financial simulations.

2008 / xii + 128 pages / Softcover / 978-0-898716-67-2
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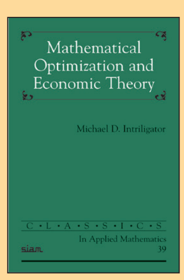
Mathematical Optimization and Economic Theory

Michael D. Intriligator

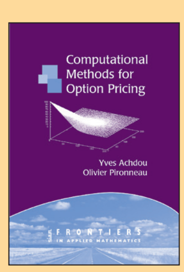


This book provides a self-contained introduction to and survey of mathematical programming and control techniques and their applications to static and dynamic problems in economics, respectively. It shows the unity of the various approaches to solving problems of constrained optimization that all stem back directly or indirectly to the method of Lagrange multipliers.

2002 / xvi + 499 pages / Softcover / 978-0-898715-11-8
List $78.00 / SIAM Member $54.60 / CL39

Computational Methods for Option Pricing

Yves Achdou and Olivier Pironneau



Here is a book for anyone who would like to become better acquainted with the modern tools of numerical analysis for several significant computational problems arising in finance. The authors review some important aspects of finance modeling involving partial differential equations and focus on numerical algorithms for the fast and accurate pricing of financial derivatives and for the calibration of parameters.

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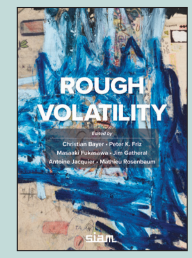
Rough Volatility

NEW!

C. Bayer, P. K. Friz, M. Fukasawa, J. Gatheral, A. Jacquier, and M. Rosenbaum, Editors



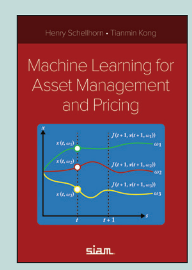
Volatility has traditionally been modeled as a semimartingale, with consequent scaling properties, but a new paradigm has emerged, whereby paths of volatility are rougher than those of semimartingales. According to this perspective, volatility behaves as a fractional Brownian motion with a small Hurst parameter. *Rough Volatility* is the first book to offer a comprehensive exploration of the subject, organizing the material to reflect the subject's development and progression. It equips readers with the tools and insights needed to delve into rough volatility models, and explores the motivation for rough volatility modeling and provides a toolbox for its computation and practical implementation.

2023 / xxviii + 263 pages / Soft / 978-1-61197-777-6
List $85.00 / SIAM Member $59.50 / FM02

Machine Learning for Asset Management and Pricing

Coming Soon

Henry Schellhorn and Tianmin Kong



This textbook covers the latest advances in machine-learning methods for asset management and asset pricing. Cutting-edge material is integrated with mainstream finance theory and statistical methods to provide a coherent narrative. Coverage includes an original machine learning method for strategic asset allocation; the no-arbitrage theory applied to a wide portfolio of assets as well as other asset management methods; and techniques other than neural networks.

2024 / xxiv + 264 / Softcover / 978-1-61197-789-9
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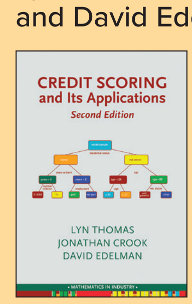
Credit Scoring and Its Applications, Second Edition

Lyn Thomas, Jonathan Crook, and David Edelman



Recognized as the bible of credit scoring, this book contains a comprehensive review of the objectives, methods, and practical implementation of credit and behavioral scoring. New to the second edition are lessons that can be learned for operations research model building from the global financial crisis, current applications of scoring, discussions on the Basel Accords, and much more.

2017 / xiv + 367 pages / Softcover / 978-1-611974-55-3
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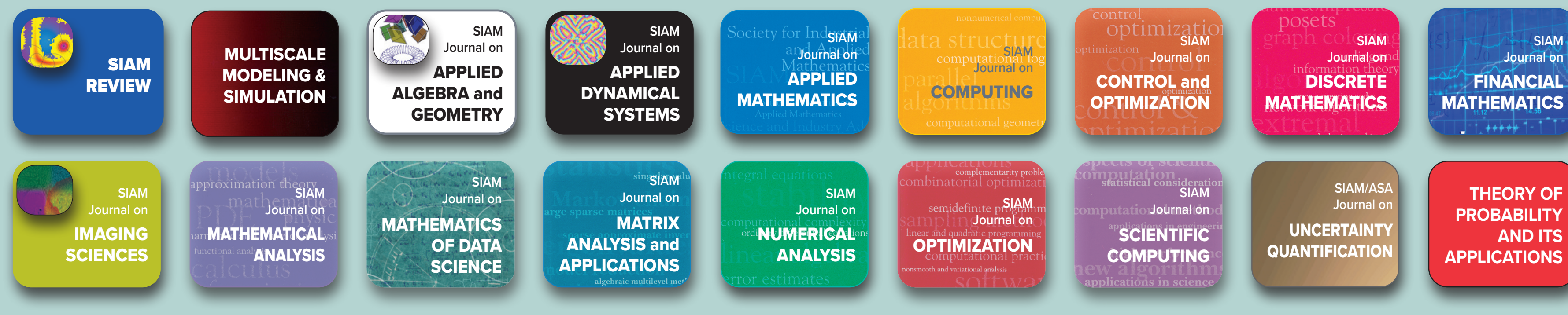

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