The SIAM Report on

Mathematics in Industry

Society for Industrial and Applied Mathematics
3600 University City Science Center
Philadelphia, Pennsylvania 19104-2688

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Executive Summary

Business, industry, and government provide not only a fertile domain for application of advanced mathematics, but also employment for a significant community of highly trained mathematical scientists. This first phase of the Mathematics in Industry (MII) study, performed by the Society for Industrial and Applied Mathematics (SIAM) with support from the National Science Foundation and the National Security Agency, seeks to

1. examine the roles of mathematics outside academia;
2. characterize the working environments of nonacademic mathematicians;
3. summarize the views of nonacademic mathematicians and their managers on the skills needed for success and the preparation provided by traditional graduate education;
4. suggest strategies for enhancing graduate education in mathematics, nonacademic career opportunities for mathematicians, and application of mathematics in nonacademic environments.

The findings of this report involve both mathematics as a discipline and mathematicians as practitioners of that discipline.

The MII steering committee, which directed and conducted much of the study, consists of seventeen applied mathematicians from industry, government, and academia. Approximately 500 mathematicians, scientists, engineers, and managers in the United States participated in the three-year MII study reported here. The findings and suggestions are derived from telephone interviews with several hundred recent advanced-degree holders (master's and Ph.D.) in mathematics working in nonacademic jobs; follow-up telephone interviews with many of their managers; and in-depth site visits by groups of steering committee members to commercial and industrial organizations and federal laboratories, chosen because they use mathematics, modeling, and computational simulation.

The study's first result is to confirm the remarkable range and variety of the applications of mathematics in industry and government. Many different success stories testify to the crucial value-added of mathematics in important real-world problems, including materials processing, automobile design, medical diagnosis, development of financial products, network management, and weather prediction. We stress that mathematics in these settings is often not labeled explicitly as "mathematics"; a final product represents a deliberately indissoluble blend of several disciplines. As expressed during one of the site visits, "Mathematics is alive and well, but living under different names".

The overwhelmingly interdisciplinary nature of nonacademic mathematics has obvious implications about the work environment for mathematicians in industry and government, as well as about qualities considered desirable by employers; these provide the focus of our second set of findings. Some of the most important traits in nonacademic mathematicians include

- skill in formulating, modeling, and solving problems from diverse and changing areas;
- interest in, knowledge of, and flexibility across applications;
- knowledge of and experience with computation;
- communication skills, spoken and written;
- adeptness at working with colleagues ("teamwork").

The qualities that distinguish these mathematicians from other scientists and engineers are seen by their managers as falling into two broad categories:

- highly developed skills in abstraction, analysis of underlying structures, and logical thinking;
- expertise with the best tools for formulating and solving problems.

Some interesting and mainly consistent views emerged about graduate education in mathematics. The mathematicians surveyed tended to agree that they were well educated for several important aspects of nonacademic jobs: thinking analytically, dealing with complexity, conceptualizing, developing models, and formulating and solving problems. However, many felt inadequately prepared to attack diverse problems from different subject areas, to use computation effectively, to communicate at a variety of levels, and to work in teams.

Based on these results, the MII steering committee offers several sets of suggestions and strategies guided by two related purposes: (1) broadening the graduate curriculum and educational programs, and (2) creating mechanisms for actively connecting academic and nonacademic mathematical scientists. These suggestions are intended not only to provide students with increased flexibility in their career choices, but also to develop a deeper understanding of real-world applications of mathematics. Some suggestions are straightforward and small-scale, while others involve cooperation among academic departments and formal affiliations with nonacademic institutions. Our objective is to present a range of strategies that can be adapted to suit particular needs and circumstances.

The topics of nonacademic employment and applications of mathematics have recently received great attention because of their relationship with two phenomena: the current crisis in the academic job market, and the perceived sharpened attention of U.S. funding agencies to work on applications. In some instances, discussion of these issues conveys grudging acceptance of unpleasant necessities that will, if all goes well, pass away; then the mathematics community can return to business as usual. The MII steering committee emphatically does not take this view. Even if the academic job market improves and funding pressure eases, we are convinced that mathematics and mathematicians should change permanently along the lines indicated in our multiplicity of suggestions. We also believe that the traits valued in nonacademic mathematicians are important and worthwhile in a far wider context.

In many areas of mathematics, history shows clearly that the flow of ideas and inspiration between mathematics and applications runs strongly in both directions. The richness of real-world applications of mathematics as well as the contributions and insights of nonacademic mathematicians should be encouraged to enhance research, teaching, and practice throughout mathematics, science, and engineering.

1. Introduction

1.1. Purposes and methodology of the SIAM Mathematics in Industry study

Business, industry, and government provide not only a fertile domain for application of advanced mathematics, but also employment for a significant community of highly trained mathematical scientists. This first phase of the SIAM Mathematics in Industry (MII) study seeks to

1. examine the roles of mathematics outside academia;
2. characterize the working environments of nonacademic mathematicians;
3. summarize the views of nonacademic mathematicians and their managers on the skills needed for success and the preparation provided by traditional graduate education;
4. suggest strategies for enhancing graduate education in mathematics, nonacademic career opportunities for mathematicians, and application of mathematics in nonacademic environments.

The findings of this MII report involve both mathematics as a discipline and mathematicians as practitioners of that discipline.

The MII steering committee, which directed and conducted much of the study, consists of seventeen applied mathematicians from industry, government, and academia; their names and affiliations are listed in the Acknowledgments section. Approximately 500 mathematicians, scientists, engineers, and managers in the United States participated in the MII project over a three-year period. The findings and guidance for the suggestions of Section 5 are derived from telephone interviews with 203 recent advanced-degree holders (master's and Ph.D.) in mathematics working in nonacademic jobs; follow-up telephone interviews with 75 of their managers; and 19 in-depth site visits by groups of steering committee members to industrial and governmental organizations.

The telephone survey of mathematicians covered only those with highest degrees from departments formally labeled as "mathematics"; this includes "applied mathematics" and "mathematical sciences" but not "statistics", "operations research", or "computer science", although we certainly encountered many individuals who were trained in statistics and operations research within mathematics departments. Our definition of "nonacademic" institutions includes government laboratories (some of which are managed by universities on behalf of government agencies) and business or industrial organizations; we do not include academic research institutions.

A major aim throughout the telephone surveys was to gather quantitative data about working environments, important skills, and the value of graduate training for nonacademic mathematicians. The site visits provided impressions, anecdotes, and extended interchanges about the nature of applied mathematics and what it means to be a nonacademic mathematician.

1.2. Context for the SIAM MII report

The present report does not arise in a vacuum; its themes have been explored in myriad forms and contexts, and are especially timely because of a recent confluence of trends and events.

Within the mathematical sciences, the past five years have seen a crescendo of articles devoted to the prospects for nonacademic careers, driven by a substantial mismatch between the number of new Ph.D.'s and the number of academic jobs in mathematics. See, for example, [Lot95, McCl95].

Addressing the same phenomenon in a broader setting, the National Research Council (NRC) Committee on Science, Engineering, and Public Policy (COSEPUP) produced a widely discussed report, *Reshaping the Graduate Education of Scientists and Engineers* [NRC-Grad], in April 1995. Part of the impetus for that report was a growing impression that, throughout science and engineering, large numbers of U.S. Ph.D.'s cannot find jobs, especially in academia. The COSEPUP report explores two issues of particular relevance to the MII study: the nature of industrial employment and the resulting implications for graduate education.

Of course, these issues are not recent discoveries. For more than a decade, substantial and thoughtful studies have been written about U.S. mathematics education; see, for example, [CBMS92, David84, David90, NRC90, NRC-Doc]. Perspectives on nonacademic mathematics and the preparation required for nonacademic jobs have been considered in, for example, [Ben94, BKTSLD, Boy75, Ch91, Davis91, Fry41, Ster95, Weyl52]. Several authors from outside mathematics have also discussed the nature of industrial jobs in other disciplines and connections with graduate education; see, for example, [Hans91, Hold92, Horn92].

