Rapid Testing in COVID and Modified SIR Model

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Abstract: The COIVD pandemic has swept the globe since 2019, posing a grave threat to human life. There are multiple ways for the government to control the pandemic, including promoting the vaccination, limiting the number of people in public places, requiring people to wear masks in public places, and suggesting infected people isolate themselves. In this paper, we used a compartmental model to analyze the spread of COVID-19 under the promotion of rapid tests. The result shows that popularization of rapid tests may have a significant impact on controlling the pandemic. With an estimated minimum requirement for the use of rapid tests, we are able to put forward suggestions on reasonable ways to curtail the pandemic.

I. INTRODUCTION

Starting in December 2019, a novel coronavirus known as COVID-19 has caused an outbreak of viral pneumonia worldwide and has had a drastic effect on people's lives since. At the start of the pandemic, due to the limitation of medical resources and diagnostic technology, the pandemic took the lives of many. Even with more refined treatments and testing procedures today, the pandemic still poses an enormous risk to public health, holding people back from returning to school or to the workplace.

The PCR test (Polymerase Chain Reaction test) is widely trusted for accurately identifying infections by SARS-CoV-2. Despite its reliability, it takes a period ranging from 1 to 3 days to receive the results. Rapid testing, which detects antibodies, has been developed to provide a faster and cheaper way of detecting COVID cases. As the name suggests, the test can be conducted at home and deliver results within 15 minutes. Although this may come with a drawback [3] of lower accuracy, it still has the potential of significantly curtailing the pandemic's spread. The purpose of this article is to discuss how the popularization of rapid tests could influence the dynamics of the pandemic. How sensitive does it need to be, and what fraction of the population needs to be tested? To answer these questions, we built a numerical simulation to determine the potential the κ factor of rapid tests in the COVID pandemic.

II. BACKGROUND

The *SIR Model* is a commonly used tool in epidemiology. It divides the population into three compartments: S (susceptible), I (infectious), and R (recovered). The population flow between these compartments is described by the following system of differential equations [4]:

$$\begin{aligned}\frac{dS}{dt} &= -\frac{\beta IS}{N}\\ \frac{dI}{dt} &= \frac{\beta IS}{N} - \gamma I\\ \frac{dR}{dt} &= \gamma I\end{aligned}$$

where N denotes the total population, β the transmission factor, and γ the recovery rate.

When the total population N is assumed to be constant (i.e. no vital dynamics and no exchange of population with other communities), the third equation can be discarded as R is simply $N - S - I$. The two remaining equations constitute a special case of Lotka-Volterra system, which can be solved implicitly.

$$I + S - \frac{\gamma}{\beta}\log S = I_0 + S_0 - \frac{\gamma}{\beta}\log S_0$$

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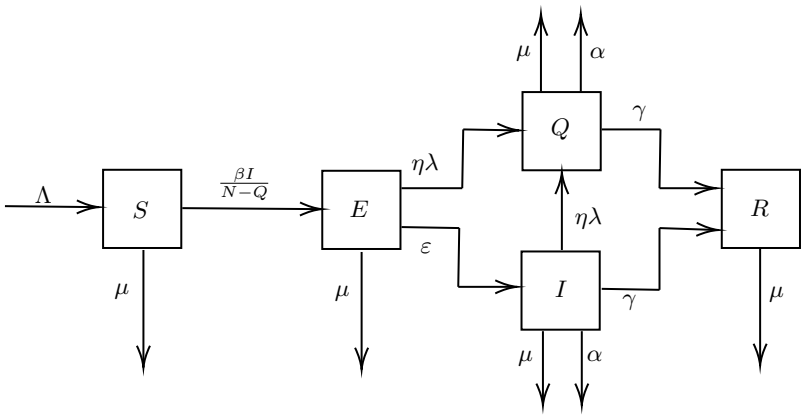

FIG. 1: SEIQR Model Flow Chart

However, there are a few discrepancies between the assumptions and the situation that we wish to model.

1. This model assumes no vital dynamics (that is, assuming N is constant). In other words, it works best when the disease induces little to no fatalities, which is not the case for the current COVID-19 pandemic.

2. This model assumes no latent period (the time interval between patients are infected and become infectious). It is observed that COVID-19 has a non-negligible latent period [1].

3. This model does not consider quarantine.

Our goal is to modify the SIR model to reflect the effect of rapid testing on curbing the COVID pandemic.

III. THE MODEL

The following five-compartment model is our attempt to modify the SIR model to better describe the situation. (To be consistent with the naming convention of SIR, we refer to it as the SEIQR model in this paper). In this section, we shall take a closer look into its properties. The dynamics of the system is illustrated in Figure 1.

While "S", "I" and "R" have the same definition as in the SIR, this model takes two additional groups into account. One of these is "E", represents the "Exposed" people who are infected but not yet contagious. In other words, this compartment contains patients in the latent period. "Q" is for "Quarantined". People who have tested positive may quarantine themselves at home. Symptomatic patients who self-isolate also go into this compartment.

