MATH MATTERS

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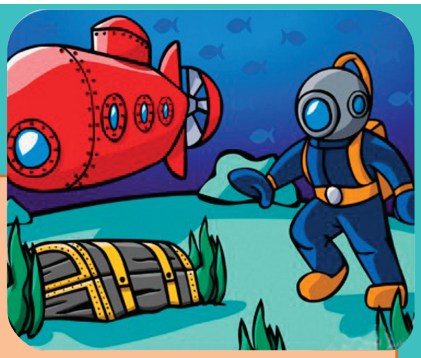


The math behind... Global Seafloor Topography

Technical terms used:

Fourier transform, potential theory, data analysis, band-pass filtering, signal processing, statistical analysis.

Uses and applications:

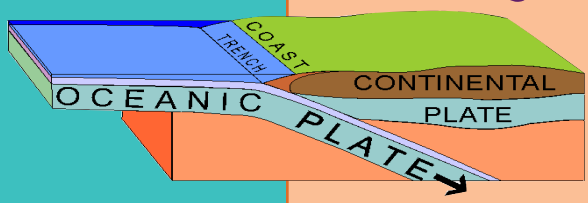
Deriving marine gravity anomaly from satellite altimetry, predicting seafloor topography from the satellite-derived gravity anomaly, combining the predicted seafloor topography with shipboard depth measurements, and providing a realistic view of global seafloor topography.

How it works:

Access to the ocean floor is greatly limited by water, which covers about 70% of the Earth. Ships equipped with echo sounders provide high-resolution depth measurements (100 m resolution in deep oceans); however mapping all the oceans using ships is not achievable within our lifetime. Fortunately, satellite altimetry can cover the oceans uniformly within 1.5 years. Satellite altimetry measures how much the sea surface departs from the reference ellipsoid that approximates the geoid of the Earth. Because the ups (e.g. underwater volcanoes) and downs (e.g. trenches) of the seafloor topography affect the Earth's gravitational field, the sea surface mimics the seafloor undulations. While the slopes of the sea surface in the x- and y-directions are the east and north components of vertical deflection, the gravity anomaly at sea level is the vertical derivative of the gravitational potential. Because these derivatives must satisfy the Laplace's equation, we can relate these quantities to one another and obtain the marine gravity anomaly.

A spectral gravity-to-topography transfer function can be derived from isostatic models for seafloor topography. However, the gravity data at short (< 12 km) and long (> 400 km) wavelengths contain little seafloor information because the attenuated signals of the former and the complete isostatic compensation of the latter result in no anomaly at sea level. To circumvent various limitations, including this limited wavelength band, and construct regionally calibrated topography, quality-controlled ship depth soundings are combined with the predicted topography. This global seafloor topography is now accessible through Google Earth and helps the public to understand Earth better.

Interesting facts:



The satellite-derived global seafloor topography provides dense and uniform data coverage over the oceans. However, we must know its limits in resolution and precision. In early 2009, Google Earth users found a rectangular feature off the coast of Africa and related it to the mythological sunken city of Atlantis. This crisscross pattern, which was initially perceived as a street grid of Atlantis, instead reflects the shipboard depth measurements incorporated into the global topography data.

References:

Satellite Geodesy Group at the Scripps Institution of Oceanography, UCSD: *http://topex.ucsd.edu.*

Submitted by Seung-Sep Kim, University of Hawaii at Manoa, third place, *Math Matters, Apply It!* contest, January 2010.

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