This MII report complements and extends the COSEPUP and other reports by concentrating in detail on applications of mathematics in industry and on the working environment for nonacademic mathematicians. It is widely perceived that graduate education in mathematics focuses almost exclusively on preparation for traditional academic research careers. Until now, however, reports have not systematically examined perceptions of industrial environments by mathematicians and their managers, nor asked for ratings by nonacademic mathematicians of their graduate education.

1.3. Possible audiences

The steering committee believes that this report may be of interest to multiple audiences for various reasons.

1. **Mathematical sciences departments.** The report contains information about nonacademic applications of mathematics and future opportunities for mathematics; a detailed characterization of traits valued in nonacademic mathematicians; an analysis of how well graduate education prepares students for nonacademic careers; ideas for broadening the graduate curriculum to provide students with greater flexibility in career choices as well as a deeper understanding of real-world applications of mathematics; and suggestions for faculty and departments to help build closer ties to industry.

2. **Deans and university officials.** Implicit in the report are policies and strategies that might be useful if universities wish to encourage shifts in curriculum or closer ties to industry.

3. **Students in mathematics and related disciplines.** A picture emerges from the report of careers in industrial mathematics, along with guidelines about academic preparation. We also suggest actions for several kinds of students: those interested in applications; those who wish to consider the option of a nonacademic career; and those who wish to develop connections outside academia.

4. **Industrial and governmental organizations who use or could use mathematics.** The success stories described in the report indicate the many ways, some unexpected, in which mathematics can be applied to produce concrete and measurable results. The managers surveyed, most of whom were not mathematicians, consistently felt that mathematics could provide a competitive edge for their organizations. We hope that this report suggests new and evolving roles for mathematics in industry.

5. **Federal and private agencies concerned with educational preparation.** The report presents data about mathematical careers and graduate education, and suggests possible strategies to broaden mathematics curricula and programs.

6. **Academic departments in disciplines where applied mathematics is important.** Some of the findings clearly reveal close connections, ripe for expansion, between mathematics and other disciplines. The portrait of the industrial environment for mathematicians is likely to contain many points of similarity for graduates in other disciplines, and our suggestions indicate generic approaches to issues of concern throughout science and engineering—in particular, developing interdisciplinary programs, teaching communication skills, and creating links with industry.

1.4. Outline of the report

Section 2 describes the roles of nonacademic mathematics in general and specific terms, including a list of success stories. The working environment for nonacademic mathematicians is discussed in Section 3, and a detailed analysis is given of the traits valued in nonacademic settings. Section 4 summarizes the views of nonacademic mathematicians and their managers about graduate education as preparation for nonacademic jobs. Based on these findings and their own experiences, the steering committee offers a variety of strategies and suggestions in Section 5, followed by a brief conclusion in Section 6.

Within this report, we sometimes use the term "industry" to denote business and commercial firms, federal research and development laboratories, and commercial and not-for-profit research, development, and production facilities, i.e., activities outside the realm of education and academic research. It should always be clear from context when "industrial" refers specifically to industry.

2. The Roles of Mathematics

2.1. Mathematics within nonacademic organizations

Mathematics appears in industry and government in a remarkable variety of forms, only some of which are clearly labeled as mathematics. To convey this diversity, we note first that during site visits, the MII steering committee visited groups working on

- research in and development of mathematical tools and algorithms;
- creation and support of mathematical and computational techniques associated with a specific product or service (e.g., computational fluid dynamics in aerodynamic design, stochastic partial differential equations in financial studies);
- consulting or modeling for internal or external customers; and
- products, processes, services, or research in which mathematics plays a useful but secondary role.

The 203 mathematicians (102 master's and 101 doctoral graduates from 1988–1992) and 75 managers who participated in the telephone surveys represent a reasonably broad spectrum of nonacademic organizations. (See the Appendix for more information about the survey sample.) Table 1 shows the distribution of graduates surveyed in five major sectors of industry, based on the Standard Industry Classification codes of the United States Office of Management and Budget.

Nonacademic sector	**Ph.D.**	**Master's**
Government	28%	22%
Engineering research, computer services, software	19%	18%
Electronic, computers, aerospace, transportation equipment	17%	12%
Services (financial, communications, transportation)	13%	22%
Chemical, pharmaceutical, petroleum-related	6%	2%

Table 1: Distribution across five major industry sectors of participants in telephone survey of graduates, broken down by terminal degree.

Mathematicians and their managers were asked in the telephone survey about the status of advanced mathematics in their overall organizations, where "advanced" means at the level of the respondent's highest degree. Those responses are summarized in Table 2 and show the consistent importance of mathematics not only for its practitioners, but also for their managers.

Importance of advanced mathematics	**Ph.D.**	**Master's**	**Managers**
Primary	43%	28%	51%
Secondary	43%	40%	37%
Only for general utility	11%	32%	12%

Table 2: Average perceived importance of mathematics in respondents' overall organizations.

The managers interviewed by telephone offered a range of descriptions of the role of mathematics in their groups. Nearly half (49%) characterized mathematics as an underlying requirement or tool for their groups' work. Three main functional roles for mathematics were mentioned by managers: development of algorithms and numerical methods (27%); modeling and simulation (23%); and statistical analysis (15%).

Table 3 shows the diverse educational backgrounds of the managers who participated in the telephone survey, and prompts two immediate observations: the managers' favorable perception of the importance of mathematics does not arise because they are predominately mathematicians; and mathematicians often report to, and hence must communicate effectively with, nonmathematicians.

Areas of managers' degrees	**Ph.D.**	**Master's**
Mathematics	16%	11%
Engineering	13%	6%
Physics	13%	3%
Statistics/biostatistics	9%	5%
Business/management	0%	11%
Computer science	0%	6%
Chemistry/biology	0%	3%

Table 3: Percentages of managers' graduate degrees in various areas.

2.2. Applications of mathematics

The site visits, telephone surveys, and experiences of steering committee members in industry build a picture in which mathematics participates in many ways in the overall enterprise of industrial and government organizations. Table 4 indicates selected associations between areas of mathematics and applications encountered in the site visits.

Mathematical Area	**Application**
Algebra and number theory	Cryptography
Computational fluid dynamics	Aircraft and automobile design
Differential equations	Aerodynamics, porous media, finance
Discrete mathematics	Communication and information security
Formal systems and logic	Computer security, verification
Geometry	Computer-aided engineering and design
Nonlinear control	Operation of mechanical and electrical systems
Numerical analysis	Essentially all applications
Optimization	Asset allocation, shape and system design
Parallel algorithms	Weather modeling and prediction, crash simulation
Statistics	Design of experiments, analysis of large data sets
Stochastic processes	Signal analysis

Table 4: Mathematical areas and industrial applications encountered during site visits.

Mathematics is a key player in numerous success stories heard during site visits. Common themes are the technical advantages and cost savings that accrue from clever modeling, analysis, and computation by mathematicians working with other professionals. The mathematician's logical, problem-solving approach is widely seen to provide a noticeable competitive edge.

Highlights of a few of those success stories are summarized next, with company names removed and certain proprietary details omitted. (Throughout this report, displayed quotations are taken from site visit reports or focus group discussions.)