This system is described by a system of ordinary differential equations, listed below (see table 1 for a summary of parameters):

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S - \beta S \frac{I}{S+E+I+R} \\
\frac{dE}{dt} &= \beta S \frac{I}{S+E+I+R} - (\mu + \epsilon + \eta\lambda)E \\
\frac{dI}{dt} &= \epsilon E - (\mu + \gamma + \alpha + \eta\lambda)I \\
\frac{dQ}{dt} &= \eta\lambda(E+I) - (\mu + \gamma + \alpha)Q \\
\frac{dR}{dt} &= \gamma(Q+I) - \mu R
\end{aligned}$$

In this model, natural death and birth are taken into account. Λ represents birth rate per day and is added to the susceptible compartment. From each section, a fraction μ, representing the natural death rate (that is, death not

Name	Description	Value
Λ	Birth rate per unit time (day)	1094
μ	Death rate per person per unit time (day)	2.05×10^{-5}
β	Probability of disease transmission per contact times the number of contacts per unit time	0.07
α	Fatality rate of COVID-19	0.0336
ϵ	Rate of progression from exposed to infectious, reciprocal of the latent period	0.33
γ	Recovery rate of infectious individuals, reciprocal of the infectious period	0.2
η	Sensitivity of rapid test	$[0, 1]$
λ	Percentage of population that take rapid tests per day	$[0, 1]$

TABLE I: Description and Values of Parameters

induced by COVID) per day per capita, is deducted.

For people in the susceptible compartment, there is a probability $\frac{I}{S+E+I+R}$ of contacting someone infectious. Because β represents the product of transmission probability and the number of contacts per unit time, $\beta\frac{I}{S+E+I+R}S$ will flow from S to E per unit time. For the exposed people, the rate of moving into section I will be ϵ (the reciprocal of latent period). Let λ be the proportion of people taking the rapid test and let η be the sensitivity of the rapid test (sensitivity means the probability that people who have tested positive do have COVID-19). Therefore $\lambda\eta E$ is the rate at which exposed people flows into the Q component. Combined with the death rate, there is an outflow flow of $(\mu + \epsilon + \eta\lambda)E$ from E in total.

In the infectious state, people recover at a rate of γ, which is reciprocal of the infectious period. Death (both natural and COVID-induced) may occur. People may also be quarantined after testing positive by rapid tests. Thus, the outflow from infectious population is $(\mu + \gamma + \alpha + \eta\lambda)I$. The outflow from the quarantined component Q consists of recovery and death (both natural and disease induced) with the same parameter as we described before.

For the removed population, we assume that they are immune and no longer infectious. Thus, the only outflow is natural death.

For the whole population, the net inflow is natural birth and the net outflow is natural death and death caused by COVID-19. This is consistent with our previous assumptions.

$$\begin{aligned} N'(t) &= S'(t) + E'(t) + I'(t) + Q'(t) + R'(t) \\ &= \Lambda - \mu(S + E + I + Q + R) - \alpha(I + Q) \end{aligned}$$

IV. RESULTS

Because η (sensitivity) and λ (compliance rate) both affect the inflow into Q linearly, we denote this linear factor by $\kappa = \eta\lambda$. It is worth noting that because both η and λ are within the unit interval, κ should also be within this range. To see how the value of the κ factor influences the change of transmission dynamics of COVID-19, we choose κ to be 0, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0 and simulate the dynamics (solve the ODE) using each of these values of the κ factor respectively.

Initial conditions and parameters are chosen as shown in table 1. Note that 38050000 (N_0) is the estimated total population in Canada in the first quarter of 2021, as obtained from Statistics Canada [8]. For simplicity, we choose $E_0 = 1000$ and $I_0 = 1000$. Birth rate and death rate are gathered from population dynamics information posted by the Department of Economics and Social Affairs, the United Nations [7]. COVID first appeared in 2019, thus we use a birth rate of 10.5 per 1000 people per year and a death rate of 7.3 per 1000 people per year in 2019 so that there is no influence of pandemic. Assuming 365 days per year, we can compute $\Lambda = 10.5/(1000 \times 365) \times 38050000 \approx 1094$ and $\mu = 7.3/(1000 \times 365) \approx 2.05 \times 10^{-5}$. Shim estimated the Case Fatality Rate(CFR) of COVID in Canada to be 3.36% and transmission rate to be 0.2 if the effective reproduction number (R_E) is 1. We used these parameters to solve this system of ODE numerically using `odeint` in `scipy` python package.

