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The Weak Form is Stronger Than You Think

By Daniel Messenger, April Tran, Vanja Dukic, and David Bortz

For a broad class of differential equations, one can obtain the *weak form* by multiplying both sides of the equation with a sufficiently smooth function ϕ, integrating over a domain of interest Ω, and using integration by parts to obtain a new equation with fewer derivatives. It is a ubiquitous, well studied, and widely utilized tool in modern computational and applied mathematics. Yet while the convolution of data with ϕ (or $\partial_t\phi$, $\partial_x\phi$, etc.) can filter noise, conversion to the weak form is more powerful than just smoothing the data; the choice of a test function asserts a topology or scale through which to view the equation. Indeed, recent advances suggest that with a data-driven topology (encoded in the form of ϕ), weak form versions of equation learning, parameter estimation, and coarse graining offer surprising noise robustness, accuracy, and computational efficiency.

Governing Equations

Researchers have been studying computational methods for scientific model discovery for decades, and recent years have seen an explosion of activity based on the sparse identification of nonlinear dynamics (SINDy) method [2]. SINDy learns the nonzero weights $\{w_j\}_{j=1}^J$—which correspond to discovered terms in a library of candidate functions $\{f_j\}_{j=1}^J$—by using an *equation error* (EE)-based sparse regression $\|\partial_t \mathbf{U} - \Sigma_{j=1}^J w_j f_j(\mathbf{U})\|_2^2$ for data $\mathbf{U}$.

Although EE regression methods are computationally efficient, the use of noisy data presents a significant challenge due to a known bias in the resulting parameter estimates and the need to approximate derivatives of the data (e.g., $\partial_t\mathbf{U}$). Several groups have built upon SINDy and independently discovered that learning by means of the weak form of the model both bypasses the derivative approximation question and is highly robust to noise [3]. The core idea is that multiplying both sides of an equation with a compactly supported test function $\phi \in C_c^\infty(\Omega)$ allows the movement of derivatives from the state variables to the test function. To illustrate this concept, consider a feature library that consists of spatial derivatives up to order K that act on polynomials up to order P. In this case, the weak form EE residual is $\|\langle\partial_t\phi, \mathbf{U}\rangle + \Sigma_{k=0}^K \Sigma_{p=0}^P (-1)^k w_{k,p} \langle \partial_x^k \phi, \mathbf{U}^p\rangle\|_2^2$.

Figure 1 details the use of the weak form equation learning framework to discover the Kuramoto-Sivashinsky partial differential equation (PDE) in the presence of 50 percent additive independent and identically distributed Gaussian measurement noise, with a

See **Weak Form** *on page 3*

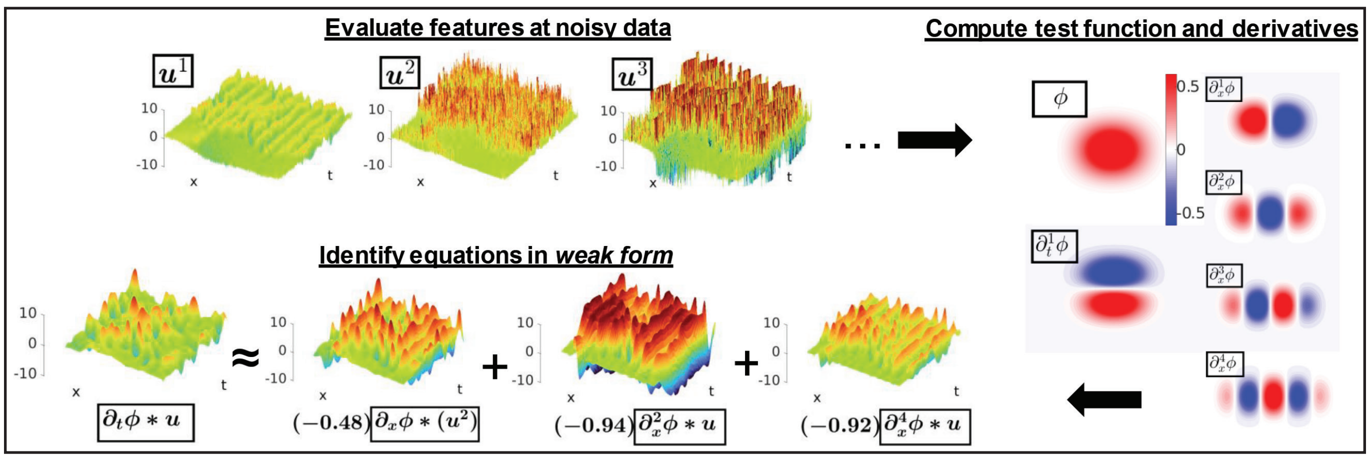

Figure 1. *Weak form partial differential equation (PDE) identification via the weak sparse identification of nonlinear dynamics (WSINDy) PDE algorithm. We collect solution data from the Kuramoto-Sivashinsky equation with 50 percent added noise; the z-axis is limited to $[-10, 10]$ for clarity. Based on noisy feature evaluations, we identify a reference test function ϕ to balance noise filtering with accuracy. We then use convolutions against ϕ and its derivatives to construct weak form features. The governing equations approximately hold in this weak form space, allowing for the accurate identification of model terms and coefficients. Figure courtesy of the authors.*

Feeling Lucky? The Relative Roles of Skill and Chance in Sports

By Anette Hosoi

As I wrote this article, I was watching the televised women's team archery gold medal competition at the 2024 Summer Olympics in Paris. I had never watched team archery before, and I was riveted. The captivating suspense—a signature of all great sports competitions—arises in part from the delicate balance between skill and chance. As spectators, we prize excellence and want highly skilled athletes to be rewarded for their efforts; however, we also love to root for an underdog. It is this tension between supremacy and uncertainty that makes sports so compelling.

The outcomes of competitions—athletic or otherwise—and many other activities are determined by a blend of skill and luck. A variety of factors impact the relative importance of these two effects, including rules and regulations and physical and biological considerations. For example, you likely arrived safely at your destination when you last drove to the office. Though this result is largely due to driving skill, you were also lucky that no other cars collided with you. If you live in an area where car accidents are relatively uncommon, then driving to work lies near the skill end of the skill-luck spectrum.

During the early 2000s, the rise of online poker and the accompanying legal debate[1] heightened mathematicians' collective interest in the estimation of different activities' placements on this spectrum. In a 2013 paper, Thomas Miles, Steven Levitt, and Andrew Rosenfield proposed a series of mathematical questions to assess the role of chance in the outcome of poker tournaments [2]. An appealing aspect of their framework is that users can formulate the questions purely in terms of inputs (player or team actions) and outputs (win-loss records). We can hence view the game itself as a black box, which eliminates the need to articulate the detailed mathematics that are associated with the rules.

Here, I will focus on a particular question that explores the persistence of skill as a metric for the placement of activities on the skill-luck spectrum [2]. Roughly speaking, if someone is good at a skill-based activity today, they will likely be good at it tomorrow. In contrast, the result of a chance-based activity—e.g., coin flipping—on one day is in no way indicative of future outcomes.

In a 2018 *SIAM Review* article, my collaborators and I proposed a metric to quantify the persistence of skill that begins with the following hypothesis [1]: Skill is an intrinsic quality of an athlete or team and does not change significantly over the course of a season. If this is true, in games of skill we expect the win fraction of each player or team in the first half of the season to correlate with that player or team's win fraction in the second half. To intuitively understand the signatures of skill and luck within this framework, consider an idealized scenario wherein an infinite number of players are playing an infinite number of games. In contests of pure luck (i.e., coin flipping), the anticipated outcome of every player is the same. If we thus plot each player's first-half win fraction against their second-half win fraction—assuming a zero-sum game in which participants compete

See **Skill and Chance** *on page 4*

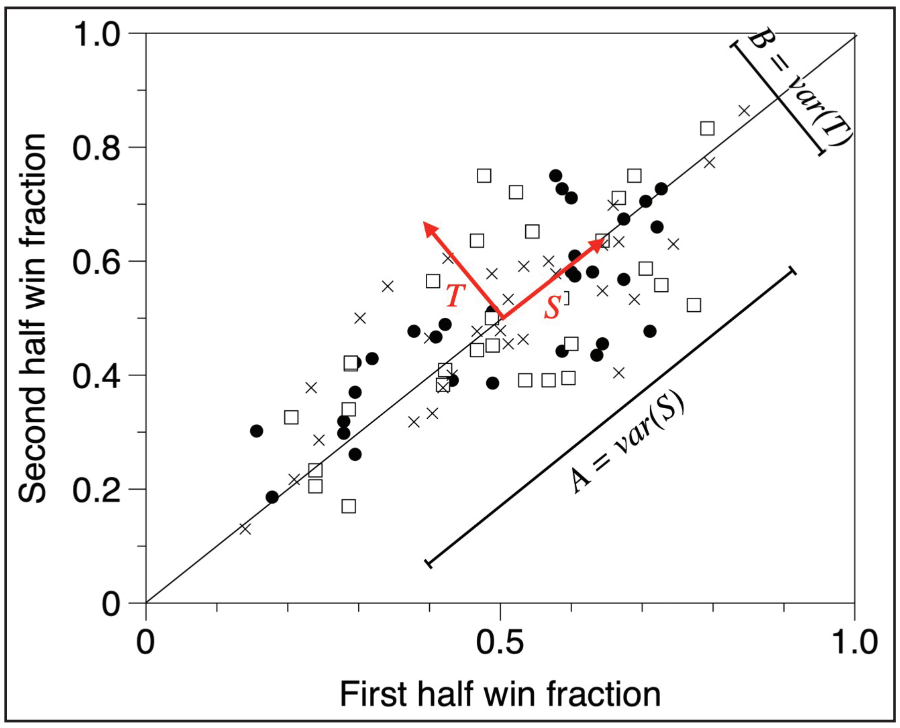

Figure 1. *Win fraction for the first half of the season versus the second half for five years of data from the National Basketball Association. Each point represents one team's win-loss record in a single season, and different symbols correspond to different years. The red arrows indicate the rotated S-T axes. Figure courtesy of the author.*

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[1] If a game's outcome is determined predominantly by chance, then the activity is typically classified as gambling and subject to laws such as the Unlawful Internet Gambling Enforcement Act of 2006.

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From Theory to Advocacy: The SIAM Science Policy Fellowship Experience

By Bashir Mohammed, Victor Churchill, and Arielle Carr

Since its inception, the SIAM Science Policy Fellowship Program[1] has offered postdoctoral researchers and early-career scientists a unique and invaluable opportunity to immerse themselves in the complex processes that shape scientific funding and influence key decisions in U.S. federal policy. Participants of this prestigious program gain a deeper understanding of the mechanics of policymaking while still pursuing their research and teaching commitments.

The Fellowship equips recipients with the necessary skills to effectively engage with federal officials and congressional staff. Science Policy Fellows take part in comprehensive training webinars about key budget considerations, legislative issues, and the appropriations process; meet with lawmakers, legislative staff, and federal agency officials in Washington, D.C.; physically attend the biannual spring and fall meetings of the SIAM Committee on Science Policy[2] (CSP); and ultimately complete an independent policy project on a topic of their choice.

Each year, three to five Fellows are selected for this transformative, two-year experience that empowers them to represent the industrial and applied mathematics community and advocate for federal support in applied mathematics, computational science, and data science. Here, three SIAM Science Policy Fellows from the 2023 and 2024 cohorts share their personal journeys within the program, reflect on their newfound perspectives, and discuss their advocacy efforts for enhanced U.S. federal support in science, technology, engineering, and mathematics (STEM).

Bashir Mohammed, Intel Corporation

Five years ago, I was invited to represent Lawrence Berkeley National Laboratory at the International Year of the Periodic Table Elemental Slam on Capitol Hill,[3] where I presented my work to U.S. legislators in Washington, D.C. This defining moment in my career offered a rare chance to interact with members of Congress and explore the intricacies of policymaking and scientific funding. Inspired by this experience, I began to actively seek other opportunities to connect with federal officials, expand my knowledge of science policy, better understand pivotal legislative matters in STEM, familiarize myself with the U.S. federal budget and appropriations processes, and advocate for applied mathematics and computational science in an impactful way. After applying for the SIAM Science Policy Fellowship Program, I was awarded the Fellowship in January 2023 alongside four other early-career researchers.

As a Science Policy Fellow, I participate in the biannual CSP meetings and promote applied mathematics research. The meetings, which take place in Washington, D.C., ensure that SIAM's voice is part of the policymaking processes that influence scientific funding in the U.S. During my first CSP meeting in spring 2023, I engaged with senior SIAM members from various sectors, attended orientations about SIAM's history with science policy, and participated in training sessions on federal budgeting and legislative advocacy. These events were guided by experts from Lewis-Burke Associates[4]—SIAM's governmental relations partner in Washington, D.C., that connects the Society with federal agencies and congressional offices—and provided valuable insights about effective strategies for communicating with policymakers. While in D.C., I even conversed with legislators and agency officials about budget allocations that significantly affect quantum and computational science. I also spoke with policymakers who support key initiatives like the National Quantum Initiative[5] and the Department of Energy's (DOE) Science for the Future Act[6] — both of which closely align with my research interests.

In addition, I completed two policy projects over the last two years. In May 2023, I spoke about urgent priorities within the SIAM community at the White House Office of Science and Technology Policy Open Science Listening Session.[7] Furthermore, I am privileged to be part of the steering committee for the upcoming 2024 SIAM Quantum Intersections Convening,[8] which will take place from October 7-9 in Tysons, Va. This interactive three-day workshop will unite quantum-curious mathematical researchers with leading experts in quantum science to increase the visibility of applied mathematics in the quantum field.

My involvement with the SIAM Science Policy Fellowship Program[9] has strengthened my commitment to advocacy and deepened my appreciation of science policy's critical role in research funding. I look forward to continuing this work at the upcoming CSP meeting later this fall.

Victor Churchill, Trinity College

After completing my Ph.D. at Dartmouth College and a postdoctoral appointment at The Ohio State University (both R1 universities), I accepted a tenure-track position at Trinity College: a small liberal arts college in Hartford, Conn. The desirable offer from Trinity—a primarily undergraduate institution (PUI)—helped me realize that PUIs constitute promising career paths for recent graduates, especially since the growing number of applied mathematics and computational science Ph.D.s is far outpacing the stagnant number of available R1 tenure-track positions. However, certain challenges that are associated with smaller departments—such as additional responsibilities and a limited workforce (i.e., no graduate students)—make it harder for PUI educators to remain current and competitive in the research world. These challenges inspired me to apply for the SIAM Science Policy Fellowship. As a class of 2023 recipient, I have learned how to advocate for policies that will allow PUI faculty to further their own research while simultaneously training the nation's future STEM workforce.