> *Beginning in the mid-1970s, a chemical manufacturer began developing models of atmospheric reactions and transport. A team of mathematicians and atmospheric physicists used state-of-the-art techniques for stiff ordinary differential equations that allowed integration to a dynamic steady state that no one else could achieve. This advance provided the manufacturer with scientific credibility and a voice in the debate with regulatory agencies. Management developed sufficient confidence in the modeling results that it broke ranks with its industrial colleagues and became the first to cease manufacture of the products shown to be harmful to the environment.*
>
> *A manufacturer of large industrial equipment developed a software system that provides a functional representation of surfaces so that "design data can be quickly moved from computer-aided design to numerically controlled machining and prototype production", thus cutting the cost of design by shortening the prototype design cycle time.*
>
> *Safety testing of its product is a critical issue for one transportation manufacturer, which routinely uses nonlinear finite element models and large-scale computing to replace a "million-dollar prototype with a ten-thousand-dollar computer run".*
>
> *One consulting organization contracted with a paper manufacturer to develop a scheduling system for paper production. The initial stages of this contract involved mathematical modeling of the production process, which eventually led to a turn-key system with a sophisticated user interface. The initial application of the modeling-based production system produced a 4% increase in revenue for the paper company, resulting in 6 million dollars per year in increased profit.*
>
> *Device simulation is important to the semiconductor industry because it is very expensive to design and prototype next-generation devices. One chip manufacturer has been so successful with simulation and modeling that "we wouldn't build a chip without modeling it first".*
>
> *Rising production costs threatened the profitability of one company's key product. Developing a process optimization methodology cut manufacturing costs so much that the product remained competitive and the company stayed financially viable.*

Nearly every manager interviewed by telephone cited a particular combination of application and mathematics in which mathematics had made a significant contribution; in fact, 13% agreed that "We couldn't have done it without a mathematician". The following list gives a subset of the cited applications:

- wavelets in the analysis of brain processes;
- Brownian motion algebra in modeling "limit" orders for financial products;
- representation and manipulation of complex geometry in computer-aided design of aircraft;
- analysis and modeling in turbulence studies and global warming;

- a numerical method for quantifying ultrasonic Doppler readings to allow analysis of regurgitative flow in heart valves, spinal anesthetic fluid migration, and thermal increases in a growing fetus;
- modeling of satellites and algorithms for centimeter-accurate orbit determination;
- matrix algebra applied to optimize managed portfolios and determine an "accuracy quotient";
- algorithms for classification of blocking and its costs in a railroad system.

The mathematical functions of greatest value in these and other successful applications were characterized by managers as

modeling and simulation;
mathematical formulation of problems;
algorithm and software development;
problem-solving;
statistical analysis;
verifying correctness;
analysis of accuracy and reliability.

Further success stories involving combinations of mathematics with industry, materials, and chemistry are presented in the NRC reports [NRC-Tech], [NRC-Mat], and [NRC-Chem].

Despite such favorable results, mathematics is often invisible outside the technical work group because its role in a successful project is not highlighted or publicized, especially to higher management within the organization. In some instances, mathematicians and managers commented that higher management was not interested in or would not understand the mathematical details. Others suggested that managers could not be expected to appreciate the contributions of all the disciplines reporting to them. In any case, the word "mathematics" is often disguised, or mathematics is described in nonmathematical terms. For example, one mathematician commented, "We never present anything to management below the level of modeling and simulation".

The contributions of mathematics as a separate, disjoint discipline are also difficult to discern because scientists and engineers in nonacademic environments necessarily join together to produce a single result. As we shall see in Section 3, industrial mathematicians tend to work in groups not entirely devoted to mathematics, and to collaborate with scientists and engineers from other disciplines. Thus, although mathematics is often a basic and crucial ingredient in industrial products and decisions, its role as such may not be explicitly recognized or understood. As expressed during one of the site visits, "Mathematics is alive and well, but living under different names".

2.3. Opportunities for mathematics

In telephone interviews and site visits, we heard many views about opportunities for new applications of mathematics. A selection of these, grouped by business area, is listed here.

Manufacturing:
Dimensional tolerancing, digital preassembly, and nominal components
Modeling of manufacturing systems, reactive ion etching, and thermal processes
Pattern placement and throughput in electron beam technology
Process optimization (reducing time to market)

Product design:
Shape optimization
Simulation of functionality

Materials:
- Predicting damage and degradation of polymers
- Nondestructive testing
- Simulation of material properties

Environmental management:
- Modeling to guide decisions about hazardous products or processes

Information science:
- Bio-informatics (optimization, neural networks, Markov models, dynamical systems).

Recent graduates interviewed by telephone indicated that they saw substantial new opportunities for mathematics in industry and government. Computing, electronics, and software were listed by 32% of the Ph.D.'s, financial analysis by 30%, engineering by 28%, and operations research by 20%. The master's graduates selected the same categories in the top four with different frequencies. Only 11% of the Ph.D.'s and 6% of the master's graduates thought that opportunities for mathematics were very limited.

The managers surveyed had a similar outlook: 59% thought that there are definitely or probably opportunities in their own organizations for increased contributions from mathematics. (Those seeking such opportunities will find suggestions in articles like [Ben94, Davis94, Davis95].) Only 17% of managers thought that there were definitely or probably no additional opportunities for mathematics in their organizations.

In the next section, we show how the nature of industrial mathematics has obvious and immediate implications for those who practice it.

3. The Working Environment

The applications of nonacademic mathematics discussed in Section 2 arise from a mixture of fields tied to the missions of industrial and governmental organizations. We now describe how the nature of nonacademic mathematics shapes and defines the working environment for its practitioners—industrial mathematicians. Successful industrial mathematicians contribute to their organization's mission, are interested in working on new areas of application, possess both breadth and depth in mathematics, have good interpersonal skills, and are adept at computation.

3.1. Mathematicians as part of their organization

The value of mathematicians to a nonacademic institution depends on their contributions to the institution's mission.

> *Mathematicians are part of the infrastructure; mathematics cannot be viewed as an end in itself.*

> *Managers evaluate their people by what they contribute to the company.*

Within nonacademic organizations, mathematicians frequently work in groups whose primary missions include, but are not limited to, mathematics. The five major work group missions mentioned by the surveyed mathematicians are indicated in Table 5.

Mission	**Ph.D.**	**Master's**
Mathematical specialty (such as modeling)	40%	24%
Computing, computer services, software	35%	42%
Research, research and development	23%	11%
Consulting	9%	6%
Engineering, risk analysis	8%	9%

Table 5: Work group missions of mathematicians surveyed.

A small number of nonacademic mathematicians—most in government laboratories, some in large corporate laboratories—spend part, or occasionally all, of their time performing basic mathematical research similar to academic research. But even groups with a research charter are increasingly called upon to make a business case for their work.

> *Research often has a serious difficulty: too much understanding and too little transfer. It needs examples of success to justify continued support by the business part of the company.*

Mathematicians in industry and government are almost always part of an interdisciplinary group. One site visit participant commented,

> *Although a few mathematicians are clustered in one group that has a mathematical charter, most are scattered among engineers, physicists, and computer scientists, where they often function as "hunter-gatherers", seeking a share of their support from mission-oriented project groups. Mathematicians here must extract the mathematics from the projects that need it.*

The data in Table 6 illustrate the interdisciplinary character of the groups in which industrial mathematicians work. Both Tables 6 and 7 show that industrial mathematicians seldom hold a majority of the positions in their immediate work groups. Mathematicians also blend in with their colleagues because their titles rarely reflect the presence of mathematics in their jobs; only 20% of the graduates interviewed by telephone hold positions with mathematical titles. In addition, many mathematicians hold positions that do not require a degree in mathematics and hence could be filled by graduates of another discipline. Among Ph.D.'s surveyed by telephone, only 31% stated that an advanced degree in mathematics was required for their position; among master's graduates, the analogous figure was 14%. Once in an industrial position, many mathematicians find that meeting the demands of their organizations' missions takes precedence over their disciplinary identities.

Discipline	Ph.D.	Master's
Mathematics	25%	16%
Computer Science	27%	24%
Engineering	23%	15%
Physical Sciences	8%	2%

Table 6: Average percentages of major disciplines in work groups of mathematicians surveyed.

Industry sector	Ph.D.		Master's	
Government	6.6	37%	5.3	10%
Services (financial, communications, transportation)	4.1	49%	5.5	38%
Chemical, pharmaceutical, petroleum-related	4.0	25%	1.5	50%
Engineering research, computer services, software	4.2	15%	2.0	16%
Electronic, computers, aerospace, transportation equipment	2.6	15%	2.0	24%
Other	2.7	18%	2.5	13%

Table 7: Average numbers and percentages of mathematicians in work group, by industry sector.

