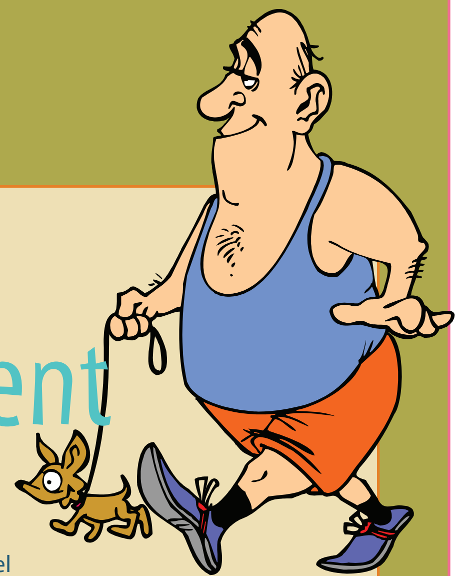
Math Matters

Apply It.

The math behind... Pedestrian Movement



Technical terms used:

Simulation, modeling, differential equations, density functions, social force model

Uses and applications:

Evacuation plans and the position of exits for pedestrians are necessary for the design of any building or major outdoor event in order to prevent tragedies like the death of 21 people at the music festival *Love Parade* in 2010. This is particularly important for large buildings and structures with high occupancies, such as malls, stadiums, or hotels, as well as events, like parades and outdoor concerts. Mathematical models allow descriptions and simulations of the expected pedestrian flow in both normal and extreme situations.

How it works:

The behavior of pedestrians is more complex than that of vehicles. Pedestrians move in many directions and may stop unexpectedly. For a mathematical model, this complexity has to be simplified. This can be done by comparing pedestrian dynamics to fluids or gases. Instead of the motion of individuals, the density and velocity of the crowd (macroscopic model) can be described with partial differential equations.

Since these macroscopic models do not reflect the motivation for the behavior of individuals, an alternative that is often used is the microscopic *Social Force Model*. The idea is that the motion of an individual is the result of the sum of psychological and physical forces acting on him. For instance, the distance pedestrians keep from a wall can be modeled with a repulsion force, a stage in a concert with an attraction force. In mathematical terms, this results in an equation of motion.

The difficulty in applying this equation to a specific situation consists of finding suitable values for the unknown parameters and translating specific pedestrian phenomena into mathematical terms. The behavior of individuals in a crowd in extreme situations serves as a good example. It is while in a panicked state that people tend to follow groups, sometimes leading to infrequently used emergency side exits.

Video analysis can help identify new phenomena in the behavior of pedestrians, improving the models substantially; however, the effort to obtain video data for specific crowd situations is costly. In an experiment in Dusseldorf (Germany) in 2013 more than 1000 volunteers participated.

Interesting fact:

Surprisingly, small changes can have huge impact. For example, placing an obstacle, such as a bank, in the middle of an intersection prevents chaotic behavior and often will lead to a rotary pedestrian flow, which will be counterclockwise in most countries.

References:

1. D. Helbing, P. Molnar, Social Force Model for Pedestrian Dynamics. *Physical Review*. E 51, 4282-4286 (1995)

2. P. Saboia, S. Goldenstein, Crowd Simulation: Improving Pedestrians' Dynamics by the Application of Lattice-Gas Concepts to the Social Force Model. *Graphics, Patterns and Images (Sibgrapi) 24th SIBGRAPI Conference on pp. 41-47. IEEE.* (August 2011)

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First place, *Math Matters, Apply it!* contest, February 2014

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