The Fellowship's learning and networking opportunities have been especially invaluable to my personal goals. During the biannual CSP meetings, I absorb and connect with a figurative cornucopia of

See **Science Policy** *on page 5*

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[1] https://www.siam.org/programs-initiatives/programs/siam-science-policy-fellowship-program

[2] https://www.siam.org/get-involved/connect-with-a-community/committees/committee-on-science-policy-csp

[3] https://www.congressweb.com/events/index.cfm?action=Event_Page&eventcode=mAGykC&bypass=tr

[4] https://lewis-burke.com

[5] https://www.quantum.gov

[6] https://www.congress.gov/bill/117th-congress/house-bill/3593

[7] https://www.whitehouse.gov/ostp/news-updates/2023/07/11/readout-of-ostp-open-science-listening-sessions-with-early-career-researchers

[8] https://www.siam.org/conferences-events/workshops/siam-quantum-intersections-convening

[9] https://www.siam.org/publications/siam-news/articles/behind-the-lab-coat-a-scientist-s-journey-to-influence-science-policy-in-washington-dc

From left to right: Andrew Salinger of Sandia National Laboratories (member of the SIAM Committee on Science Policy), Jonas Albert Actor of Sandia National Laboratories (2024 SIAM Science Policy Fellow), and Bashir Mohammed of Intel (2023 SIAM Science Policy Fellow) gather in Washington, D.C., for the spring 2024 meeting of the SIAM Committee on Science Policy. Photo courtesy of Bashir Mohammed.

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Weak Form

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three-decibel signal-to-noise ratio. In this example, the candidate library encompasses all unique operators $u \mapsto \partial_x^k(u^p)$ for $0 \leq k$, $p \leq 6$ — a total of 43 terms that contain the true three-term model. It is important to note that making a mathematically justified choice for the test function ϕ—*and hence the topology*—is critical to performance. Here, we match the spectral properties of the test functions to those of the data [3] to filter high-frequency noise and preserve the solution signal. By centering shifted copies of test functions on each sample point, we can create a system of equations—i.e., a regression problem for the coefficients $\mathbf{w}$—and yield a method for accurate PDE discovery from highly noisy data in less than a second on a modern laptop. This ability is in direct contrast to strong form methods; for example, data with more than one percent noise will prevent (strong form) SINDy from learning the Navier-Stokes equation.

The discovery capabilities of the weak form are broader than simply finding a canonical PDE or ordinary differential equation to describe the data. For example, asymmetric force potentials that model attraction/repulsion, alignment, and drag can be learned for *each particle* in a deterministic interacting particle system (IPS) model of collective motion. In Figure 2a, the gray unlabeled trajectories illustrate the motion of a heterogeneous population wherein a common force model governs subsets of particles. In less than 10 seconds, a weak form method—in this case, weak SINDy (WSINDy)—can rapidly and parallelizably learn particle-specific potentials that lead to accurate trajectory predictions. This method can even cluster models to discover population structures (e.g., the teal curves in Figure 2a are from a single subpopulation), thus serving as a novel tool with which biologists can study cell population heterogeneity based on movement trajectories [8].

The weak form can also augment existing techniques, such as the creation of reduced order models (ROMs) from noise-corrupted or stochastic data. For example, we can extend the latent space dynamics identification (LaSDI) method via the weak form (WLaSDI) to robustly learn ROM dynamics [12]. Figure 2b illustrates the results of WLaSDI's application to noisy measurements of a reaction-diffusion system's solution, which yields a ROM with 200 times speedup and roughly four percent solution error (with the same data, a LaSDI ROM has more than 100 percent solution error). Even when we increase the noise level to 100 percent, WLaSDI still returns a ROM with less than 10 percent relative error; in contrast, a LaSDI ROM has more than 200 percent error [12].

Parameter Estimation

Researchers have utilized regression with EEs for parameter estimation since at least the 1950s; in fact, Marvin Shinbrot proposed a weak form of the system equations in 1954 [10]. The successor of this approach is the *modulating function* method. However, several factors have prevented widespread adoption of this class of weak form methods: (i) the challenge of selecting the test function ϕ, (ii) a known statistical bias in EE-based inference, and (iii) the ready availability of software that uses output error methods to match a model solution to data. We recently proposed the weak form estimation of nonlinear dynamics (WENDy) parameter inference method, which includes an automated strategy for the creation of orthogonal ϕs from multiresolution C_c^∞ functions that are merged with a generalized least squares approach to address statistical issues [1]. The combination of these two techniques generates substantial improvement in both computation time and inference accuracy. Figure 2c portrays the relative errors versus walltime in the use of WENDy to estimate parameters for the Kuramoto-Sivashinsky PDE from data with 20 percent noise. In most cases, WENDy is at least an order of magnitude more accurate and more than an order of magnitude faster than conventional output error methods [1].

Coarse Graining

Coarse graining is the process of mapping a first principles model to a lower-order one; the technique is characterized by effective descriptions of small-scale dynamics via larger-scale quantities of interest. In many cases, we derive a solution to the coarse-grained model as a limit of solutions to the first principles model (converging in a suitable *weak topology*). This process naturally leads to questions about the role of weak form equation learning in coarse-graining applications. For a first-order stochastic IPS, WSINDy can discover the governing PDE that corresponds to its mean field McKean-Vlasov process based on histograms of discrete-time IPS samples at the N-particle level [4]. And in the context of diffusive transport with a highly oscillatory spatially-varying diffusivity, WSINDy similarly identifies the correct homogenized equation [4]. Figure 3a depicts histograms (in gray) from an N-particle system that diffuses with a large but finite spatial frequency ω, from which WSINDy can identify the correct $N \to \infty$, $\omega \to \infty$ homogenized system (the learned system is in teal).

For nearly-periodic Hamiltonian systems, WSINDy robustly identifies the correct leading-order Hamiltonian dynamics of reduced dimension that result from averaging around an associated periodic flow that commutes with the full dynamics to leading order [6]. In Figure 3b, noisy observations from an eight-dimensional coupled charged particle system (in white) enable the identification of a four-dimensional coarse-grained Hamiltonian system (in blue), complete with accurate identification of the ambient electric field $\hat{V}_{\mathbf{E}}$ (background contours).

Future Opportunities

This article seeks to highlight the successes and opportunities of weak form methods; notable recent works suggest that many more advances are yet to be made. Computationally, the narrow-fit and trimming approach in WeakIdent can improve sparse regression [11]. On the theoretical side, we see both a novel proof of convergence (in a reproducing kernel Hilbert space) of WSINDy-created surrogate models [9] and an asymptotic result that finds the model classes for which the correct model will be recovered with probability 1 [5].

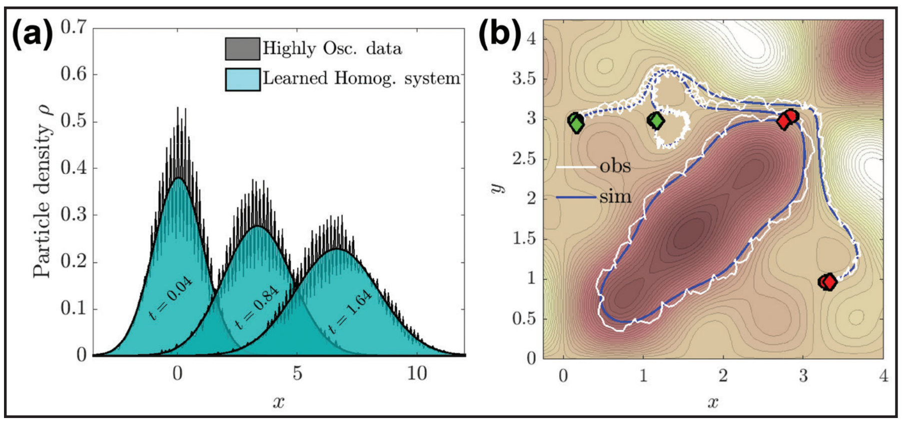

Figure 3. *Coarse graining in action.* **3a.** *Homogenization of a highly oscillatory Fokker-Planck equation from particle data.* **3b.** *Reduction of noisy, coupled, charged particle motion (in white) to coarse-grained Hamiltonian dynamics (in blue), including inference of background electric potential $\hat{V}_{\mathbf{E}}$ (contours). Particles begin at the green markers and end at the red markers. Note the proximity of the full dynamics (circles) to the coarse-grained model (diamonds). Figure courtesy of Daniel Messenger and inspired by [4, 6].*

Finally, we note that all of the advances in this article are based on conventional techniques of statistics, applied analysis, and numerical analysis. Applications of these techniques yield versions of equation learning, parameter inference, and model coarse graining that offer substantial robustness and accuracy and demonstrate the weak form's broad utility beyond well-known theoretical and computational methods.

An expanded version of this article with a more complete set of references is available online [7], and the code with which to reproduce the results is accessible on our group's webpage.[1]

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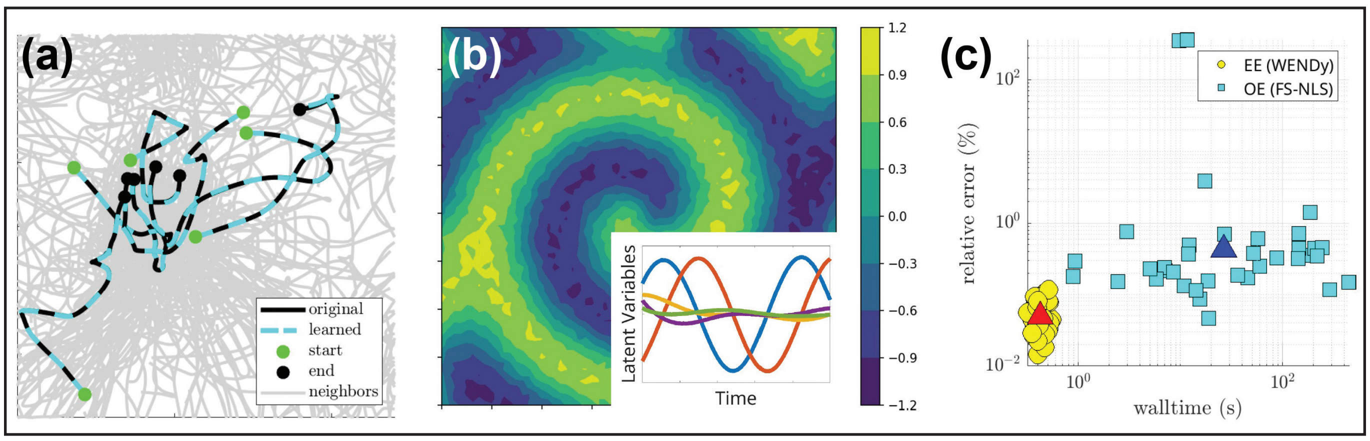

Figure 2. *Equation learning and parameter estimation.* **2a.** *For unlabeled particle trajectories (gray) in a multi-species population, force potentials are learned for each particle and then sorted into species. Teal trajectories share a common learned model. [7].* **2b.** *Contour snapshot of the noisy measurements of the activator in a reaction-diffusion system with five-dimensional reduced order model latent space (pictured in the inset) [11].* **2c.** *Comparison of parameter estimation performance on the Kuramoto-Sivashinsky equation via equation error (EE) and output error (OE) methods [1]. Yellow circles represent weak form estimation of nonlinear dynamics (WENDy), teal squares signify forward-solver nonlinear least squares (FS-NLS), and triangles depict the corresponding geometric means. Figure 2a courtesy of [7] and 2b and 2c courtesy of the authors.*

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[1] https://github.com/MathBioCU

Skill and Chance

Continued from page 1

against one another—all players converge to a single point at $(1/2,\ 1/2)$. But in a skill-dominated contest, we expect to observe a distribution of skill across the playing population; for instance, players who win 80 percent of their games in the first half of the season should also win 80 percent in the second half. In this case, the data will converge to a line with slope 1.

In reality, however, we have neither infinite players nor infinite games. As such, we anticipate the presence of scatter in the data — even in the extremes of all-skill or all-luck settings. And because neither skill nor luck exclusively determine the outcomes of most activities, the data will likely resemble an elongated cloud; in this format, higher levels of elongation correspond to the increasingly large role of skill in influencing the outcome.

Figure 1 (on page 1) illustrates five years of National Basketball Association (NBA) data, where each point represents one team's win-loss record in a single season. To quantify the relative importance of skill, we must determine the point cloud's degree of elongation. To do so, we rotate the coordinate system by $\pi/4$ so that it becomes an S-T coordinate system (shown in red in Figure 1) and consider the ratio of the variance in the T direction versus the variance in the S direction. Specifically, we define

$$R=1-B/A,$$

where A is the variance along S and B is the variance along T. For an outcome that is primarily determined by skill, $B\rightarrow 0$ and $A\rightarrow$ a finite number; therefore, $R\rightarrow 1$. For an outcome that is primarily determined by luck, the point cloud approaches a symmetric blob — meaning that $B\rightarrow A$ and $R\rightarrow 0$.[2] Note that this analysis does not judge the difficulty level of the sport in question (e.g., we do not claim that basketball requires more skill than hockey). Rather, we measure whether the rules of the game or tournament are designed to reward skill; at the end of the season, are the more skilled players or teams ranked the highest, or is the order random?

Armed with this metric, we can calculate the R value for different sports and place them on a skill-luck spectrum. Figure 2 displays R values for 10-year spans of five professional sports leagues: the NBA from 2008 to 2018 (excluding 2011), Premier League soccer from 2012 to 2021, Major League Baseball (MLB) from 2012 to 2021, the National Hockey League (NHL) from 2007 to 2017 (excluding 2013), and the National Football League (NFL) from 2014 to 2023.[3] The plot suggests a possible "sweet spot" for sports that balances the roles of skill and chance (indicated by the gray box). In this regime, talent is rewarded but surprises are possible and underdogs can still prevail.

If a league's objective is to hit this sweet spot and make the game more compelling to fans, several strategies can nudge competitions into this region of the spectrum. For instance, leagues could adjust the number of scoring opportunities, as "even tiny differences in skill manifest themselves in near certain victory if the time horizon is long enough" [2]. If two competitors are separated by a small amount of skill, the lesser player may win a single game but will struggle to prevail in four out of seven games, for example. Increasing the number of games and/or the scoring opportunities per game increases the odds that the more skilled player will triumph.