An immediate consequence of these findings is that mathematicians seeking nonacademic positions will almost certainly be competing with advanced-degree holders in engineering, computer science, or the physical sciences. Because nonacademic slots are rarely reserved for "mathematics", it is essential for such mathematicians to possess background and skills that enhance their value. Sections 3.2–3.6 provide details about qualities considered beneficial for industrial mathematicians.

3.2. Depth and breadth in mathematics

Not surprisingly, wide variations occur in mathematical specialization within nonacademic working environments. Mathematicians working in large departments with a specific mathematical function are almost by definition required to be experts in that area. Similarly, mathematicians' work is highly specialized if the institution's core business is closely linked to a mathematical area like computational fluid dynamics.

For a few Ph.D.'s, their area of expertise in industry remains that of their dissertation. However, managers often view completion of a Ph.D. dissertation as evidence of ability to exert sustained effort to solve a difficult problem rather than as training in a particular specialty that will occupy a professional lifetime.

If mathematicians are functioning largely as consultants or if the demands of the organization's mission lead to shifts in technical requirements, it may be impossible or professionally undesirable for a mathematician to work on only a single specialty. Many of the mathematicians interviewed indicated that, soon after starting work, they were shifted to projects much different in mathematical content from those for which they were originally hired. Several speakers at the 1994 SIAM Forum [Davis94] cited instances in which responsibilities were changed not only by management, but also by the employees themselves, to improve career prospects or job security.

Telephone survey responses regarding needed mathematical specialties are summarized in Table 8; also see Table 9. It is evident that many of these positions require a variety of mathematical knowledge.

Mathematical specialty	Ph.D.	Master's
Modeling and simulation	73%	68%
Numerical methods/analysis	65%	47%
Statistics	55%	61%
Probability	50%	55%
Engineering analysis/differential equations	50%	28%
Operations research/optimization	38%	42%
Discrete mathematics	26%	24%

Table 8: Percentages of mathematicians surveyed who mentioned mathematical specialties as a primary technical requirement of their positions; multiple mentions were permitted.

Possession of broad mathematical skills is valued in nonacademic settings because technical problems driven by business needs can change rapidly, unpredictably, and dramatically, and because the work of some groups cuts across many fields.

> *We never know what kind of mathematics is the right kind, so an "algebraist for life" is not the right kind of mathematician.*

Furthermore, regardless of their areas of expertise, mathematicians may be seen by colleagues and managers as a resource to answer general mathematical questions. One industrial mathematician noted, "If you are a mathematician, everyone expects you to know statistics and operations research".

As seen in Section 2, the problems confronting industrial mathematicians arise from the needs of their institution, and cannot be chosen to fit a predetermined palette of mathematical tools. Hence breadth and depth are both important.

> *You can't isolate yourself in just one area. Even a simple project has many aspects. But you do need to be an expert in one area.*

3.3. Interest in and knowledge of other areas

Because of both the interdisciplinary and varied natures of their technical problems, nonacademic employers strongly prefer mathematicians with an interest in applications.

> *Mathematicians working here must like to apply real solutions in the real world.*

> *What kind of mathematicians are attractive? Mathematicians with the right mind set who want to solve problems rather than just do mathematics.*

Knowledge of technical areas outside mathematics is regarded as extremely helpful in nonacademic positions. Only one of the 203 recent graduates interviewed worked in a position for which a knowledge of mathematics was the sole requirement.

> *A metaphor for success that we heard from more than one manager was the letter T, which meant that a successful mathematician must have depth in an area of specialization but at the same time develop a broad understanding of technical and business issues in the company.*

The balance between depth and breadth as well as the need for a genuine interest in applications are issues outside mathematics as well; see, for example, [Hans91, NRC-Grad, Natr89].

Table 9 lists the percentages of mathematicians surveyed who mentioned other disciplines as crucial in their jobs. These data demonstrate the importance of interdisciplinary knowledge.

Discipline	**Ph.D.**	**Master's**
Computer Science	69%	88%
Physics	32%	28%
Electrical Engineering	29%	19%
Mechanical Engineering	19%	10%
Chemistry	6%	10%
Biology	9%	6%
Materials Engineering	5%	3%
Chemical Engineering	5%	2%
Civil Engineering	3%	2%

Table 9: Percentage of mathematicians surveyed who mentioned other disciplines as a primary technical requirement of their jobs; multiple mentions were permitted.

It is clear from Table 9 that the most significant second discipline is computer science. Table 10 presents responses to a telephone survey question about computation; its perceived importance by all groups is unmistakable and striking. One manager commented during a site visit, "It's hard to envision a pencil-and-paper mathematician here".

Role of advanced computation	**Ph.D.**	**Master's**	**Managers**
Essential	54%	31%	65%
Very important	28%	24%	19%
Somewhat important	12%	15%	8%
Not particularly important	7%	31%	8%

Table 10: Perceived importance of advanced computation.

The most valued computational skills obviously depend on the context and cannot be prescribed in advance. Often, however, software embodies a mathematician's contribution to a problem, and it may be impossible or undesirable to delegate implementation to others. Thus expertise in both programming and numerical analysis is essential. As one manager noted, "Unless mathematics is put into software, it will never be used". Computing is also central in managing and analyzing large sets of data, connecting and coordinating all pieces of a problem, and displaying results graphically in a meaningful, compact form.

In groups where computing is important, on average the Ph.D.'s estimate that they spend nearly half their time (44%) on computational tasks, with 47% of those tasks requiring advanced mathematics. The master's graduates estimate that they spend on average 33% of their time computing, with 26% of those tasks requiring advanced mathematics.

Comments from nonacademic mathematicians and other sources (see, for example, [Hold92]) suggest that, in addition to basic sciences and engineering, business and finance are increasingly important disciplines for analysis of technical questions about markets, pricing, and related issues.

3.4. Formulating problems and finding solutions

In the life of a nonacademic mathematician, two themes not traditionally associated with core academic mathematics emerged clearly from our surveys, site visits, and discussions:

- problem formulation as an interactive and continuing process;
- collaboration and communication on several levels.

The first of these themes was emphasized repeatedly during our site visits. Industrial problems are almost never stated in mathematical form when first presented to a nonacademic mathematician; and even if they are posed initially in mathematical terms, alternative formulations may eventually turn out to be preferable. Consequently, successful nonacademic work demands the ability to understand problems couched in terminology from another field, and to discern and analyze the important underlying mathematical structures and questions.

> *The hardest task for a mathematician is developing the real problem requirements. The user doesn't usually know what the solution will look like in the end.*

> *Sometimes customers recognize the problems. In other cases, all they can do is express their frustration and you must figure out the problem.*

> *Problems never come in formulated as mathematics problems. A mathematician's biggest contribution to a team is often an ability to state the right question.*

For a nonacademic mathematician, "solving" a problem usually does not mean a tidy theorem or counterexample, or even one-shot numerical results. Industrial problems typically evolve over time, as inadequacies in the original model are revealed or data and assumptions become more precise. Once a mathematician successfully obtains a theoretical or numerical solution to an initial, possibly simplified problem, he or she is frequently asked to analyze and solve an extended, sometimes fundamentally different, problem. In other cases, mathematicians can show that a particular mathematical formulation is flawed, but this does not dispose of the original, larger problem that remains to be solved. Arriving at the best formulation of a problem—realistic, yet mathematically reasonable—is an inherently interactive and complicated process.

> *It is essentially never the case that someone comes in and says "Here is an equation; please solve it", and then that's the end of the story. The mathematics presented in the first discussion is usually the tip of the iceberg.*

In addition to dealing with shifting problem formulations, industrial mathematicians are expected to provide "answers" even when no rigorous solution can be found. Timely, useful results, albeit incomplete, are often of critical importance, especially during the process of problem formulation. In some instances, it is more productive to expose quickly a potentially defective formulation than to work out a lengthy complete solution.

You may be ahead if you find only 80% of the solution if this takes 20% of the work required to find the complete solution.

Most problems must be "solved" in hours or days; this often means finding an adequate solution rather than a perfect one.