To confirm this strategy's consistency with the previously computed R values, Figure 3 summarizes the number of scoring opportunities per season for each of the five leagues. As expected, more "scores" per season per team shifts the location of the league towards the skill side of the skill-luck spectrum.

The Premier League is the sole exception to this trend, which suggests a second strategy to adjust location on the skill-luck spectrum: concentrate or diffuse the distribution of talent. Consider the aforementioned Olympic archery championship. If I were to participate in the competition, my arrow would likely not land anywhere near the target; the outcome of the contest would be a foregone conclusion (my loss), and skill—or lack thereof—would be the primary factor in determining the winner. But in the elite faceoff that I witnessed, the differences in skill are so slight that the timing of a random gust of wind could serve as a key differentiator. The role of skill in an outcome thus depends on the distribution of talent in the playing population.

This distribution can be adjusted with rules and regulations that allow or constrain the concentration of talent — e.g., by employing a salary cap, weighting draft order inversely with performance, or dividing tournaments into classes of different skill levels. Of the five leagues that we considered, the Premier League is the only one that does not have a salary cap. Given the number of scoring opportunities in soccer, one might expect the Premier League to sit closer to the luck end of the spectrum. However, the lack of salary cap enables the concentration of skill in wealthier teams and shifts the league back towards the skill end of the spectrum so that it falls solidly within the desirable region.[4]

Finally, a third strategy to manipulate the relative role of skill versus luck involves adjusting the equipment or physical environment of the game. In the 2017 MLB season, a striking increase in the rate of home runs corresponded with a decrease in the average drag of MLB baseballs. Fans may recall the "juiced" ball debate and the accompanying outcry from spectators and players alike. Interestingly, this decrease in drag coefficient was small compared to the natural variance in drag among baseballs. A beautiful graphic from Baseball Savant[5] illustrates both the variance in drag for each season and the change in these distributions from year to year. Because modern-day baseballs are still hand stitched and made of natural materials, much of this inconsistency is inevitable. Given that miniscule changes in the ball's surface can yield significant variations in home run rate, an ongoing debate pertains to the possible integration of modern materials and manufacturing techniques to decrease variance across baseballs.

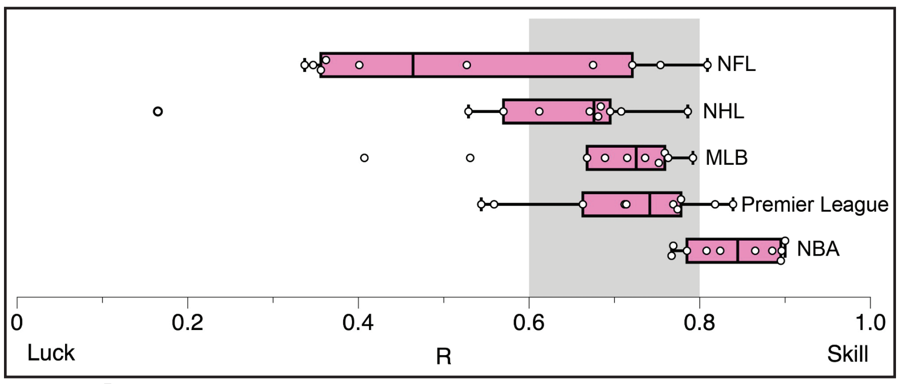

Figure 2. R *values for 10 seasons of the National Football League (NFL), the National Hockey League (NHL), Major League Baseball (MLB), Premier League soccer, and the National Basketball Association (NBA). Each white dot indicates a single season and the pink bars represent interquartile range box and whisker plots for the 10-year span. The gray region suggests a desirable balance between skill and chance. Figure courtesy of the author.*

League	# games / season / team	# "scores" / game	Salary cap?
NBA	82	57 (40 FG, 17 FT)	Y
Premier League	38	3	N
MLB	162	8	CBT
NHL	82	3	Y
NFL	16-17	4 (2.5 TD, 1.5 FG)	Y

Figure 3. *Estimates of the number of "scores" per season for the National Basketball Association (NBA), Premier League soccer, Major League Baseball (MLB), the National Hockey League (NHL), and the National Football League (NFL). The final column indicates the existence of a salary cap; the Competitive Balance Tax (CBT) for MLB is a penalty for exceeding a predetermined payroll threshold. Figure courtesy of the author.*

But in light of MLB's current placement on the skill-luck spectrum, *should* the balls be more uniform? After all, the randomness that is associated with a high- or low-drag ball is an intrinsic component of each at-bat. Removing that variance would push MLB further towards the skill end of the spectrum, and it is unclear whether such a shift would benefit the game.

When I turned back to my television, the Olympics had moved on to diving — the outcome of which seems to be largely determined by skill (the variance of talent across athletes is large due to the dominance of the Chinese divers). The competition showcased exquisite physical capabilities of extraordinary athletes. But it captivated my attention in a different way than the archery competition, which had an element of tension driven by uncertainty. So, I will end by celebrating this uncertainty. The creation of contests that are compelling to watch is ultimately an optimization problem that must strike the desired balance between determinism and chance. And if leagues perform this calculation correctly, we can all join the throngs of sports fans who support underdogs and believe that "maybe this year will be different."

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Anette (Peko) Hosoi is the Neil and Jane Pappalardo Professor of Mechanical Engineering and a professor of mathematics at the Massachusetts Institute of Technology (MIT). She is also co-founder of the MIT Sports Lab, which connects the MIT community with professional teams and leagues to address data and engineering challenges in the sports domain.

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[2] We originally performed this analysis as part of a legal argument in *The People of the State of New York v. FanDuel, Inc.* [3]. In that case, we defined $R=1-B/A$ (rather than $R=B/A$) because the lawyers suggested that it is more intuitive to associate 1 with skill and 0 with luck.

[3] We omitted the 2011 NBA and 2013 NHL seasons because lockouts resulted in significantly fewer competitions.

[4] The Premier League will implement a salary cap for the 2025-2026 season, thus providing a beautiful natural experiment of the salary cap's role in setting the skill-luck balance.

[5] https://baseballsavant.mlb.com/drag-dashboard

Learning in Image Reconstruction: A Cautionary Tale

By Martin Burger and Tim Roith

The growing influence of learning methods across science, industry, and society has generated significant interest in image reconstruction and inverse problems. This research area constitutes one of the most important interfaces between mathematics and a wide range of scientific and industrial domains, including medicine, materials testing, remote sensing, and astronomy. Despite the notorious difficulties that are caused by the mathematical ill-posedness of inverse problems, several examples have exhibited striking performance in recent years. Here, we highlight some associated questions of reliability and trustworthiness for learning algorithms.

The seemingly innocent formulation of an inverse problem is commonly stated as

$$y=A(x)+\varepsilon,$$

where $x\in\mathcal{X}$ denotes the quantity of interest, A denotes a forward operator that encodes the measurement process, and $y\in\mathcal{Y}$ denotes the observed measurement data (with noise ε). Solving this inverse problem involves the retrieval of x given data y. To clarify, let us focus on X-ray computed tomography (CT) for medical imaging. In this context, A is the Radon transform, y is the sinogram, and x is the density inside some part of the human body.

Johann Radon had already derived an exact inversion formula—called *filtered back projection*—about 50 years before the construction of the first CT scanner [8]. Although his method is still utilized today, filtered back projection yields less favorable results in realistic scenarios where measurements are only partially available and corrupted by errors due to technical constraints. To tackle these issues, the inverse problems community developed *regularization methods* [2] that can approximately recover the desired quantity x in situations where the direct inversion would produce unfavorable results.

In response to the increasing availability of datasets, novel developments in learning theory, and improved hardware resources, focus has now shifted towards data-driven methods. Typical approaches include end-to-end learning (directly from the given data or as a form of post-processing), unrolling, plug-and-play, learned regularization, and the use of diffusion models as priors in Bayesian inverse problems. While the reconstruction performance is often remarkable, we must also question the resilience of the employed models. For some reconstruction method $F:\mathcal{Y}\rightarrow\mathcal{X}$, we would thus like to quantify the reconstruction error $\mathcal{E}(y,x;F):=\|F(y)-x\|$ in some norm.

Our case concerns a parametrized method F_θ, where the parameters θ are typically the weights of a neural network. For supervised learning, we utilize a dataset of input-output pairs $\mathcal{T}=\{(x_1,y_1),\dots,(x_N,y_N)\}$; we typically assume that the pairs are independent and identically distributed with respect to an unknown data distribution π. Learning the reconstruction method F_θ involves finding parameters θ that minimize the loss $\mathcal{L}(\theta)=\Sigma_{(x,y)\in\mathcal{T}}\mathcal{E}(y,x;F_\theta)$. Because we can solve the optimization problem reasonably well, the key quality indicator is the so-called *generalization behavior*, which measures the method's performance on unseen data.

To test this property, we usually employ a validation dataset $\widetilde{\mathcal{T}}$ that is disjoint from the training dataset. The first obvious but core issue is as follows: *What if the dataset does not capture the desired distribution π?*

While it is important to note that learned methods can provide functionality beyond memorization of the dataset, the quality of the data plays a crucial role in the learning process. In medical imaging, datasets consist of numerous high-resolution images and scans. What happens if certain structures (pathologies) are not present in the training or validation dataset but appear with low probability in the desired distribution π, which includes real-life diagnosis scenarios?

We explore this question for limited-angle CT, where A is the Radon transform with a restricted range of available angles $[0,\pi/2]$. The reconstruction method first applies a direct reconstruction (generalized inverse of the forward operator A) before employing a neural network f_θ with a state-of-the-art U-Net architecture [9]. To obtain

See **Image Reconstruction** *on page 6*

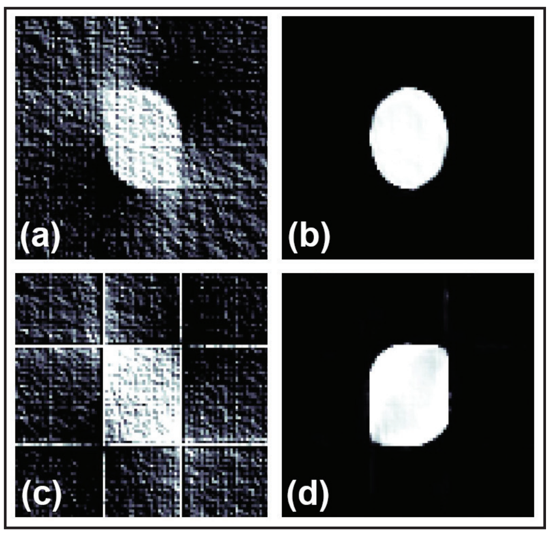

Figure 1. *Direct reconstruction and network output for ellipse data (which is in the training distribution) and rectangle data (which is not).* **1a.** *Direct reconstruction of an ellipse.* **1b.** *Network output from the data in 1a.* **1c.** *Direct reconstruction of a rectangle.* **1d.** *Network output from the data in 1c. Figure courtesy of Tim Roith.*

Science Policy

Continued from page 2

knowledge. Lewis-Burke lobbyists forecast budget and appropriations trends while leaders from federal agencies like the DOE, National Science Foundation (NSF), National Institutes of Health (NIH), and Department of Defense present their organizations' respective visions, which the CSP then communicates to Congress. On the other side of the table, SIAM's esteemed CSP members share nuanced insights and commentary that reflect their robust experience with research funding.

These meetings provide attendees with a remarkable sense of perspective on the relationship between science and government. In my case, I have observed a through line across agency priorities and committee discussions: the importance of training a STEM workforce that is globally competitive, particularly in foundational areas that support larger initiatives (such as artificial intelligence). Given the universality of this goal, CSP meetings also serve as a platform for bellwethers like me to call attention to disparities between the institutional focus on teaching and mentoring at PUIs and the dearth of research funding that is earmarked for these institutions.

Attendees of the 2024 spring meeting of the SIAM Committee on Science Policy (CSP) pose in front of the U.S. Capitol in Washington, D.C., while conducting congressional visits. From left to right: Jake Price of the University of Puget Sound (2023 SIAM Science Policy Fellow), Emily Evans of Brigham Young University (member of the CSP and former SIAM Science Policy Fellow), Miriam Quintal of Lewis-Burke Associates, and Arielle Carr of Lehigh University (2024 SIAM Science Policy Fellow). Photo courtesy of Miriam Quintal.

Ultimately, my participation in the SIAM Science Policy Fellowship Program has spurred my own advocacy efforts in multiple ways. In fact, I hosted a minisymposium and panel about research funding at PUIs[10] during the 2024 SIAM Annual Meeting,[11] which took place in Spokane, Wash., this July. I am also creating a resource and funding guide for PUI faculty and communicating directly with funding agencies about the inclusion of more PUI faculty on review boards for PUI-specific funding opportunities. I look forward to continuing these efforts throughout my career.

Arielle Carr, Lehigh University

As a 2024 SIAM Science Policy Fellow, I have the unique opportunity to advance my professional interests in advocacy and promote diversity in STEM fields while simultaneously fulfilling my academic responsibilities at Lehigh University. Because external funding from federal agencies is essential in my position as an assistant professor, I must thoroughly understand the funding process to guarantee my own research sustainability. I am also passionate about diversity and seek to bolster the experiences of underrepresented groups—particularly women in applied math and computational science—and advocate for inclusive and equitable policy in the mathematical sciences. This Fellowship allows me to meaningfully connect my academic pursuits and desire to improve the cultural landscape in my field with federal lawmaking that dramatically impacts mathematical sciences research.

In May of this year, I visited the Washington, D.C., headquarters of Lewis-Burke Associates for a three-day workshop with CSP members and the other first- and second-year Fellows. On the first day of the workshop, an informative orientation with Lewis-Burke representatives equipped us with the necessary tools to communicate complex scientific ideas to non-expert audiences. The second day involved meetings with senior leadership from a variety of federal agencies, including the NSF, DOE, NIH, National Oceanic and Atmospheric Administration, and National Aeronautics and Space Administration; the delegates spoke about their current areas of interest and corresponding calls for funding. On the third and final day, we broke into smaller groups to converse with representatives from several congressional offices and advocate for continued funding in the mathematical sciences. These interactions strengthened my own advocacy skills and underscored the crucial role of persuasive communication in policy development.

Over the last year, I have learned that we must remain informed about legislative issues that affect the research and education communities, since these issues also impact the decision-making process for scientific funding. As I begin my second year of the SIAM Science Policy Fellowship Program, I look forward to developing and executing an independent policy project that promotes a proactive—rather than reactive—approach towards broader participation and workforce development in computational science and applied mathematics. I anticipate a continued fruitful collaboration with CSP members to ensure the thoughtful integration of scientific research into legislation that impacts society.