Industrial mathematicians are almost always asked to find the best solution under time and budget constraints.

3.5. Communication and teamwork

Sections 3.1–3.4 have stressed that mathematicians in industry are not solving problems solely for themselves and other mathematicians: they must actively collaborate with colleagues and managers who are not mathematicians, and they must be able to justify the value of their work to their organization.

It is almost never possible for a nonacademic mathematician to work in isolation and communicate only with specialists in a narrow area. Successful industrial mathematicians accordingly require a high degree of communication skills in several forms—speaking, writing, and listening—and at several levels. Technical and business interactions often continue over a long period of time, so that clear exchanges of information and ideas are crucial.

You can't just toss the results over the fence.

It is essential that you follow up on the problem; check whether the customer has implemented your solution, whether it worked out, whether further tuning is necessary.

The most effective people are those who can interact, understand, translate. A key is being able to explain something outside your discipline.

The importance of communication skills is emphasized equally strongly in other, more general studies such as [IRI91, NRC-Grad, Natr89].

During our site visits, we heard frequently that industrial mathematicians must possess a closely related quality: the ability to work effectively in close collaboration with diverse groups.

Lack of interpersonal and team skills is the primary cause of failure in industry.

An example of failure was a mathematician who would talk only to other mathematicians, and whose attitude was "Just tell me what the problem is and I will solve it".

Recent studies have highlighted a similar need for teamwork among Ph.D.'s of all varieties employed outside academia; see, for example, [IRI91, NRC-Grad]. A closely related issue is the need for an attitude of flexibility and a willingness to use different techniques and work in new areas.

3.6. How mathematicians are viewed

As a final element in our examination of the working environment for industrial mathematicians, we asked site visit participants and managers in the telephone survey about two issues: Why are mathematicians valued? What are their perceived strong and weak points? There was a high degree of consistency in the answers to these questions, particularly to the first.

Numerous site visit participants articulated, sometimes in almost identical words, two main reasons that industrial mathematicians are valued:

- highly developed skills in abstraction, analysis of underlying structures, and logical thinking;

- expertise with the best tools for formulating and solving problems.

As indicated in Sections 2 and 3.4, problems in industrial mathematics can arise from anywhere, most often in poorly defined and evolving forms. Mathematicians are valued because they can see and understand the inner nature of a problem; determine which features matter and which do not; and develop a mathematical representation that conveys the essence of the problem and can be solved numerically.

> *Powerful—even pure—mathematicians are better equipped to keep going when textbooks have to be left behind.*
>
> *Mathematicians do not always know the answers, but they know the right questions to ask and they know when the questions being asked are wrong.*
>
> *Mathematicians are better equipped than others in coming up with the correct definitions of problems and developing the right level of abstraction.*
>
> *Mathematicians have an ability to deal with abstraction, uncoupled from specific technology and involving many subsystems; to develop models for the abstract systems; to use a common language (mathematics) to communicate the results; and to apply well-developed skills to spot hidden gaps and identify connections.*

The key idea is not that mathematicians are ignorant of details, but that their training equips them to deal with problems at an abstract, system-wide level, independently of commitments to a particular approach or technology.

An example of these abilities was described during a site visit: mathematicians were modeling a production plant running below design capacity and began asking for data about aspects of the plant operation. Their systematic questions highlighted the root of the problem before a model was even assembled. Details that we heard of the stories sketched in Section 2.2 illustrate the crucial "edge" provided by mathematical insights and techniques.

A distinction between Ph.D.'s and master's graduates frequently mentioned during site visits was that master's graduates are willing to "look under the hood"; that they are more flexible, especially with an undergraduate degree in a second discipline; and that they are willing to approach any problem. On the other hand, a Ph.D. is seen as bringing a deeper understanding of how to solve difficult problems.

Managers interviewed by telephone were asked about the reasons for hiring mathematicians. The most frequent specific answers given for hiring Ph.D.'s and master's graduates are grouped and summarized next, with percentage of respondents shown in parentheses.

Why hire mathematics Ph.D.'s?
- Analytical and problem-solving skills (58%)
- Conceptual breadth, transitions across disciplines (28%)
- Skill with numerical algorithms (14%)

What makes mathematics Ph.D.'s special?
- Logical, sophisticated thinking (36%)
- Understanding of algorithms, techniques, and theory (27%)
- Higher level of mathematics training (24%)
- Advanced, abstract problem-solving (13%)

Why hire mathematics master's graduates?
- Logical and analytical skills (43%)
- High level of mathematical training (35%)
- Knowledge of techniques and algorithms (22%)

What makes mathematics master's graduates special?
Linking theory with applications (25%)
Knowledge of algorithms and mathematical techniques (20%)
Mathematical skills and expertise (20%)
Logical, analytical skills (18%)
Computational skills (17%)

We also asked managers about what they believed to be shortcomings or limitations of mathematicians. The responses confirmed certain common images—in some instances, full-blown negative stereotypes—of mathematicians. According to site visit participants, these views are based mainly on their encounters with mathematicians who did not understand a nonacademic environment.

A weakness of a mathematician is tunnel vision: write a paper and that's the solution.

Some managers feel that mathematicians in general have a bad image; they don't care about the real environment—realistic models, cost, implementation. They are concerned instead with proving irrelevant theorems.

It is important for mathematicians to learn that they can't continue their investigations forever. They have to learn to say "enough" in the available time.

Mathematicians are sometimes viewed as being unwilling to become involved with an organization's real business issues, perhaps because, one stereotype suggests, those problems are not sufficiently elegant or interesting mathematically to warrant attention.

Managers in the telephone survey were asked to name areas in which mathematicians could improve; the most frequent responses are shown with the percentage of respondents in parentheses.

In what areas could industrial mathematicians be improved?
Understanding of and interest in practical applications (41%)
Communication skills, interaction with others (36%)
Breadth of knowledge of other areas (23%)

In the context of describing opportunities for mathematicians, several site visit participants commented that mathematicians sometimes do not make the best possible case for either their discipline or themselves. For example, one manager observed that mathematicians who seek or already occupy nonacademic positions do not often play to what she sees as their strongest point: their background provides the ability to "see into application areas" and thus be major contributors in strongly interdisciplinary work. A manager at a different site urged mathematicians to "take advantage of the interdisciplinary nature of mathematics. Exploit it or lose". And a third manager was genuinely perplexed at the "apparent unwillingness of mathematicians to assume their rightful role in the science landscape".

4. Perceptions of Graduate Education

Several recent reports (for example, [NRC-Grad]) have suggested that U.S. graduate education is too focused on producing faculty "clones"—professors at research universities—and hence prepares students poorly for nonacademic careers. One of the aims of the SIAM MII study was to determine the views of industrial mathematicians about how well their graduate education prepared them for their jobs. The findings of our study show an interesting mixture of opinions. Certain aspects of graduate education are almost universally praised, whereas others are widely regarded as needing improvement.

As shown in Table 11, industrial mathematicians interviewed by telephone mostly believed that their graduate education had helped them to obtain and perform well in their present positions.

Effectiveness of mathematics graduate education	**Ph.D.**	**Master's**
Obtaining a highly desirable position		
Extremely effective	58%	57%
Somewhat effective	25%	23%
Not particularly effective	17%	20%
Helping in your work		
Extremely effective	59%	38%
Somewhat effective	32%	41%
Not particularly effective	9%	21%
Providing a superior career path		
Extremely effective	27%	18%
Somewhat effective	51%	58%
Not particularly effective	22%	24%

Table 11: Ratings of effectiveness of graduate education in mathematics.

The mathematicians surveyed were asked to rate how well their graduate education had provided them with specific skills. There were four categories for educational preparation: "excellent", "very good", "good", and "less than good".

Skills for which preparation was rated as "excellent" or "very good" by more than half the Ph.D.'s are given in Table 12. Except for "creating new ideas or innovative approaches", educational preparation in these qualities was also highly rated by the master's graduates.

Graduate education is perceived by a substantial majority of these mathematicians as having prepared them very well for thinking analytically, dealing with complexity, and conceptualizing. Smaller proportions of the respondents, but still a majority of Ph.D.'s, felt very well prepared in formulating problems, modeling, and creating new ideas. As discussed in Sections 3.2, 3.4, and 3.6, much of the value of industrial mathematicians depends on these skills. Hence graduate education in mathematics is highly successful in preparing students for many key elements of nonacademic jobs.

Table 13 lists the qualities for which more than 30% of the Ph.D. respondents rated their educational preparation as "less than good": working with colleagues; communicating; having broad scientific knowledge; using computer software and systems; and dealing with varied problems.

Skills very well developed by graduate education	Ph.D.	Master's
Thinking logically; dealing with complexity	87%	92%
Problem-solving	79%	84%
Conceptualizing and abstracting	76%	84%
Formulating problems; modeling	65%	72%
Creating new ideas or innovative approaches	60%	41%

Table 12: Skills for which respondents' graduate education was rated "excellent" or "very good".

Skills for which preparation was "less than good"	Ph.D.	Master's
Working well with colleagues	67%	60%
Communicating at different levels	58%	48%
Having broad scientific knowledge	47%	37%
Effectively using computer software and systems	45%	42%
Dealing with a wide variety of problems	35%	29%

Table 13: Skills for which respondents' graduate education in mathematics was rated "less than good".

This set of weaknesses in graduate training is significant because Sections 3.1 and 3.3–3.5 show that these skills, in addition to those shown in Table 12, are crucial for success in nonacademic careers. The high proportions of both Ph.D.'s and master's graduates who felt inadequately educated in working well with colleagues and in communicating at different levels are notable because of the special importance of these skills in interdisciplinary environments; see Section 3.5. It is also interesting that nearly half the Ph.D.'s felt less than well educated in knowledge of other scientific fields and in use of computer software and systems, in spite of the importance of these attributes.

Each manager in the telephone survey was asked to indicate areas in which graduate education in mathematics needs improvement; the six leading replies are shown in Table 14.

Area for improvement in graduate mathematics education	Percentage of managers
Applications of mathematics	40%
Knowledge of other disciplines	23%
Real-world problem-solving	21%
Communication; technical writing	19%
Computer skills	13%
Teamwork	8%

Table 14: Areas in graduate education suggested by managers as needing improvement. More than one area could be named by each manager.

Mathematicians in the telephone survey were asked about the importance that they attach to changing graduate education. Table 15 indicates that a majority of both Ph.D.'s and master's graduates strongly support changes in mathematics graduate education. Only a small proportion of Ph.D.'s or master's graduates believe that change is not important.

Importance of educational change	**Ph.D.**	**Master's**
Extremely or very important	60%	69%
Somewhat important	34%	26%
Not important	6%	5%

Table 15: Importance of changes in graduate mathematics education.

To summarize, our inquiries indicate that graduate education in mathematics is seen by recent graduates working in industry as excellent in certain areas, but needing improvement in others. More than 90% of these mathematicians believe that changes in graduate education are important. Furthermore, the areas for improvement highlighted by both the mathematicians and their managers (shown in Tables 13 and 14) overlap significantly with one another and with attributes of successful industrial mathematicians. Based on these results, Section 5 presents our suggestions for strategies to enhance and expand graduate education in mathematics to strengthen these areas.

5. Suggestions and Strategies

The suggestions that follow are made by the MII steering committee, based on detailed comments from telephone interviews and site visits and on our own experiences. Our intent is to embody guiding principles and values in the form of numerous specific actions ranging from small-scale to long-term, from local to institutional. They are in no sense claimed to be exhaustive; some of them, in fact, immediately suggest other possibilities. The principles guiding the educational suggestions are exposure to applications in other disciplines, real-world problem-solving, computation, and communication and teamwork.

This section lists and in some cases briefly sketches approaches that have been tried successfully, but details and references are not given here. More information can be obtained from SIAM in several ways: via Internet from SIAM's home page (`http://www.siam.org`), following the links to "Mathematics in Industry"; by sending electronic mail to `mii.info@siam.org`; or by contacting SIAM at the address given at the front of this report.

The steering committee is aware that implementation of these suggestions will require time, dedication, and persistence. But we believe strongly that the effort will be worthwhile.

5.1. Graduate education

Graduate education in mathematics serves its students well today in providing the skills from Table 12: thinking logically, dealing with complexity, conceptualizing, formulating problems, modeling, and creating new ideas. Sections 3.2, 3.4, and 3.6 indicate that the special value-added of mathematicians hinges on these same traits; training in mathematics, more than in other fields, is perceived to allow analysis at a high, system-wide level, thereby revealing underlying patterns and structure. Hence graduate education in mathematics is highly successful in preparing students for many key elements of nonacademic jobs.

In contrast, industrial mathematicians and their managers consistently mentioned several qualities, shown in Tables 13 and 14, to which greater attention could be given in mathematics graduate education:

1. substantive exposure to applications of mathematics in the sciences—physical, biological, medical, and social—and in engineering;
2. experience, both inside and outside the classroom, in formulating and solving open-ended real-world problems, preferably involving a variety of disciplines;
3. computation;
4. communication and teamwork.

We now describe curricular suggestions for developing these skills. Some of these elements can also be incorporated with suitable modifications into an undergraduate mathematics curriculum.

5.1.1. Exposure to applications of mathematics. Familiarity with applications as well as interdisciplinary problem-solving skills can be generated through courses devised in cooperation with other departments and nonacademic mathematicians. A 1995 National Research Council report, *Mathematical Challenges from Theoretical/Computational Chemistry* [NRC-Chem], makes several suggestions about graduate education. Some of its recommendations, suitably generalized, have been included in our suggestions.

- Join with other departments to create special "interdisciplinary tracks" of graduate courses that combine the most important advanced topics in several disciplines. Nonacademic mathematicians should be involved in designing and teaching such courses.

- Encourage graduate degrees that involve dual mentoring by mathematics and another department.
- Form a mathematical consulting group involving mathematics graduate students (and perhaps faculty) to be available to other researchers on campus.
- Allow and encourage mathematics graduate students to minor in another discipline.
- Join in teaching, preferably with a nonacademic colleague, mathematically based specialty courses already offered in other departments. It seems inefficient and ill advised to allow a serious divergence between advanced mathematical applications and advanced mathematics.
- Create new mathematics courses focusing on the techniques most needed in certain applications, and include substantial course material on those applications. It is natural to share teaching in such courses with at least one other department and with nonacademic mathematicians.
- Develop general courses on industrial mathematics. Several books are available that offer suggestions and guidelines.
- Incorporate nonacademic internships or other on-site problem-solving experiences into degree requirements.
- Participate in or initiate programs in computational science, which stress a combination of mathematics, computer science, and applications. Existing computational science programs typically involve nonacademic scientists from several disciplines.
- Invite speakers from other disciplines, including scientists from industry, to speak in research seminars or colloquia. Such interdisciplinary gatherings can also be organized in cooperation with other departments.
- Maintain a departmental database, with links to national databases supported by government agencies, of summer positions or internships in industry or government laboratories.

5.1.2. Problem-solving. Mathematics problems assigned in academic contexts almost never include the work already invested to strip away all but the essentials and to describe the problem in a form that suggests how it might be solved. The skills needed to perform those unseen preliminary steps are crucial for nonacademic mathematicians (see Section 3.4), and they cannot be taught except by direct experience.

The interdisciplinary and applications-oriented courses suggested in Section 5.1.1 are an obvious milieu for assignments based on open-ended, real-world problems. A growing selection of publications on problem-solving and industrial problems is available to help mathematics faculty in building course content and materials. Several relevant books, some based on joint university–industry mathematics programs, are listed in the References section.

Graduate students can experience interactive problem formulation by attending intensive seminars and workshops organized by universities in collaboration with local industrial and government organizations. In one of the most successful formats, industrial scientists with real problems describe their problems; Ph.D. students then work in small groups to understand, formulate and analyze; and both sets of participants meet frequently to explain, redefine if necessary, and evaluate the results. Preliminary reading is assigned to the students as preparation for the technical background of the problem. In addition to teaching problem-solving, activities of this kind necessarily enhance students' awareness of the applications of mathematics.