Conclusion and Call to Action

The SIAM Science Policy Fellowship Program is an exceptional opportunity for early-career researchers and postdoctoral appointees to gain invaluable experience at the intersection of science and federal policy. The firsthand accounts of current Fellows demonstrate the program's profound ability to influence policy discussions, champion the future of scientific research, amplify scientific voices, and ultimately drive change. We strongly encourage all interested candidates to apply for the Fellowship and join a growing network of scientists who are dedicated to shaping the future of U.S. science policy.

The application deadline for the next round of Fellowship recipients is **November 1, 2024**. To be eligible for consideration, successful applicants must be SIAM members in good standing, work and live in the U.S., be passionate about public policy, possess strong communication skills, and be willing and able to travel to Washington, D.C., twice a year. Applications should include a CV/resume, a candidate statement, and an issue statement about a relevant policy topic of the applicant's choice. Selected Fellows will serve a two-year term that includes training sessions, attendance at CSP meetings, communication with federal officials, and participation in an advocacy day on Capitol Hill in Washington, D.C. To learn more about the Fellowship and submit your application, visit the program's webpage.[12]

Bashir Mohammed is a Senior Staff AI Architect in Intel's Network and Edge Group, where he drives pioneering innovations in artificial intelligence (AI) for the Edge Platform. His work focuses on the development and deployment of large language models, large vision models, and AI reasoning and action agentic frameworks. These efforts deliver transformative solutions across diverse industries to enhance the efficiency, scalability, and intelligence of edge applications. Victor Churchill is an assistant professor in the Department of Mathematics at Trinity College. His research interests include scientific machine learning and inverse problems in imaging, while his policy interests focus on issues that pertain to faculty and students at primarily undergraduate institutions. Arielle Carr is an assistant professor in the Department of Computer Science and Engineering at Lehigh University. She uses techniques such as Krylov subspace recycling and preconditioner updates to develop novel and innovative iterative methods that solve large-scale linear systems and eigenvalue problems to enhance solver efficiency.

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[10] https://www.siam.org/publications/siam-news/articles/key-takeaways-from-an24-session-on-research-funding-at-primarily-undergraduate-institutions

[11] https://www.siam.org/conferences-events/siam-conferences/an24

[12] https://www.siam.org/programs-initiatives/programs/siam-science-policy-fellowship-program

Image Reconstruction

Continued from page 5

an understandable training set, we randomly sample ellipses on the domain $[0,1]^{\times 2}$ as targets x_i and their limited-angle sinograms with additive noise as inputs y_i.[1]

Figure 1 (on page 5) illustrates the output of the direct reconstruction and the learned U-Net. Although the specific ellipse in Figures 1a and 1b was not part of the training data, the network greatly suppresses artifacts and even recovers structures in places where the necessary information was absent from the data. However, Figures 1c and 1d depict the unsatisfactory results when we apply the network to structures that were absent from the training data. Missing structures or biases in the dataset therefore transfer to *out-of-distribution* (OOD) data [6].

Practitioners face two key problems in this setting: the reconstruction of the rectangle in Figure 1c (which is OOD) does not attain the desired result, and the output of the network is a very reasonable image — meaning that even experts often cannot recognize the artifacts. In addition to missing pathologies, this effect can create *hallucinations* (i.e., structures that are completely fabricated by the network). Researchers are investigating these hallucinations for far more complex scenarios, such as magnetic resonance imaging scans of brains [7]. Such cases are rare but critical and call for both OOD detection and uncertainty quantification (UQ). This problem inspires a second important question: *Can we ever quantify the reconstruction error?*

Recent studies use hallucinations to demonstrate the lack of meaningful error estimates in the context of medical imaging [1, 3, 5, 7]. To accentuate this issue, we construct *adversarial examples* [10]: small perturbations of regular inputs that yield catastrophic failure cases. For the same network as before, doing so amounts to solving the following problem:

$$\max_{\varepsilon \in B_\delta} \| f_\theta (A^+(y+\varepsilon)) - x \|_2^2,$$

where x is now an in-distribution input (i.e., an ellipse) and y is the corresponding Radon transform. The adversarial noise ε is restricted to having a small ℓ^2-norm, namely $\|\varepsilon\|_2 / (N \cdot K) \leq \delta = 0.0006$; here, N denotes the width of the image and K is the number of angles.

Figure 2 displays increased noise artifacts for direct reconstruction, though it still carries some interpretable information. The learned model output is completely corrupted, however, with densities that are close to zero and no observable structure whatsoever. Although this example employs a toy setup for illustrative purposes, it aligns with the findings of various studies [5], highlights the difficulty of providing error estimates, and accentuates the potential risk of certain learning methods for ill-posed inversion. We intend for this scenario to serve as a cautionary tale and a call to action, rather than a deprecation of learning methods.

Various advancements can outperform the employed end-to-end architecture $F_\theta = f_\theta \circ A^+$ in terms of both performance and resilience. Furthermore, OOD detection and UQ are emerging as extremely relevant topics that can mitigate the severity of hallucinations. The robustness of neural networks is already positioned as a leading issue for learning, and theoretical understanding—especially in the context of inverse problems—is continuing to grow [4]. To conclude, data-driven methods exhibit enormous potential for improved image reconstruction but will only be of real use if they can prove their resilience and lack of hallucinations; artifacts should be clearly detectable and uncertainties should be quantifiable. This objective calls for a paradigm shift in the development of these methods to focus on UQ and behavior in critical cases, rather than seemingly impressive results for favorable cases.

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Martin Burger is a leading scientist and head of the Computational Imaging Group at Deutsches Elektronen-Synchrotron (DESY), which is part of Helmholtz Imaging. He is also a professor at the University of Hamburg and an expert in inverse problems. Tim Roith is a postdoctoral researcher in the Computational Imaging Group at DESY. He holds a Ph.D. in mathematics from Friedrich-Alexander-Universität Erlangen-Nürnberg.

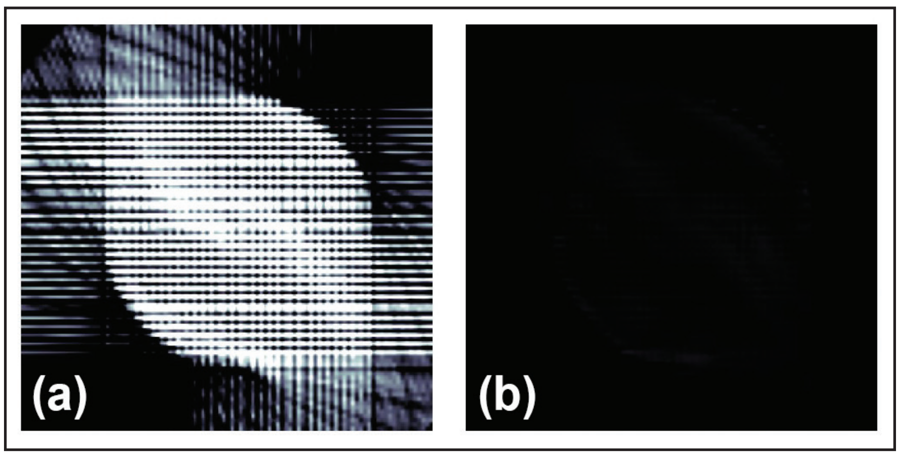

Figure 2. *Direct reconstruction and network output on adversarial inputs. The images are clipped to the valid range* $[0,1]$. **2a.** *Direct reconstruction of the adversarial data.* **2b.** *Network output on the adversarial data. Figure courtesy of Tim Roith.*

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[1] The source code for the experiment is available at https://github.com/TimRoith/LearningInImageReconstruction-SIAM-News.

Meeting in the Middle for RandNLA, Optimization, and Inverse Problems

Recapping the 2024 Gene Golub SIAM Summer School in Ecuador

By Matthias Chung, Juan Carlos De los Reyes, Petros Drineas, Rosemary Renaut, and Alex Townsend

Students and instructors gather for a group photo at Laguna de Cuicocha during the 2024 Gene Golub SIAM Summer School, which took place in Quito, Ecuador, from July 22 to August 2. Photo courtesy of the authors.

Near the equator at an altitude of 2,850 meters above sea level, a group of 50 graduate students from around the world converged for the 2024 Gene Golub SIAM Summer School[1] (G2S3), which took place in Quito, Ecuador, from July 22 to August 2. As co-organizers of G2S3 2024, we formed an instructional team that also included Carola-Bibiane Schönlieb (University of Cambridge) and Malena Español (Arizona State University). The theme of this year's school was "Iterative and Randomized Methods for Large-scale Inverse Problems," and attendees learned about randomized numerical linear algebra (randNLA), optimization, and inverse problems while simultaneously appreciating the awe-inspiring natural beauty and rich culture of Quito. Uplifting research discussions and valuable cultural exchanges became contagious amongst all participants; we dubbed this unbridled enthusiasm the "Gene Golub virus."

The locals call Quito the "Mitad del Mundo," which translates to "middle of the world." Given its central location, students hailed from 12 different countries in the Northern Hemisphere and seven countries in the Southern Hemisphere. Through a series of lectures and group activities, these students explored the latest ideas and concepts in randNLA — inspired by both theoretical computer science and traditional numerical linear algebra. They also studied the use of optimization methods for the solution of discrete, continuous, large, and challenging inverse problems.

Sketching is a popular randNLA technique that utilizes randomly generated near-isometries to efficiently represent a large matrix or dataset with a smaller one that still preserves essential properties. Although we sketched matrices, datasets, least-squares problems, and inverse problems, we never sketched our subsampling of Ecuadorian coffee, chocolate, or bananas. The banana is one of Ecuador's jewels, and we ate many surprising dishes made from the popular fruit. Our favorite was *caldo de bolas de verde*, a delicious soup made from salty bananas.

Quito was the perfect location for the 2024 iteration of G2S3. Throughout the 12-day school, Juan Carlos De los Reyes (Research Center for Mathematical Modeling) served as our incredible local guide; he and his team of graduate students ensured that we experienced all of Quito's offerings. For example, Quito's unique geographical location is known as the "Camino del Sol" or "path of the sun." Twice a year—on the solar equinoxes—the sun is directly overhead at noon and shadows vanish. Three attendees challenged themselves by practically testing out the steepest descent approach in pitch-black mountain terrain. Although they did not discover a converging path of the sun, they did encounter friendly locals who helped them down the mountain.

At the end of G2S3 2024, all of the graduate students presented their work and shared details about their ongoing projects. These presentations were certainly a highlight of the program and will likely inspire several long-lasting collaborations. Ultimately, we are proud to have delivered a thought-provoking, culturally stimulating summer school that hopefully would have made the late Gene Golub smile. We are thankful for his incredible generosity—as well as that of SIAM—for allowing us to provide 50 students with a wonderful sense of community, a strong research base, and unforgettable memories for years to come.

Juan Carlos De los Reyes of Ecuador's Research Center for Mathematical Modeling delivers a lecture at the 2024 Gene Golub SIAM Summer School, which was held in Quito, Ecuador. The 12-day program attracted 50 student attendees and focused on iterative and randomized methods for large-scale inverse problems. Photo courtesy of Matthias Chung.

The 2025 Gene Golub SIAM Summer School on "Frontiers in Multidimensional Pattern Formation" will take place at Concordia University in Montréal, Québec, Canada, from ***August 11-26, 2025****. More information—including application instructions—will be available on the G2S3 webpage[2] later this fall.*

Interested in organizing a future school? Letters of intent that propose topics and organizers for the 2026 iteration of G2S3 are due to Richard Moore, SIAM's Director of Programs and Services, at ***moore@siam.org*** *by* ***January 31, 2025****. Visit the G2S3 webpage[3] to learn more.*

Matthias (Tia) Chung is a professor in the Department of Mathematics at Emory University. His broad research interests lie in scientific machine learning, inverse problems, uncertainty quantification, numerical linear algebra, and associated large-scale applications. Juan Carlos De los Reyes is founding director and a full professor of mathematical optimization at the Research Center for Mathematical Modeling (MODEMAT) in Ecuador. His research focuses on bilevel partial differential equation-constrained optimization, inverse problems, and variational data assimilation with applications in meteorology, non-Newtonian fluids, and imaging science. Petros Drineas is a professor and head of the Department of Computer Science at Purdue University. His work focuses on randomized numerical linear algebra, which he applies extensively to data science — particularly for the analysis of genomic data. He is also a SIAM Fellow. Rosemary Renaut is a professor in the School of Mathematical and Statistical Sciences at Arizona State University. Her research explores the development of robust algorithms for the solution of large-scale inverse problems with practical applications. She is also a SIAM Fellow. Alex Townsend is a professor in the Department of Mathematics at Cornell University. His research focuses on the development of algorithms for numerical linear algebra, scientific computing, and operator learning.

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[1] https://g2s32024.github.io

[2] https://www.siam.org/programs-initiatives/programs/gene-golub-siam-summer-school

[3] https://www.siam.org/programs-initiatives/programs/gene-golub-siam-summer-school

AI in Education: A Progressive, Practical Proposal

BOOK REVIEW

By John R. Jungck

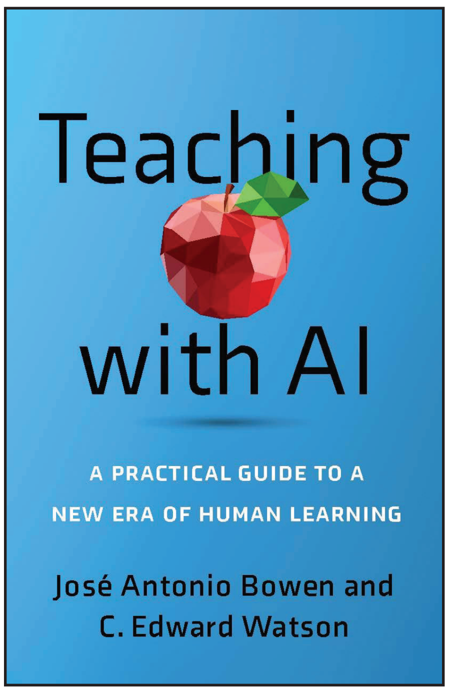
Teaching with AI: A Practical Guide to a New Era of Human Learning. *By José Antonio Bowen and C. Edward Watson. Johns Hopkins University Press, Baltimore, MD, April 2024. 280 pages, $24.95.*

As artificial intelligence (AI) continues to revolutionize mathematics [3], increasingly more students are utilizing the new generation of AI tools for their coursework. In response, educators must adopt, adapt, and implement AI-based mathematical education research that affects their pedagogy. *Teaching with AI: A Practical Guide to a New Era of Human Learning* by José Antonio Bowen and C. Edward Watson is the best, most balanced, and most insightful guide on this topic that I have found to date. Perhaps I shouldn't be surprised, as Bowen—a former president of Goucher College—is the author of several popular books on teaching and Watson is Vice President for Digital Innovation at the American Association of Colleges and Universities. The two previously collaborated on a 2017 book titled *Teaching Naked Techniques: A Practical Guide to Designing Better Classes* [2].

Bowen and Watson assert that educators should not perceive current AI as a threat, but rather welcome AI tools as catalysts for progressive change that enhance learning, develop students' creative potential, and "transform cheating into innovation." While the authors do make generic statements like any other pundits—e.g., "Previous AI helped *curate* your world ... but GPT AI will allow you to *create* your world"—they explicitly identify the impactful aspects of AI that will inspire users to generate ideas, ask better questions, evaluate assumptions, seek contradictory evidence, investigate additional sources, anticipate problems, clarify concepts, find information, create visualizations, write drafts, offer feedback, analyze data, and so forth.

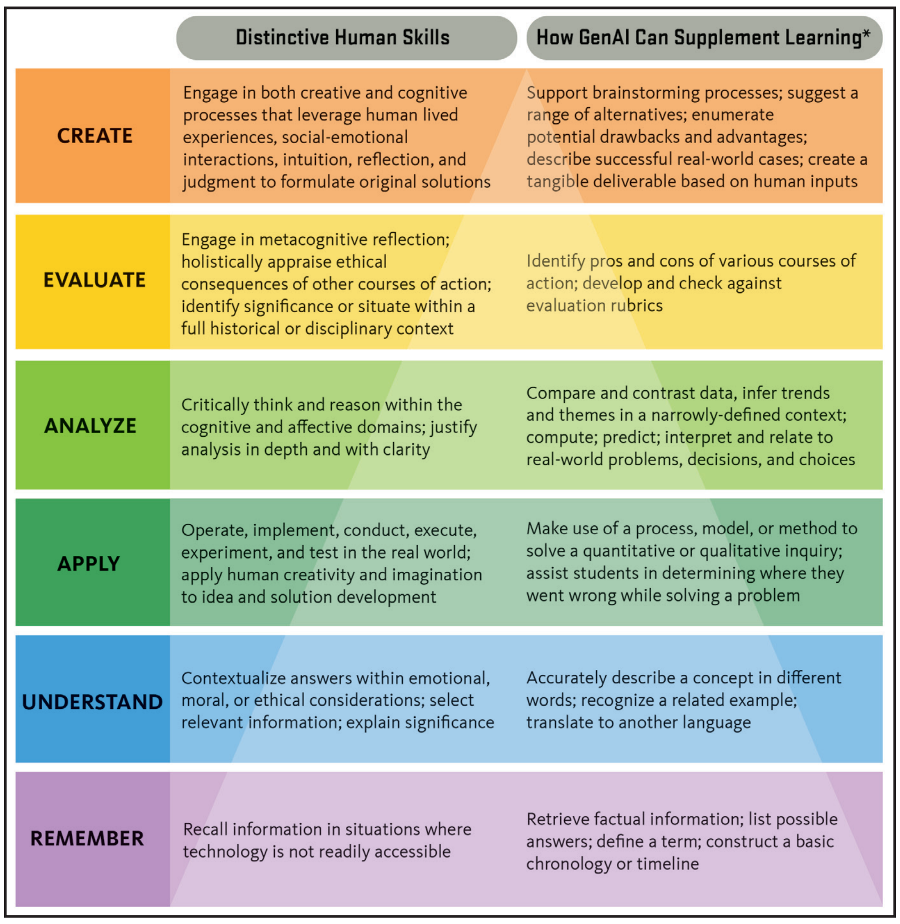
A number of sources have generated updates of Benjamin Bloom's classic taxonomy that incorporate AI; for instance, Figure 1 displays an Oregon State University revision of Bloom's taxonomy[1] that is referenced in the book. One could similarly modify my pyramid of a quantitative reasoning perspective to reflect the significance of AI (see Figure 2, on page 9). Furthermore, several recent discussions have focused on the movement from Ed 3.0 (one size fits all) to Ed 4.0 (dynamic and responsive) and even Ed 5.0 (adaptive learning) [8]. These levels represent three paradigms: (i) directed education, wherein the student is a recipient; (ii) supported education, wherein the student is a co-learner; and (iii) empowered education, wherein the student is a collaborator [7]. For adult learners, this shift marks a transition from pedagogy to andragogy to heutagogy [1].

While most *SIAM News* readers can likely skip Bowen and Watson's introductory chapter about AI and the shift from early rule-based systems to contemporary machine learning and large language models (LLMs), I still found helpful tidbits in their narrative of different AI applications that can survey the literature and possibly verify information. The authors list 12 LLMs (seven of which they describe in terms of ability and limitation) and 20 software intermediaries for specific functions such as writing, research, and coding. This first chapter also contains a glossary of significant terms like LLM, application programming interface, generative AI, and artificial general intelligence.



Teaching with AI: A Practical Guide to a New Era of Human Learning. By José Antonio Bowen and C. Edward Watson. Courtesy of Johns Hopkins University Press.

Consistent with existing math education literature on problem posing, Bowen and Watson elaborate on the axiom that "asking the right question requires first questioning the problem." They illustrate a Double Diamond innovation process that involves both divergent and convergent thinking about discoveries and definitions in problem posing as well as ideas, prototypes, and tests in problem solving. The authors write that "the more we can prepare students to question assumptions, analyze problems more deeply, tolerate the discomfort of ambiguity, find subproblems, and clearly reframe problems…the better prepared they will be not only for the first wave of AI-inspired jobs, but for the subsequent waves that are still unknown."



	Distinctive Human Skills	How GenAI Can Supplement Learning*
CREATE	Engage in both creative and cognitive processes that leverage human lived experiences, social-emotional interactions, intuition, reflection, and judgment to formulate original solutions	Support brainstorming processes; suggest a range of alternatives; enumerate potential drawbacks and advantages; describe successful real-world cases; create a tangible deliverable based on human inputs
EVALUATE	Engage in metacognitive reflection; holistically appraise ethical consequences of other courses of action; identify significance or situate within a full historical or disciplinary context	Identify pros and cons of various courses of action; develop and check against evaluation rubrics
ANALYZE	Critically think and reason within the cognitive and affective domains; justify analysis in depth and with clarity	Compare and contrast data, infer trends and themes in a narrowly-defined context; compute; predict; interpret and relate to real-world problems, decisions, and choices
APPLY	Operate, implement, conduct, execute, experiment, and test in the real world; apply human creativity and imagination to idea and solution development	Make use of a process, model, or method to solve a quantitative or qualitative inquiry; assist students in determining where they went wrong while solving a problem
UNDERSTAND	Contextualize answers within emotional, moral, or ethical considerations; select relevant information; explain significance	Accurately describe a concept in different words; recognize a related example; translate to another language
REMEMBER	Recall information in situations where technology is not readily accessible	Retrieve factual information; list possible answers; define a term; construct a basic chronology or timeline

Figure 1. *An adaptation of Bloom's taxonomy for educational goals in the context of artificial intelligence. Figure courtesy of Oregon State University Ecampus via the Attribution 4.0 International (CC BY 4.0) deed.*

While Bowen and Watson explicitly address concerns about cheating and plagiarism, they do so after a chapter titled "Reimagining Creativity" because they believe that educators must first appreciate that AI is iterative. "The goal is not to have AI do the thinking," they write, "but to have a dialogue that helps you think." After detailing AlphaFold's[2] success in the 2020 Critical Assessment of Structure Prediction competition and its subsequent prediction of a three-dimensional structure with more than 200 million proteins, they surmise that AI will significantly impact numerous scientific problems — including star mapping, climate modeling, the new generation of antibodies, and hydrogen fusion research. The authors thus view AI as a partnership, stating that "it is in the iteration, the reflection, the back and forth, and the refined questions that thinking and creativity happen."

A subsequent chapter about "AI Cheating and Detection" reveals that 22 percent of college students have utilized AI for schoolwork; 49 percent used AI tools on a daily, weekly, or monthly basis in fall 2023; and 89 percent use ChatGPT in some capacity. Given this current state of affairs, the text details a number of AI detectors with varying rates of false positives. Since many of these tools are unreliable (even if they exceed most faculty's ability to detect AI writing), Bowen and Watson assert that the alternative is better course design. Their proposed strategies include discussing academic integrity, giving "integrity quizzes," allowing students to withdraw submissions, demonstrating AI detection tools, and normalizing homework assistance. They conclude the chapter by stating that "increasing transparency, relevancy, belonging, and motivation will all reduce cheating, but none will eliminate it."

This commentary introduces a very interesting rubric with four assessment levels: Absent (0 percent), AI level (50 percent) = F, Good (80 percent) = B, and Great (100 percent) = A. When I tried this rubric on my own students, they only reacted negatively to the criteria for the harsh reality of an "F." "AI has shrunk the distance between a C grade and an A grade, but it has made articulating the distance between them more important," the authors write. "Combining high standards with high care, building trust and community, focusing on equity and inclusion, increasing motivations, and creating better, clearer, and more relevant assignments…can both increase learning and reduce cheating."

The culminating section of *Teaching with AI* focuses on "Learning with AI." Bowen and Watson concentrate on the use of AI to provide instantaneous and individualized feedback; offer responsive tutoring; serve as a debate partner with specificity and contextual knowledge; act as a discussion leader to eliminate certain social pressures; and help students run scenarios, create visualizations, and forecast

See **AI in Education** *on page 9*

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[1] https://ecampus.oregonstate.edu/faculty/artificial-intelligence-tools/meaningful-learning

[2] https://alphafold.ebi.ac.uk

MPE24 Panel Explores Mathematical Careers in Earth Science and Sustainability

By Jillian Kunze

Mathematicians who are passionate about the environment have many opportunities to bridge these two fields. Applied mathematics, computational science, and data science directly contribute to studies on climate modeling, ecological dynamics, and clean energy technologies — among numerous other application areas in science, technology, engineering, and mathematics (STEM). While these connections are not always clear from the outset, a multitude of paths within academia, industry, and the national laboratories lead to jobs that focus on important research questions that are rooted in Earth science.

During the 2024 SIAM Conference on Mathematics of Planet Earth[1] (MPE24), which took place in Portland, Ore., this past June, a panel[2] of researchers discussed career trajectories for mathematically inclined individuals who are also enthusiastic about MPE. Alex Cannon of the Meteorological Service of Canada chaired the conversation between Nicole Jackson of Sandia National Laboratories, Alice Nadeau of C.H. Robinson,[3] and Thordis Thorarinsdottir of the University of Oslo, all of whom overviewed the ways in which STEM careers can encompass both mathematical methods and domain applications in the broad realm of Earth and environmental science.

Initial conversation centered on whether graduate school is necessary to work in this space. Nadeau—who performs backend data analytics for carbon emission reporting tools at a third-party logistics company—provided an industry-based point of view. "On my team, there's a mix," she said. "Half of my team started right after undergrad as entry-level data analysts and moved up, and the other half are academic converts." In light of this flexibility, Nadeau urged students to consider their personal motivations: Would they rather enter the workforce immediately or pursue graduate school and postdoctoral appointments, given that such credentials are not strict requirements for MPE-relevant occupations? "Graduate school isn't a necessary condition in my case," she said.

CAREERS IN MATHEMATICAL SCIENCES

However, Thorarinsdottir cautioned that individuals who do not obtain advanced degrees may find themselves passed up for promotions or opportunities to tackle more interesting problems. She suggested that one might wish to earn an undergraduate degree, work for a few years to gain real-world experience, then revaluate personal and professional goals and potentially return to graduate school. Jackson noted that Sandia National Laboratories' robust internship program[4] can lead to permanent employment for successful and engaged undergraduate students. But while Sandia hires employees at all educational levels, graduate degrees may indicate a stronger aptitude for problem-solving and accelerate pathways towards leadership roles. "You shouldn't let the job market dictate your educational goals," Jackson said. "You should do it because it matters to you."

The discussion then turned to job-hunting strategies. The last several years have seen an increase in the availability of remote positions, providing additional avenues for researchers to find placements that align with their ambitions and values. Unfortunately, recent layoffs in the tech industry have simultaneously tightened the job market. As such, it is especially important that job seekers highlight specific projects in their application materials that directly relate to their desired career paths. Online platforms like Kaggle[5] are great resources for activities that can later serve as concrete interview talking points.

Although Jackson sent out a relatively small number of applications before securing a postdoctoral appointment at Sandia, she was forthright about her desire to convert to a full staff position at the lab. "If you want to have competing offers and boost your salary, there is a little bit more strategy in being aggressive and seeking lots of job offers," she said. On the other hand, Thorarinsdottir commented that the application portfolios of people who apply for too many positions may become generic and untargeted — and hiring managers will notice. She hence advised attendees to focus on positions that they genuinely find compelling.

The timeline for job application submissions compared to graduation date varies by field and company. If a soon-to-be graduate applies well before their feasible start date but submits a strong portfolio, some places may wait to hire them. "If you're good and apply to something you really want and they really want you, they will wait for you," Thorarinsdottir said. However, other organizations may operate on more rapid timelines and seek to fill open appointments as soon as possible. Either way, it never hurts to take a chance. "If you think a place might work for you, submit your things and just see where the chips land," Jackson said.

Nadeau began the job search process by chatting with all of her contacts in industry and inquiring about their responsibilities. When one of her colleagues asked her what *she* wanted to do, Nadeau reframed her thinking and determined that she felt motivated by

See **MPE24 Panel** *on page 11*

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[1] https://www.siam.org/conferences-events/past-event-archive/mpe24

[2] https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=80664

[3] https://www.chrobinson.com

[4] https://www.sandia.gov/careers/career-possibilities/students-and-postdocs/internships-co-ops

[5] https://www.kaggle.com

At the 2024 SIAM Conference on Mathematics of Planet Earth, which took place in Portland, Ore., this past June, a panel of researchers explored career opportunities at the intersection of mathematics, environmental science, and sustainability. From left to right: moderator Alex Cannon of the Meteorological Service of Canada and panelists Thordis Thorarinsdottir of the University of Oslo, Nicole Jackson of Sandia National Laboratories, and Alice Nadeau of C.H. Robinson. SIAM photo.

AI in Education

Continued from page 8

and test ideas. They note that "if we are to prepare students for a world where collaboration with AI is required rather than prevented, then helping students leverage AI to produce better work should become a signature pedagogy of higher education."