In evaluating problem-solving experiences, especially internships, it is important to remember that the first priority of an industrial mathematician is problem-solving, not exercising a predetermined set of mathematical ideas. Consequently, it is difficult to guarantee in advance that students working on industrial problems will use a certain kind or level of mathematics. A successful student experience is one that attacks an important problem and produces a solution that is helpful to the user.

5.1.3. Computation. Members of the steering committee are convinced that computing as a discipline has enormous intellectual content. The view that "anyone can write computer programs" is just as wrong as the statement that "anyone can write"; the task of transforming a mathematical procedure into a correct computer program involves attention to structure, accuracy, efficiency, convergence, and reliability. Hence time spent learning about computation is time well spent.

The most obvious way to ensure that mathematics students understand and experience computation is to impose course requirements involving computer science, but students should encounter serious computational work in their mathematics courses as well. Almost all computer science departments offer courses in programming, numerical analysis, data structures, and algorithms. Since many of these courses have a high mathematical content, their inclusion in a graduate mathematics curriculum is entirely appropriate.

We know from Section 3.3 that computation involving advanced mathematics is extremely important in nonacademic settings. It may be less widely appreciated that courses combining traditional pure mathematics and computation have been created in several universities to encourage new patterns of mathematical thinking and new mathematics. Even students wishing to prepare entirely for an academic career in the purest of mathematics would, in the steering committee's view, benefit from expanded opportunities for course work that links mathematics and computing.

5.1.4. Communication and teamwork. "Communication" may seem at first to be a skill far removed from the traditional mathematics curriculum, but it is simply too important to be ignored or left for others to teach. Writing and speaking skills are important for all mathematicians, not only those in industry; and careful listening is essential in formulating complex problems. The following suggestions have been applied with proven success to teach communication within mathematics courses.

- Assign term papers in selected advanced mathematics courses, and grade the papers for exposition and clarity as well as for content. To help students learn to present advanced material for nonspecialists, two versions of a paper can be required, one of which must be understandable to nonexperts. For at least one paper, require an "executive summary" that justifies the importance of the work in nontechnical terms. Have students give short (10-minute) presentations explaining the nature of the work and its (nontechnical) justification; invite nonspecialists, such as faculty or graduate students from the business school, to critique the presentations.
- Require a course (or mini-course) on technical writing. Such courses can be internal to the mathematics department, or jointly offered by several departments; excellent textbooks are available. Reinforce the message with uniformly high expectations for written communication throughout the curriculum.
- Require at least one "projects" course in which work is entirely performed and graded in teams, or incorporate team projects with written and oral presentations into conventional courses. An interdisciplinary course is well suited to this format, but such a course can also be centered around open-ended problems mixing several areas of mathematics.
- Hold regular seminars, preferably with other departments, in which students are required to read a technical paper outside their area and present an oral report on its contents.

Course grading should be based on evaluation of the presentation by other students. Such programs have a recognized track record of producing students highly skilled in oral presentations.

- Require each student to give a technical presentation that is videotaped and then graded by him/herself.

5.2. Suggestions for faculty

It is widely perceived, and there is some evidence, that graduate education in mathematics is inordinately concentrated on preparation for traditional academic research careers; see, for example, the article [Jack95] about the general study [NRC-Grad]. Recent degree recipients in mathematics have commented in print and privately that some faculty do not seem to support or understand career choices outside academia; see, for example, [Lot95]. We therefore offer further suggestions for faculty because committed faculty involvement is an implicit but essential element in broadening graduate education.

As in the NRC report [NRC-Chem], our suggestions are designed to enhance connections between mathematics faculty members, other faculty members, and nonacademic colleagues working in mathematics and related disciplines. Mathematicians of all varieties acknowledge the stimulation resulting from technical discussions; our suggestions focus on ways to create occasions for such discussions.

- Actively encourage mathematicians, scientists, and engineers from industry to speak in and attend mathematics seminars and colloquia.
- Invite at least one highly visible nonacademic mathematician to visit for one or two days during each term; the visit should include presentation of a talk in your departmental colloquium. Provide ample opportunities for this visitor to meet with students and faculty. The "visiting lecturer" programs of scientific societies can help to supply names of outstanding speakers in designated fields.
- Organize joint colloquia and seminars with other departments, especially those strongly tied to outside mathematical scientists, and with mathematics-intensive groups from industry.
- Maintain a database of departmental graduates working outside academia. Ask them for suggestions about the curriculum, and invite them to return and give a colloquium.
- Include nonacademic mathematicians on departmental advisory committees.
- Appoint a focused nonacademic advisory committee to review curriculum, meet with students, suggest career opportunities, provide industrial contacts, and so on.
- Create institutional connections with mathematics-intensive groups in local industry. Invite appropriate members of such groups to participate in the intellectual life of your department by, for example, serving as adjunct faculty or teaching short courses.
- Seek opportunities for interested faculty to consult with or spend time in industry. Federal programs are in place that support faculty sabbaticals and summers in industry; many government laboratories will support extended faculty visits or sabbaticals.
- Arrange extended visits to your department by industrial mathematicians.
- Encourage interested faculty to seek collaborations with colleagues in other departments or industry. If these collaborations develop, it is important that the resulting work be valued in the departmental rewards process. The MII steering committee recognizes that, for many academics, the path to interdisciplinary work is filled with obstacles, often erected by their own departments. (Some of these barriers are described in reports by

the Joint Policy Board on Mathematics [JPBM94] and the National Research Council [NRC-Chem].) Nonetheless, we hope that the evident scientific rewards of interdisciplinary, nonacademic mathematics will accelerate the engagement of mathematicians in such problems; positive intervention by deans and university officials also has an important role to play.

5.3. Suggestions for students

Many strategies for mathematics graduate students are implicit in our suggestions for graduate education. Section 3 provides clear guidance about skills that are important in nonacademic environments, and students can, to the extent possible, organize their programs to build those skills. Students can also act along lines similar to those indicated for faculty:

- Organize, preferably with graduate students from other departments, a regular interdisciplinary seminar or colloquium focusing on applications of mathematics, and invite scientists from industrial or government organizations to speak.
- Invite nonacademic mathematicians to meet with current students for informal discussions about their work and to present a high-level description of an important practical problem.
- Organize technical talks in which students are critiqued by their fellow students.
- Request videotaping services for student talks from an appropriate office at your institution.
- Organize visits by students in your department to local industry or government laboratories to meet with mathematical scientists.
- Suggest names of nonacademic mathematicians or scientists to be invited to speak in your departmental colloquia.
- Pursue opportunities for summer jobs, cooperative employment, or internships in industry or government laboratories, or for participation in applications-focused workshops. Besides providing valuable experience, these activities are likely to be helpful in finding a permanent job. Much of this information is available online—for example, from the home pages of major government laboratories.
- Attend talks by nonacademic mathematicians at meetings of scientific societies.

5.4. Suggestions for business, industrial, and government organizations

Not surprisingly, the MII steering committee believes that the discipline of mathematics, acting in large part through trained mathematicians, can contribute enormously to solving important problems. The success stories in Section 2.2 and the characterizations in Section 3.6 provide ample evidence of the measurable value added by mathematics and mathematicians. Connections, formal and informal, between nonacademic organizations and academic mathematics departments can build pathways for a two-way flow of both concepts and results. We have therefore included a set of suggestions that can, we believe, help industrial and government organizations to make better use of available mathematical resources. For reasons of convenience, cost, and ease of access, these strategies focus mainly on local academic institutions, but links over a wider area may also be appropriate.

- Become acquainted with academic mathematical scientists by attending colloquia, meeting with students, offering to give talks, and inviting students and faculty for informal visits to your organization.
- Invite mathematicians to give technical seminars at your institution and to meet informally with interested employees.