As a faculty member, I was surprised by the many ways in which AI already influences my professional life. For instance, some journals and grant agencies are deploying AI in a variety contexts that extend beyond checking for plagiarism, quality, and adherence to guidelines even before they share a manuscript or grant application with reviewers. Unfortunately, *Teaching with AI* was already complete when Lisa Messeri and M.J. Crockett published their "taxonomy of productivity and objectivity" with four aspects of considerable concern about the use of AI for research as an (i) oracle, (ii) surrogate, (iii) quant, and (iv) arbiter [5]. Each of these AI collaborations engenders epistemic, ethical, social, and cultural susceptibilities to illusions of understanding. Messeri and Crockett argue that "the proliferation of AI tools in science risks introducing a phase of scientific enquiry in which we produce more but understand less."

So what's missing? Applied mathematics educators may feel that Bowen and Watson are simultaneously too generic and too focused on humanities and social science. With 20 pages of references, readers have plenty of opportunities to peruse further literature. In fact, many SIAM members are already exploring the issues and potentialities of *Teaching with AI* in their own classrooms. In *SIAM News* earlier this year, Alvaro Ortiz Lugo summarized a SIAM Education Committee session about AI and education[3] at the 2024 Joint Mathematics Meetings [6]. The eight talks in the session addressed the use of AI in theorem proving; data analysis; modeling; coding and computation; visualization; parameterization; spatial reasoning; audio, video, and text processing; emotion detection and classification; lesson planning; report writing; and more. As AI continues to advance in the coming months and years, increasingly more members of the SIAM community will undoubtedly witness and adapt to its effects in a variety of academic settings.

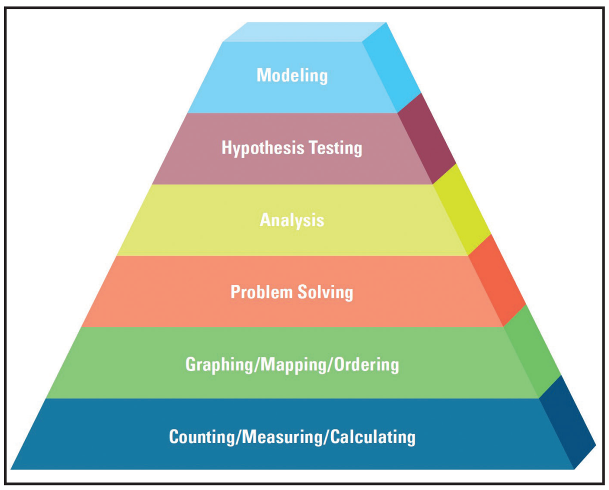

Figure 2. *Bloom-like taxonomy pyramid of quantitative reasoning. Figure courtesy of [4].*

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[3] https://www.siam.org/publications/siam-news/articles/siam-session-at-jmm-2024-addresses-artificial-intelligence-in-the-education-sector

Pseudomagnetism in Photonics: From Mathematical Theory to Experiment

By Mikael C. Rechtsman and Michael I. Weinstein

While magnetic fields deflect the paths of charged particles like electrons, they do not directly influence the path of light. In 2021, we developed a mathematical theory that demonstrated how to engineer a two-dimensional (2D) nonmagnetic photonic crystal in which photons of light move much like electrons under the influence of a magnetic field [5]. In particular, we showed that a nonuniformly deformed photonic (or other wave-propagating) continuous 2D medium with honeycomb spatial symmetries gives rise to effective pseudomagnetic and electric fields, which influence the propagation of light waves. A strained pattern that produces a constant perpendicular pseudomagnetic field induces *photonic Landau levels* within the structure. We can use these infinitely degenerate states of light to enhance light-matter interactions; potential applications include increased emission from quantum emitters such as quantum dots, and more efficient generation of quantum light in the form of entangled pairs of photons. Our study also suggested strategies for inducing topological phenomena in photonic and other wave systems that are analogous to the phenomena in quantum materials. Recent experiments observed and confirmed our predictions [1, 2].

Our work was motivated by an analogous effect in condensed matter physics for the 2D material *graphene*: a monolayer of carbon atoms with the symmetries of a honeycomb tiling. After graphene's discovery, researchers realized that appropriately strained graphene causes electrons to behave as though they were flowing in the presence of an out-of-plane magnetic field — thus exhibiting Landau-level electronic spectra with a high density of electronic states [9].

Is such an effect possible for photons as well? A previous study used the *tight-binding approximation*, which underlies condensed matter predictions for graphene, to anticipate this effect for photons in optical waveguide arrays [10]. However, tight-binding theory is not valid in 2D *photonic crystals* [6]; instead, we proposed a continuum theory that is applicable to 2D Maxwell's equations [5]. As a complement to our experimental work [2], we extended this theory to encompass the vectorial effects of three-dimensional Maxwell's equations. This vectorial theory yields excellent agreement with experiment, with no free fitting parameters.

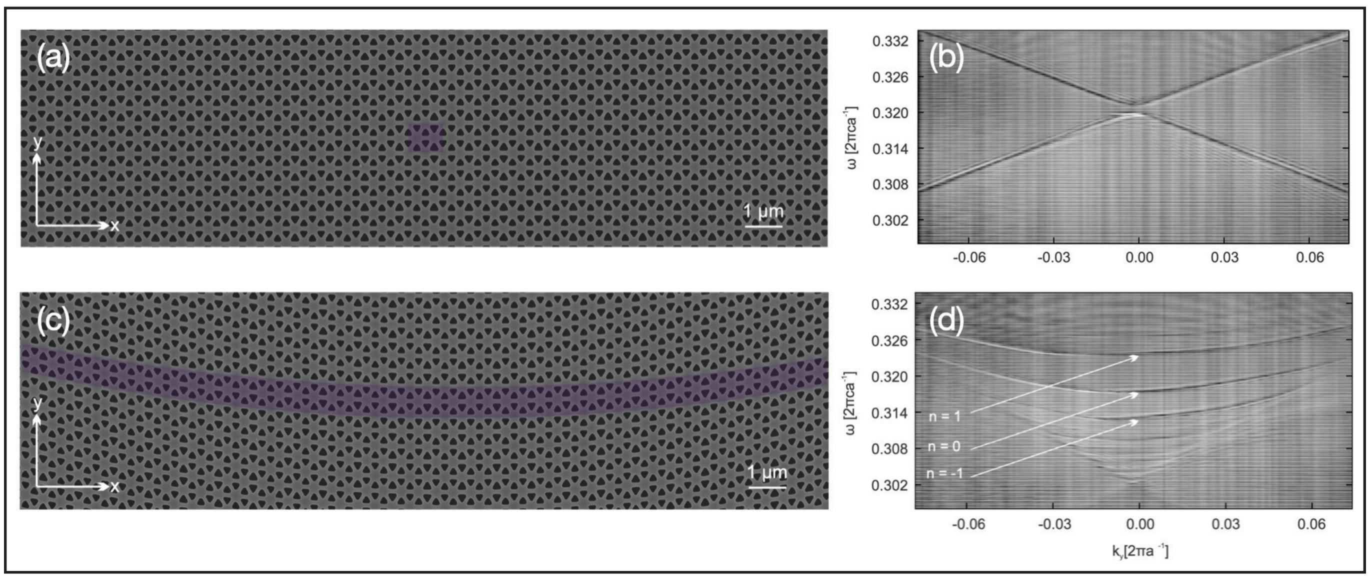

Figure 2. *Unstrained and strained photonic crystals and their associated spectra.* **2a.** *Electron microscope image of photonic crystal structure, with a unit cell in purple.* **2b.** *An experimentally observed reflectance spectrum that shows the Dirac point as a small gap that results from perturbations to the structure.* **2c.** *A photonic crystal structure that is similar to 2a, after the application of a coordinate transformation that corresponds to the strain.* **2d.** *The resulting spectrum with Landau levels. Here,* k_y *is the* y*-component of the wavevector,* n *indicates the Landau level indices, and* a *is the lattice constant. Figure courtesy of [2].*

The Unstrained Honeycomb Medium and Dirac Quasi-particles

Time-harmonic transverse electric modes of Maxwell's equations, which model electromagnetic waves of frequency ω in a dielectric medium, are determined by solutions of the scalar Helmholtz equation

$$H\psi = -\nabla_x \cdot \xi(x)\nabla_x\, \psi(x) = E\psi(x),$$
$$x \in \mathbb{R}^2;\;\; E = \left(\frac{\omega}{c}\right)^2.$$

Here, $\xi(x) = [\varepsilon(x)]^{-1}$ and $\varepsilon(x)$ denotes the medium's electric permittivity. We assume that the medium is nonmagnetic and set the vacuum value for the magnetic permeability as $\mu = \mu_0$. The scalar function $\psi(x) = \psi(x_1, x_2)$ denotes the x_3 (longitudinal) component of the magnetic field: $\mathbf{H}(x) = \psi(x)\hat{\mathbf{x}}_3$. We also assume that *the dielectric constant has the symmetries of honeycomb tiling on the plane* (see Figure 1a). Precisely, we suppose that $\xi(x)$ is periodic with respect to the equilateral triangular lattice, invariant under parity inversion ($\mathcal{P}[\xi](x) = \xi(-x) = \xi(x)$), and real-valued ($\mathcal{C}[\xi](x) = \overline{\xi(x)} = \xi(x)$).

Since $L = -\nabla_x \cdot \xi(x)\nabla_x$ is a self-adjoint operator on $L^2(\mathbb{R}^2)$ with periodic coefficients, L has a *Floquet-Bloch band structure*: a generalization of the usual spectral decomposition of $L^2(\mathbb{R}^2)$ into the plane-wave eigenstates of the Laplacian that arise for $\xi(x) \equiv 1$. We can express the wave propagation in a honeycomb medium via a continuous weighted superposition of Floquet-Bloch modes $\Phi(x,k) \in L^2_{\mathrm{loc}}$ that solve the family of self-adjoint elliptic eigenvalue problems

$$H\Phi = E\Phi, \quad \Phi(x+v,k) = e^{ik\cdot v}\Phi(x,k), \tag{1}$$

which are parameterized by the quasi-momentum (or k pseudo-periodicity parameter) $k \in \mathcal{B}$. Here, $\mathcal{B}$ is the Brillouin zone: a fundamental period cell that is associated with the dual lattice Λ^*. For each $k \in \mathcal{B}$, (1) has a discrete spectrum of $E_1(k) \le E_2(k) \le \ldots \le E_b(k) \le \ldots$ with corresponding k pseudo-periodic eigenstates $\Phi_b(x,k)$ of (1). The graphs $k \mapsto E_b(k)$ are called *dispersion surfaces* or *bands* and are the analogues of dispersion relations that arise in the solution of constant coefficient wave equations via the Fourier transform.

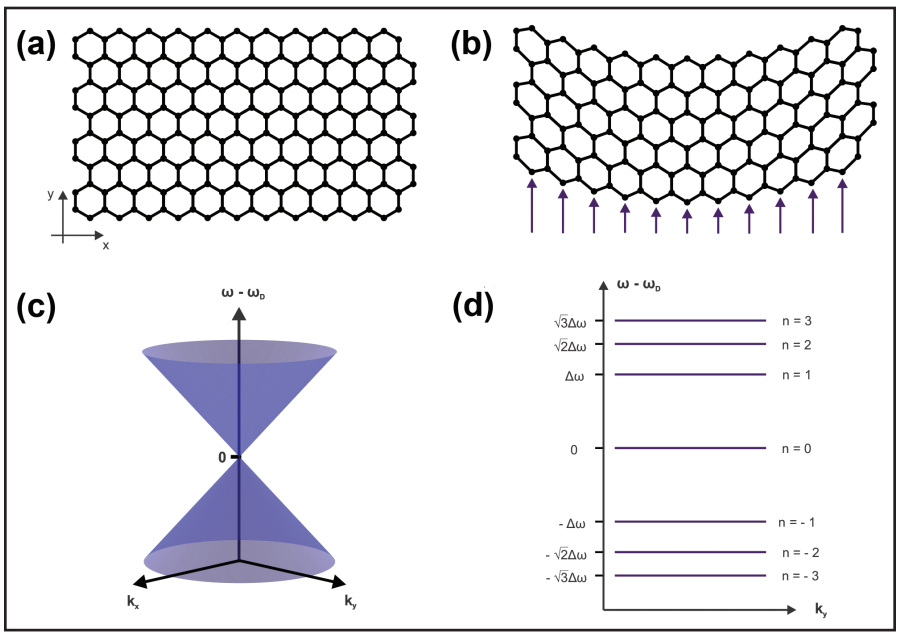

Figure 1. *Undeformed and deformed honeycomb media and their local band structures. Figure courtesy of Zeyu Zhang.*

Remarkably, the symmetries of honeycomb media give rise to *Dirac points* in the band structure. At these energy/quasi-momentum pairs (E_D, k_D), two adjacent dispersion surfaces touch in a locally conical manner:

$$\begin{aligned} &E_\pm(k) - E_D \\ &\quad = \pm v_D\, |k-k_D|\, (1 + \mathcal{O}(|k-k_D|)), \end{aligned}$$

where $v_D > 0$ and $E_D = (\omega_D / c)^2$ [3, 8]. Figure 1a illustrates a medium with honeycomb symmetry, while Figure 1c depicts the local behavior of two dispersion surfaces that touch in a Dirac point. Around $E = E_D$, the spectrum of H is continuous with zero density of states at E_D. An important consequence of this local band structure—which underlies many of graphene's important properties—is the fact that the envelope of wavepackets (quasi-particles) that are spectrally localized about a Dirac point propagates according to a system of 2D Dirac equations with propagation speed (Dirac/Fermi velocity) v_D. The effective Dirac Hamiltonian is as follows [4]:

$$\not{D}_{\mathrm{eff}} \equiv v_D\; \sigma \cdot \mathbf{P}.$$

Here, $\mathbf{P} = (-i\partial_{Y_1}, -i\partial_{Y_2})$, $\sigma_0 = \mathrm{Id}_{2\times 2}$, $\sigma = (\sigma_1, \sigma_2)$, and σ_j are the standard 2×2 Pauli matrices.

Inducing Effective Magnetic and Electric Field Potentials

Now imagine a nonuniform distortion of the honeycomb medium (a strain) on a length scale of κ^{-1}, which is large when compared with the triangular lattice period a: $x \mapsto T_\kappa(x) = x + \mathbf{u}(\kappa x)$, where $\kappa a \ll 1$. Figure 1b displays a strain deformation that only varies along x_1; it maintains discrete translation symmetry with respect to x_2. The modes of the strained medium are governed by the deformed Helmholtz equation

$$-\nabla_x \cdot \xi'(x)\nabla_x \psi(x) = \left(\frac{\omega}{c}\right)^2 \psi(x), \;\; x \in \mathbb{R}^2,$$

with dielectric parameter $\xi'(x) = (\xi \circ T_\kappa^{-1})(x)$, where $\xi(x) = 1/\varepsilon(x)$.

So, how do wavepackets evolve in such a distorted medium? We found that the envelope of wavepackets evolves under $i\partial_T \alpha = \not{D}_{A,\mathrm{eff}}\, \alpha$: a Dirac equation where

$$\not{D}_{A,\,\mathrm{eff}} \equiv v_D \sigma \cdot (\mathbf{P} - \mathbf{A}_{\mathrm{eff}}) + \sigma_0 W_{\mathrm{eff}}.$$

$\mathbf{A}_{\mathrm{eff}}(\mathbf{Y})$ and $W_{\mathrm{eff}}(\mathbf{Y})$ are effective magnetic and electric potentials that are given explicitly in terms of the deformation $x \mapsto T_\kappa(x)$ [5].

Choice of Strain Gives Rise to Photonic Landau Levels

Next, we describe the connection between the choice of strain and the scenario in Figures 1b and 1d. For a strain with a single direction of discrete translation symmetry (as in Figures 1b and 1d), the band structure consists of dispersion *curves* that are parameterized by a single quasi-momentum parameter k_y. If this distortion is taken to be quadratic—that is, $T_\kappa(x) = (x_1, x_2) + (0, (\kappa x_1)^2)$—then we claim that for small κ, the dispersion curves are well-approximated in an asymptotic sense by the flat k_y independent bands (see Figure 1d). The eigenspaces that correspond to the flat bands are spanned by states of the form $g(x_1)e^{ik_y x_2}$, which are Gaussian localized in the x_1 direction (around the structure's center of symmetry) and similar to plane waves in the x_2 direction. Because the group velocity vanishes on a flat band, wavepackets that are formed from a superposition of these states will neither transport nor dispersively spread light. Such states are therefore candidates for enhanced light-matter coupling, including nonlinear optical effects.

Experiments

We used a silicon-on-insulator platform to conduct our experiments and employed electron beam lithography to fabricate a periodic photonic crystal by etching small triangles into a silicon slab—220 nanometers thick with a refractive index of 3.5—on a silica substrate with a refractive index of 1.5 (see Figure 2). The periodic structure is a honeycomb lattice with six-fold rotational symmetry. Experimentally, we injected a focused beam (several microns in width) onto the slab and captured the reflected light through the focusing lens. The slab's band structure corresponds to resonant modes at particular wave vectors $\mathbf{k}$ and frequencies ω that absorb the incoming light and produce clearly observable features in the reflectance spectra at the appropriate angles and wavelengths.

We fabricated this structure in both the fully periodic honeycomb lattice geometry and the strained geometry, wherein the lattice is distorted according to the transformation T_κ (described above). In the

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Understanding Jupyter as a Valuable Student Resource for Computational Discovery

FROM THE SIAM BOOKSHELF

By Neil J. Calkin, Eunice Y.S. Chan, and Robert M. Corless

The following is a brief description from the authors of Computational Discovery on Jupyter,[1] which was published by SIAM in 2023. The text uses Python, a popular programming language, to expose students to mathematical ideas outside of the standard curriculum. Multimedia materials, associated Jupyter Notebooks, and sample projects facilitate an active learning approach and encourage students to interact and play with the concepts.

Computational Discovery on Jupyter utilizes deliberately nonstandard and offbeat computational topics to motivate the notion of mathematical proof and inspire students to write small computer programs. Each of the seven chapters contains multiple "activities" that promote further exploration, and the appendix includes a complete report on every activity. With guidance from these examples, the book prompts students to ask their own questions.

The Target Audience

The text is meant for students who are just entering university. It contains almost no proofs because we concur with the dictum of Ed Barbeau—professor emeritus at the University of Toronto—that "there should be no proof without doubt" (in this case, doubt on the student's part about the result). Similarly, during a conference at the Pennsylvania State University in 1994, the late physics professor Henry Abarbanel stated that "I have absolutely no interest in proving things that I know are true." This attitude is widespread throughout the community.

Teachers generally wish to *motivate* their students to actually want to prove things, but they must overcome several obstacles in order to do so. We wrote this book—which we originally conceptualized as an online educational resource,[2] rather than a full publication—to help surmount some of these challenges. Because many high school curriculums have eliminated proofs from their lesson plans, the first hurdle is that incoming college students have much less experience with proofs than in the past.

Therefore, we decided to sneak up on the idea of proof from a different angle. We also want students to discover things on their own, so we encourage *active learning* with rich mathematical topics (like continued fractions) that leave room for experimentation. Writing small programs that perform unusual computations quickly leads to "bugs," as such programs often do not behave as expected. This, we find, is a natural place to plant the seeds of doubt.

Students are typically eager to learn to code, and use of the popular language Python (as well as Maple) inside the Jupyter Notebook environment[3] allows for some economy of effort on their part. The skills within *Computational Discovery on Jupyter* are easily transferable to other topics, and the exploration of simple mathematical concepts is perhaps the most straightforward path to programming that we are likely to find.

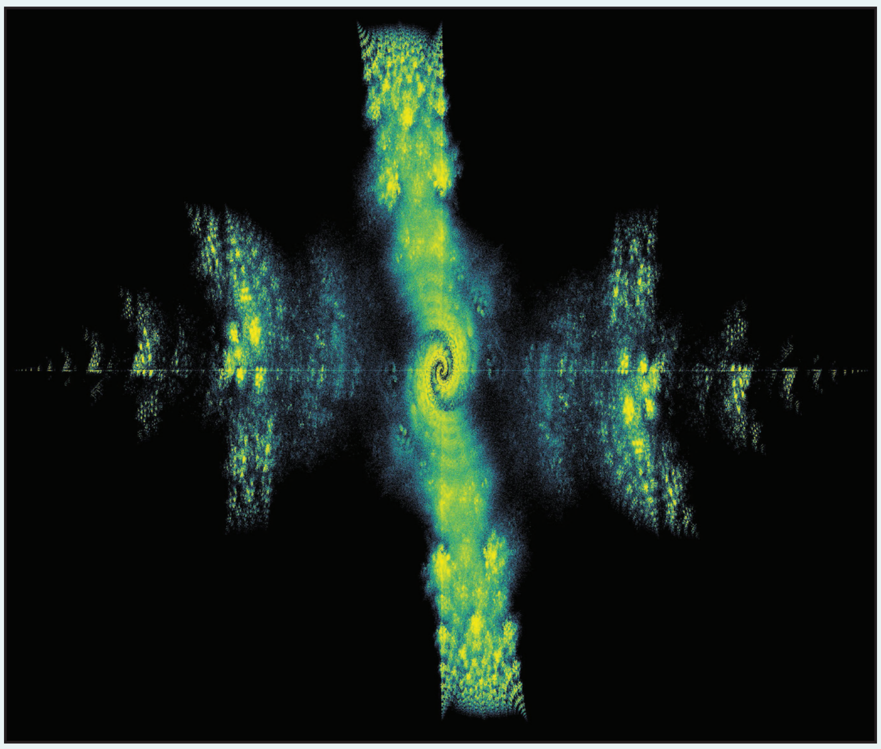

Figure 1. *An eigenvalue density plot for skew-symmetric tridiagonal matrices with population* $(2,\ 5i,\ 2+i)$, *shown on* $-10 \leq \Re(\lambda) \leq 10,\ -5 \leq \Im(\lambda) \leq 5$ *(axes are not identically scaled). Eigenvalues of all* $3^{13} = 1{,}594{,}323$ *dimension-14 matrices are computed and imaged in the Maple™ programming language. What can be said about the "vortex" in the middle? We certainly don't understand it! Figure courtesy of Robert Corless.*

Purpose and Intent

By highlighting offbeat topics with motivating applications and appealing images, we hope to attract a wide pool of students who are potentially interested in mathematical careers. For instance, students really seem to delight in the inclusion of Bohemian matrices[4] [1] and appreciate the project-based nature of the work. While instructors probably *could* use this book for an exam-based course, we have not done so.

Given its many possibilities, this book is *not* going to simplify your life as an instructor. It will, however, make coursework more fun for both you and your students.

Open Problems

Another goal of the text is to introduce students to open problems, which appear in every chapter. To be sure, the majority of them are not "serious" problems — they are mostly just problems for which we do not have answers. For instance, Figure 1 contains a visible spiral in the center even though the image depicts a plot of the density of all eigenvalues of a particular Bohemian family of matrices. Each matrix is 14×14, skew-symmetric, and tridiagonal; they draw their nonzero entries from the finite population 2, $5i$, and $2+i$, where $i^2 = -1$. Why should the eigenvalues spiral? We simply do not know.

In addition to witnessing their teachers' occasional bafflement with existing images, students like to generate their *own* images that also sometimes baffle us. In fact, some of the figures on our Bohemian matrices website[5] were created by students and gave rise to questions (e.g., about circulant matrices) that still remain unanswered.

We had a lot of fun writing *Computational Discovery on Jupyter* and enjoy teaching with this material. We believe that other instructors can come up with creative new ways to use the text, and we wish you joy and success in your teaching endeavors.

Enjoy this passage? Visit the SIAM Bookstore[6] to learn more about Computational Discovery on Jupyter[7] and browse other SIAM titles.

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[1] https://epubs.siam.org/doi/book/10.1137/1.9781611977509

[2] https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter

[3] https://jupyter.org

[4] https://www.siam.org/publications/siam-news/articles/rhapsodizing-about-bohemian-matrices

[5] http://www.bohemianmatrices.com/gallery

[6] https://epubs.siam.org/bookstore

[7] https://epubs.siam.org/doi/book/10.1137/1.9781611977509

MPE24 Panel

Continued from page 9

sustainability questions. She then began asking individuals in her network if they could connect her with people at their companies who worked in her targeted domain, which led to interview opportunities.

Reaching out to professional acquaintances in this capacity can be intimidating, but referrals to new contacts are almost always helpful. Even without a direct connection at a company of interest, crafting a tailored message and demonstrating legitimate enthusiasm increases the likelihood of a response. Additionally, potential employers find it easier to respond if an initial message inquires more broadly about general opportunities, rather than immediately requesting an interview. "If it's a place that you're really interested in, it also doesn't hurt to follow up," Nadeau said, adding that the recipient might have missed the original message.

While many people naturally transition to industry after graduate school, former academics should nonetheless make a concerted effort to market themselves for industry positions. Nadeau recommended that candidates craft a one-minute elevator pitch that serves as a condensed introduction to their personal story, skills, and goals. "Have some sort of motivation of why you're interested in the types of jobs that you're applying to," she said. "People want to connect with you on a human level. They want to know that they can work with you and understand your motivation."

To effectively communicate these incentives, candidates must first explicitly define their career goals and articulate them in a succinct manner. "You have to explain why you, as a person, want this job," Thorarinsdottir said. "That holds for whatever job you're applying for." Doing so often requires pivoting between different levels of technical descriptions and breaking down ideas for non-experts in an approachable, non-condescending way.

The panelists next addressed potential tensions between the pursuit of novel mathematical methods versus the use of established techniques to answer real-world questions. In many settings—particularly when simultaneously managing the varying expectations of multiple different people—researchers must trust themselves to strike a balance between these two sides. Jackson especially feels this distinction when writing proposals, as funding is typically bucketed into either fundamental or applied science. Difficulties can arise when a project is potentially too fundamental for the applied fund or too applied for the fundamental fund; applicants must then decide which aspect of their work to emphasize. Regardless, Jackson first determines whether a call for funding involves a topic area that excites her. If it does, she crafts a proposal that responds to the specifics of the funding opportunity and addresses the corresponding need within the scientific community.

Thorarinsdottir typically tries to balance applications for funding opportunities that have high acceptance rates with the pursuit of projects that most interest her. "You have to be a bit flexible here because the best money you're going to apply for is the money you're going to get," she said. Nadeau brought a different perspective, as she does not have to apply for funding in her industry role. However, other groups at C.H. Robinson occasionally ask her team about particular analyses or tools, which ignites a conversation about the requisite funding for completing a task internally versus hiring an external contractor or buying a product.

To conclude the panel session, Nadeau confirmed that numerous industry positions focus on both analytics and environmental concerns. "If sustainability is important to you, those jobs exist and should continue to grow," she said. She also reassured listeners that many people successfully find work that is not directly related to their Ph.D. dissertation if their interests have shifted. Jackson encouraged current mathematics students who are passionate about environmental science to seize the flexibility of the academic setting and take classes beyond the realm of mathematics and statistics. "Having some experience in the language of the domain side will help you approach it on the math side," she said.

Ultimately, building a career based on one's interests and passions is always a personal journey. "If I had to give my former self advice, it is to not worry so much — everyone's path is winding," Thorarinsdottir said. "Do not worry so much about closing doors. There are many doors."

Jillian Kunze is the associate editor of SIAM News.

Chinese Academy of Sciences SIAM Student Chapter Hosts 13th Annual Meeting

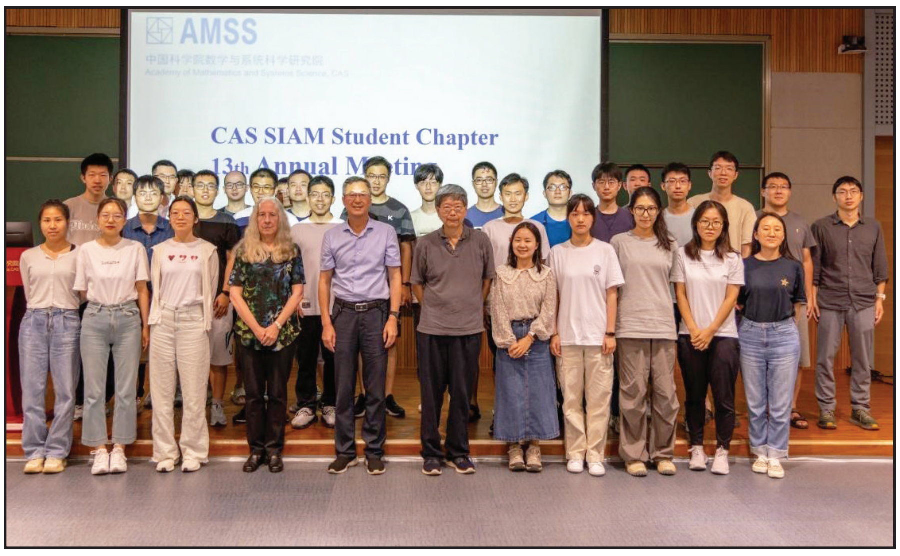
By Yan Yang and Jingru Zhu

In July 2024, the Chinese Academy of Sciences (CAS) SIAM Student Chapter[1] successfully held its 13th Annual Meeting. The conference—which explored computational mathematics and its real-world applications—attracted more than 40 attendees from various institutions, including the CAS Academy of Mathematics and Systems Science (AMSS), the Central University of Finance and Economics, Peking University, Tsinghua University, and even Louisiana State University (LSU). This event served as a valuable opportunity for student attendees to chat about cutting-edge research, seek academic advice from distinguished scientists with experience in the field, and exchange ideas with colleagues from other academic institutions in Beijing and beyond.