- Arrange to present a technical problem of interest to a group of mathematics faculty and graduate students.
- Identify academic mathematicians who specialize in areas of interest to your organization. A variety of arrangements can then be made in which your organization might call upon these mathematicians for advice, either short- or long-term.
- Invite mathematics faculty and scientific societies to present continuing-education short courses at your organization on mathematical topics of interest.
- Arrange for visiting lecturers from mathematical societies to visit and speak to your organization.
- Spend a short or longer-term visit in a university department containing one or more mathematicians with interests closely connected to your organization's.
- Try to find talented students who could participate in a summer internship and faculty who might wish to establish collegial relations with members of your organization.
- Encourage interested mathematics graduates to apply for positions, even those not labeled as "mathematics".
- Volunteer to organize a session at a meeting of a mathematical society.
- Become active in scientific societies; volunteer to serve on committees, particularly organizing committees of meetings; suggest topics for short courses at scientific meetings.

6. Conclusions

A substantial part of this report has explored the applications of mathematics in industry, business, and government, as well as many aspects of nonacademic careers for mathematicians. These topics have quite recently received great attention in the mathematics community because of their relationship with two phenomena: the current crisis in the academic job market, and the perceived sharpened attention of U.S. funding agencies to work on applications. In some instances, discussion of applications and nonacademic jobs conveys grudging acceptance of unpleasant necessities that will, if all goes well, pass away; then the mathematics community can return to business as usual.

The MII steering committee emphatically does not take this view. Even if the academic job market improves and funding pressure eases, we are convinced that mathematics and mathematicians should change permanently along the lines indicated in our multiplicity of suggestions. We also believe that the traits valued in nonacademic mathematicians are important and worthwhile in a far wider context.

Throughout the history of mathematics, ideas and inspiration have flowed strongly in both directions between mathematics and applications. Nonacademic applications offer opportunities not simply for mathematicians to solve practical problems, but to enrich and deepen mathematics as well as a wide variety of other fields, including science, engineering, medicine, and business.

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Appendix: MII Study Methodology

The MII study was initiated by SIAM in 1993 to examine the role of mathematics and mathematicians in nonacademic environments. The focus of the study was on recent graduates of mathematics departments—departments with American Mathematical Society (AMS) classification codes I–III and IVa, including applied mathematics departments—working in industry. By "industry" we mean generically employment outside education and academia; we thus include business, industry, and government (including federal and national laboratories). Excluded from our definition are research institutes associated with universities.

The MII study was performed in four stages: focus groups of mathematicians from industry and government; a telephone survey of Ph.D. and master's graduates of mathematical sciences departments working in industry; a follow-up survey of a set of their supervisors; and visits to industry and government sites conducted by members of the MII steering committee. Overall, we spoke with approximately 500 mathematicians, scientists, and engineers in industry, including more than 175 managers, covering a range of experience levels.

A.1. Focus Groups

In the first stage, we held a series of exploratory focus groups with 40 mathematical and computational scientists working in industry and government. Focus group members had a variety of experience, from new hires to managers, and were employed by a wide array of companies. We also visited several industry sites to speak with mathematicians. The results of this preliminary stage are reported in [Davis91].

A.2. Telephone Survey of Recent Graduates

The second stage of the study involved a telephone survey of Ph.D. and master's graduates of U.S. mathematical sciences departments from 1988–1992 who currently held jobs in industry or government in the United States. Graduates of statistics and operations research departments were not included in the telephone survey, although graduates of mathematics departments working in statistics and operations research were included.

The MII database of recent graduates working in industry was developed as follows. An initial database of Ph.D. graduates was obtained from data collected by the AMS-MAA Data Committee for its annual surveys of graduates in 1988–1992. We then wrote to 210 mathematical sciences departments: 135 Ph.D.-granting departments, chosen as a representative sample using AMS classification codes; and 75 departments granting only master's degrees. Our letter asked for contact information about graduates from 1988–1992 working outside academia. Eighty percent of the departments responded, either providing some of the requested information or giving a negative response (that there were no graduates in industry or that the information was not available). Twenty-five percent of the responding departments were able to supply current information on master's graduates. The Ph.D. information received from departments was then checked against the AMS-MAA Data Committee records, and new or updated records were merged into the database.

As a result, a database was created of 335 Ph.D. and 271 master's graduates working in nonacademic positions. From this database, two samples—101 Ph.D.'s and 102 master's graduates—were selected for telephone interviews. The samples were structured to be representative by both type of department and employer, using the AMS classification codes for departments and an employer code based on the Standard Industry Classification (SIC) codes of the U.S. Office of Management and Budget. Government employers, including government and national laboratories, were assigned employer code 1. The remaining employer codes correspond to the first two digits of the SIC codes: code 2 for engineering research and computer services, including software (87xx and 73xx); code 3 for financial, communication, and transportation services (40xx and 60xx); code 4 for manufacturing, including electronic, computer, aerospace, and transportation equipment (35–38xx); code 5 for manufacturing, including chemical and allied products, petroleum, and petroleum extraction (28xx, 29xx, 13xx); and code 6 for all other codes and cases in which no code could be determined.

Table 16 gives an indication of the numbers of graduates in various categories. Note that the total number of Ph.D.'s is based on the AMS–MAA surveys of Ph.D.'s, 1988–1992, and excludes graduates from departments of statistics and operations research, whereas the total number of master's graduates is taken from [SEI93, page 280], and includes graduates from departments of statistics and operations research. (No sources of data were available to provide consistent values for these numbers.) The estimates of the numbers of graduates working in industry—25% of Ph.D.'s and 44% of master's graduates—are based respectively on [SEI93, page 283] and [SEI91, page 283].

Category	Ph.D.	Master's
Total number of graduates	3,701	17,780
Estimated number of graduates in industry	925	7,823
Entries in MII database	335	271
Telephone interviews	101	102

Table 16: Ph.D. and master's graduates, 1988–1992.

The MII Ph.D. sample is reasonably large relative to the estimated number of mathematics Ph.D.'s working in industry, while the master's sample is much smaller relative to the estimated population. This reflects the great difficulty of obtaining current information about master's graduates working in industry.

A questionnaire was designed by members of the MII steering committee and tested in a pilot telephone survey of 15 graduates. The full telephone survey of graduates was conducted by Business Science International (BSI) in December 1994.

A.3. Telephone Survey of Managers

Following the telephone survey of graduates, 75 managers representing a good sample of industry classification codes were selected from the "immediate supervisors" of the mathematicians interviewed in the telephone survey. These managers were then interviewed by telephone by BSI.

The telephone questionnaire for managers was intended to obtain further details about the organizations in which the graduates were employed, confirm some of the information obtained from the graduates, and solicit opinions about graduate education in the mathematical sciences.

A.4. Site Visits

The final stage of the MII study consisted of visits by steering committee members to 19 sites in industry and government that employ a significant number of mathematical scientists. During these site visits we spoke with 175 mathematicians, engineers, scientists, and other professionals, of whom 100 were managers at the level of group leader or above. At twelve sites we spoke to managers at the level of director and above; at four sites we spoke to vice-presidents of research. The sites were selected to cover the range of industry codes. We visited five government sites, including four laboratories; five engineering, consulting, or software companies; one financial services company; five aerospace, transportation, or electronics companies; and three companies in chemical and allied products, including one pharmaceutical company. In addition to these site visits, we conducted a special focus group consisting of three managers from small companies (one consulting and two software firms).

Although site visits focused primarily on mathematical and computational scientists, we also spoke with nonmathematicians, particularly managers, who were familiar with the uses of mathematics and computing at the site and in the company at large, as well as with the contributions of mathematical and computational scientists.

Before the site visits, interviewees were sent an overview of the study and its goals, along with a list of detailed questions. In conducting the site visit discussions, we tried to address six general areas: background information on the site, the role of mathematics, the role of mathematicians, factors for success in the company for a mathematician, interaction with academia and government in conducting business, and opinions about academic programs in mathematics. Detailed notes on each visit were taken by members of the site team (typically two to three members), and a report was prepared by one member of the visiting team.

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