The CAS SIAM Student Chapter was founded in June 2011 thanks to the initiative of then-SIAM President Lloyd Nick Trefethen of the University of Oxford and faculty advisor Ya-xiang Yuan of CAS. The chapter has continued to grow steadily over the years and currently services roughly 200 members.

The CAS SIAM Student Chapter was honored to host SIAM Past President Susanne Brenner of LSU, who delivered a public lecture at the meeting that traced computational mathematics' fascinating history—including scientists, machines, and algorithms—from ancient times to the modern day. She also discussed the goals, practices, challenges, and opportunities that are associated with computational math. Brenner's engaging lecture, which was accompanied by illustrative videos, sparked lively conversations among audience members.

SIAM Past President Susanne Brenner of Louisiana State University overviews historical computational instruments during the 13th Annual Meeting of the Chinese Academy of Sciences SIAM Student Chapter, which was held in July 2024. Photo courtesy of Yan Yang.



Speakers and attendees of the Chinese Academy of Sciences (CAS) SIAM Student Chapter's 13th Annual Meeting, which took place in July 2024, gather for a group photo at the CAS Academy of Mathematics and Systems Science. Photo courtesy of Renfeng Peng.

Additionally, two early-career researchers gave presentations at the conference about their recent work. Biqiang Mu of AMSS spoke about the use of two-step estimators to solve non-convex localization and pose estimation problems, and Jinting Wang of the Central University of Finance and Economics introduced value-at-risk-based queueing game systems. These talks addressed innovative research topics in applied mathematics and encouraged student attendees to further investigate related areas of study.

The 13th Annual Meeting also featured a discussion session that allowed students to ask Brenner and Yuan both academic and career-related questions. During this segment, Brenner shared valuable suggestions for organizational strategies and future student chapter activities based on the practices of the LSU SIAM Student Chapter[2] (for which she is a faculty advisor). She also distributed commemorative SIAM pens to all participating students, which made the interaction even more memorable. Yuan then offered his perspectives on effective research methodologies and career development in applied mathematics.

The CAS SIAM Student Chapter is grateful to both the speakers and attendees for their valuable contributions and looks forward to similarly successful events in the coming months.

Yan Yang is a Ph.D. student at the Chinese Academy of Sciences' (CAS) Academy of Mathematics and Systems Science (AMSS) and president of the CAS SIAM Student Chapter. His research interests include low-rank optimization, bilevel optimization, and reinforcement learning. Jingru Zhu is a Ph.D. student at AMSS of CAS and vice president of the CAS SIAM Student Chapter. Her research interests include nonlinear system control, adaptive control, and reinforcement learning.

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[1] https://lsec.cc.ac.cn/~siamstuc

[2] https://www.math.lsu.edu/~siam

Pseudomagnetism

Continued from page 10

periodic case, the Dirac cone is associated with the six-fold rotational symmetry of the structure. And in the strained geometry, the spectra did indeed break into Landau levels, reflecting the pseudomagnetic field that resulted from the strain. The spacing between Landau levels closely matched predictions from our theoretical description based on a two-scale expansion [2, 5] (see Figure 2, on page 10).

In practice, the Landau levels are not perfectly flat. Some dispersion exists in k_2 because the strain locally shifts the frequency of the Dirac cone up and down — a phenomenon that does not arise in the tight-binding model for solid-state graphene. We subsequently found that we could mitigate this dispersion and flatten the Landau level bands by adding an *additional* strain with a different functional form, namely $T_\kappa(x)=(x_1,x_2)+(0,(\kappa x_1)^2)+((\beta x_1)^3,0)$. This strain induces an additional pseudoelectric potential and locally shifts the Dirac cone's frequency in the opposite direction, effectively flattening the Landau levels (see Figure 3).

Concluding Remarks and Future Work

Since their very conception [7, 11], photonic crystals have been used to engineer the density of states — either decreasing it in a band gap to suppress emission or increasing it to achieve stronger light-matter coupling. Periodicity-breaking strains offer a new design principle to engineer photonic crystals and improve their properties. This capability can find purpose in a variety of applications, including the design of lower-threshold on-chip lasers; the shaping of the spectral and spatial profiles of these lasers' modes; the realization of enhanced nonlinear optical effects, such as photon pair generation; and strong coupling to quantum emitters. Ultimately, the use of gauge fields to mathematically describe the effects of aperiodic photonic crystals will facilitate a better analytical understanding of the effects of deformation without resorting to brute-force numerics. This form of physical intuition will likely enable the design of new structures whose relevant physical properties bridge very distinct length scales.

Acknowledgments: We gratefully acknowledge funding from the Office of Naval Research's Multidisciplinary University Research Initiatives (MURI) program under agreement N00014-20-1-2325, the Air Force Office of Scientific Research's MURI program under agreement FA9550-22-1-0339, the Ewing Marion Kaufman Foundation under grant KA2020-114794, and the David and Lucile Packard Foundation under grant 2017-66821. This research was also supported in part by National Science Foundation grants DMS-1620422, DMS-1620418, DMS-1908657, and DMS-1937254, as well as the Simons Foundation's Math+X Investigator Award #376319.

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Mikael C. Rechtsman is a professor of physics at the Pennsylvania State University. Michael I. Weinstein is a professor of applied mathematics and a professor of mathematics at Columbia University.

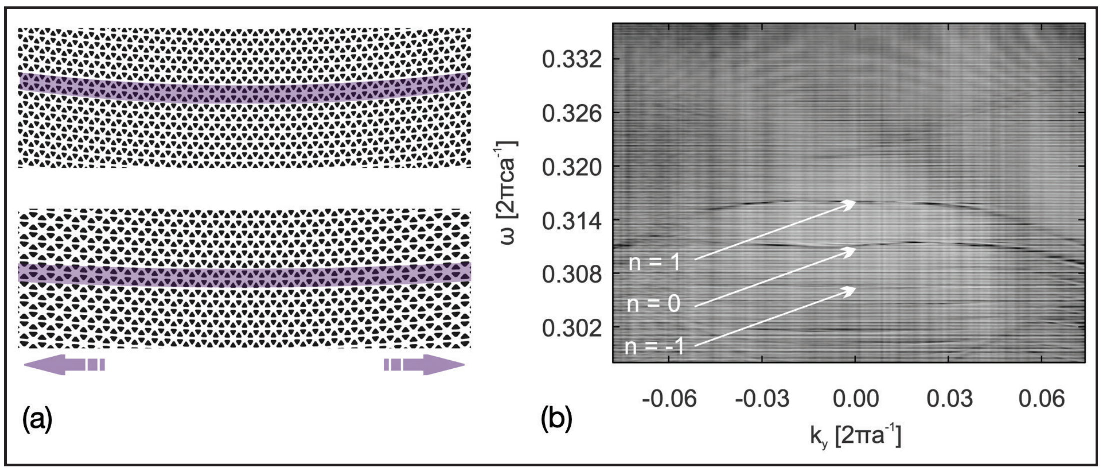

Figure 3. *Additional strain flattens the Landau level bands.* **3a.** *Lattice distortion due to the strain.* **3b.** *Photonic spectra that show flat Landau levels, in contrast to the dispersive levels in Figure 2d. Figure courtesy of [2].*

InsideSIAM

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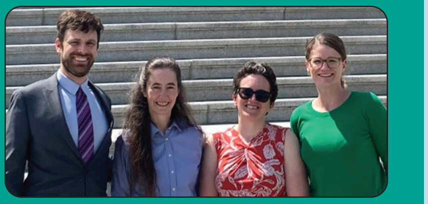

The SIAM Science Policy Fellowship Program develops postdoctoral fellows and early career researchers into strong advocates for U.S. federal support in applied mathematics and computational science. The program enables participants to gain in-depth knowledge of the policy processes that determine science funding and policy decisions while still pursuing their research and teaching.

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SIAM is calling for Letters of Intent submitted in advance of possible proposals of sites, topics, and organizers for the Gene Golub SIAM Summer School (G2S3) offered over two weeks to approximately 40 graduate students in 2026.

Courses in the G2S3 are expected to be at the research level, have a computational component, and cover topics not usually found in regular university courses. The G2S3 should have an overall theme of current interest, with lectures and exercises/project sessions on complementary topics for this area.

Information about the 2025 G2S3 on *Frontiers in Multi-dimensional Pattern Formation*, an archive of prior summer schools, and the full call for proposals for the 2026 G2S3 can be found at the URL above.

The deadline for Letters of Intent is January 31, 2025.

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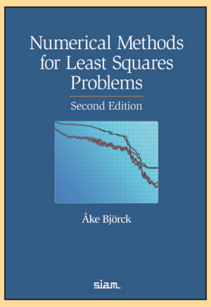

The first edition was the leading reference on the topic for many years. The updated second edition provides an in-depth and up-to-date treatment of direct and iterative methods for solving different types of least squares problems and for computing the singular value decomposition. It covers generalized, constrained, and nonlinear least squares problems as well as partial least squares and regularization methods for discrete ill-posed problems. The bibliography of over 1,100 historical and recent references provides a comprehensive survey of past and present research in the field.

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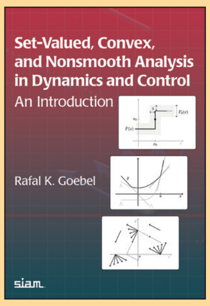

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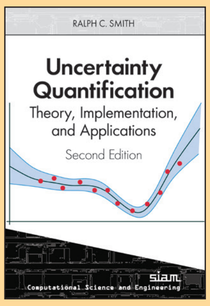

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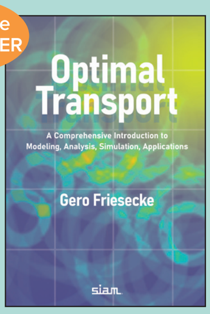

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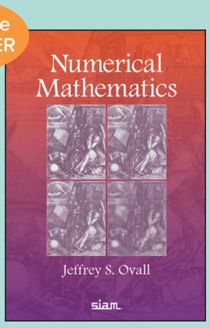

This textbook introduces key numerical algorithms used for problems arising in three core areas of scientific computing: calculus, differential equations, and linear algebra. Theoretical results supporting the derivation and error analysis of algorithms are given rigorous justification in the text and exercises, and a wide variety of detailed computational examples further enhance the understanding of key concepts. It includes topics not typically covered in similar texts at this level, such as a Fourier-based analysis of the trapezoid rule, finite volume methods for the 2D Poisson problem, and more.

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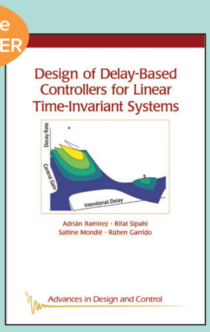

This book provides the mathematical foundations needed for designing practical controllers for linear time-invariant systems. The authors accomplish this by incorporating intentional time delays into measurements with the goal of achieving anticipation capabilities, reduction in noise sensitivity, and a fast response. Also included: a thorough survey of the field and the details of the analytical approaches needed to design delay-based controllers. In addition, readers will find accessible mathematical tools and self-contained proofs for rigorous analysis, numerous examples and comprehensive computational algorithms to motivate the results, and experiments on single-input single-output systems and multi-agent systems using real-world control applications to illustrate the benefits of intentionally inducing delays in control loops.

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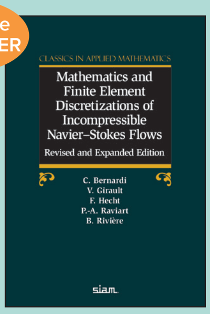

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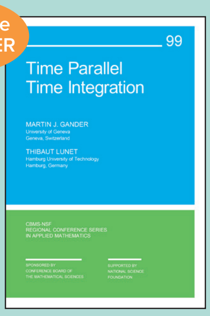

How does one predict the far future before the near future is known? Time parallel time integration, also known as PinT (parallel in time) methods, aims to predict the near and far future simultaneously. In this self-contained book, the first on the topic, readers will find a comprehensive and up-to-date description of methods and techniques that have been developed to do just this. The authors describe the four main classes of PinT methods and provide historical background for each of the method classes, complete convergence analysis for the most representative variants of the methods in each class, and illustrations and runnable MATLAB code.

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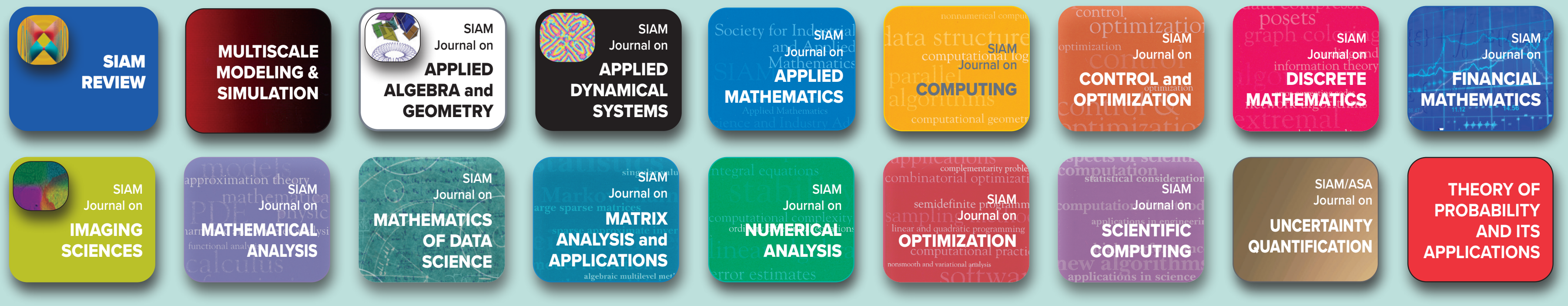

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