Figure 2 consists of 5 plots that illustrates each compartment of the population as a function of time. Note that the first plot for the susceptible population is nearly linear (that is, its rate of change is almost constant). This rate

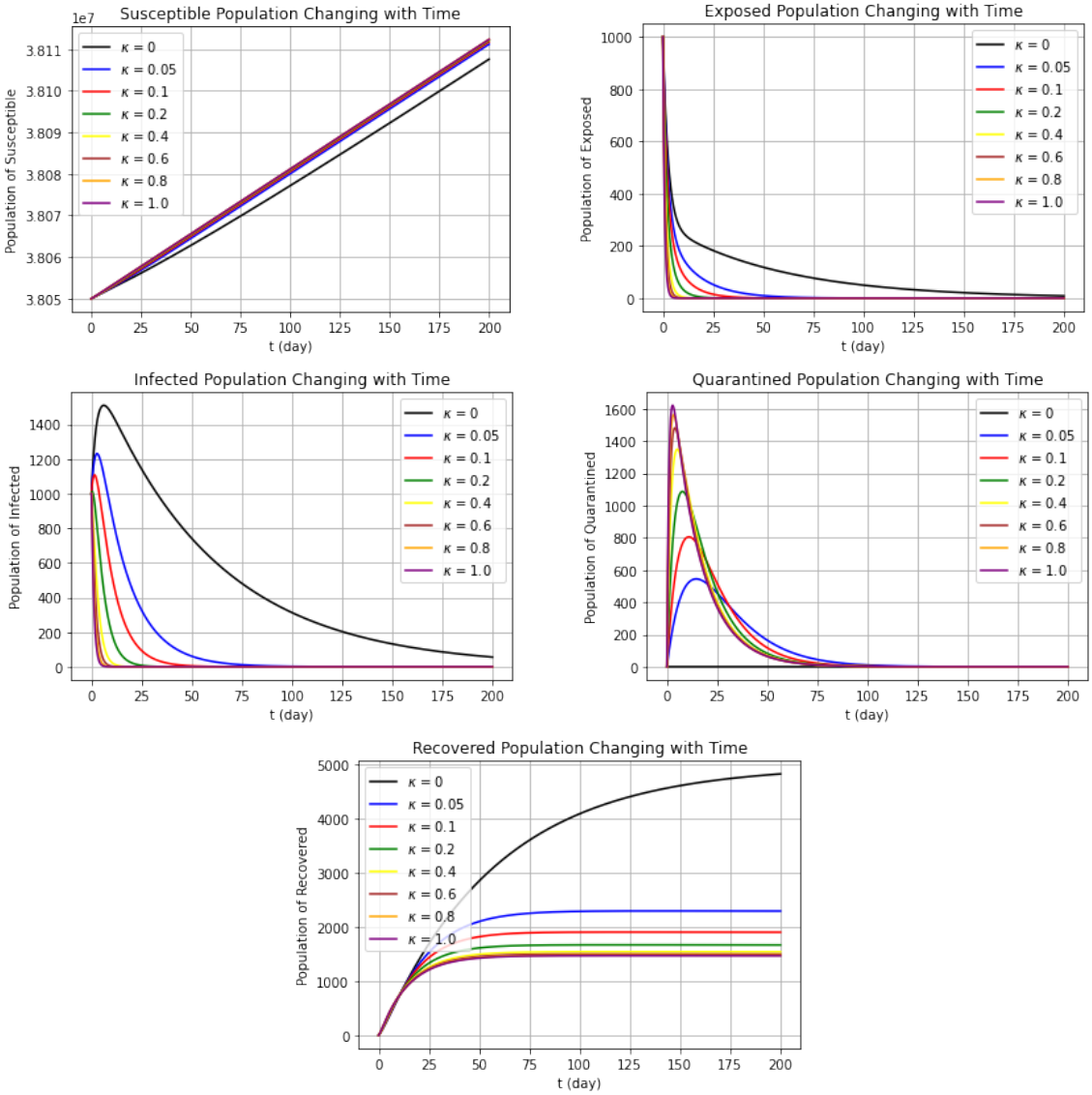

FIG. 2: Change of Population Size

of change is dominated by the inflow of susceptible people per day, the birth rate, which is 1094, a relatively large number compared to the outflow every day. For the infected population, the black curves represent the situation without rapid test. As the κ factor increases, the number of infected number is smaller.

V. DISCUSSION

To control the pandemic, infected people need to decline with minimal disease induced deaths. From the figure "Exposed Population Changing with Time", we can see that without rapid test, the exposed population decreases slowly and is still over 0 on day 200. If the κ factor was 0.05, the exposed population would go down quickly in the first 50 days and approach 0 at time 75 days. In addition, we can observe from the figure "Infected Population Changing with Time" that without rapid test ($\kappa = 0$), the number of infected people reaches a peak much higher compared to that of the $\kappa = 0.05$ curve. With even a marginal κ factor, the peak is almost brought down to the initial infected population. Similarly, the $\kappa = 0.05$ curve reaches 0 in about 75 days, while the curve of $\kappa = 0$ is still above 0 on day 200. With those two figures, we can conclude that the rapid test could have a significant effect in helping mitigate the pandemic.

Another success of the rapid test is on the cumulative number of infected individuals. From plot "Recovered Population Changing with Time" (in FIG. 2), we can see that, in the long run, the total of infected people is less than half of that of the situation without rapid test, which is still growing after 200 days.

A natural question to investigate is: whether there exists a critical κ for daily increase in the infected compartment? There are many criteria to evaluate different values of the κ factor; for simplicity, we chose the maximum of the infected population. From the figure "Maximum Infected Number under the κ factor (0 to 1)", we can see that the curve goes down quickly from 0 to 0.258 and flattens out after 0.258. In this case, 0.258 can be selected as the the κ factor value we want to achieve to control the pandemic.

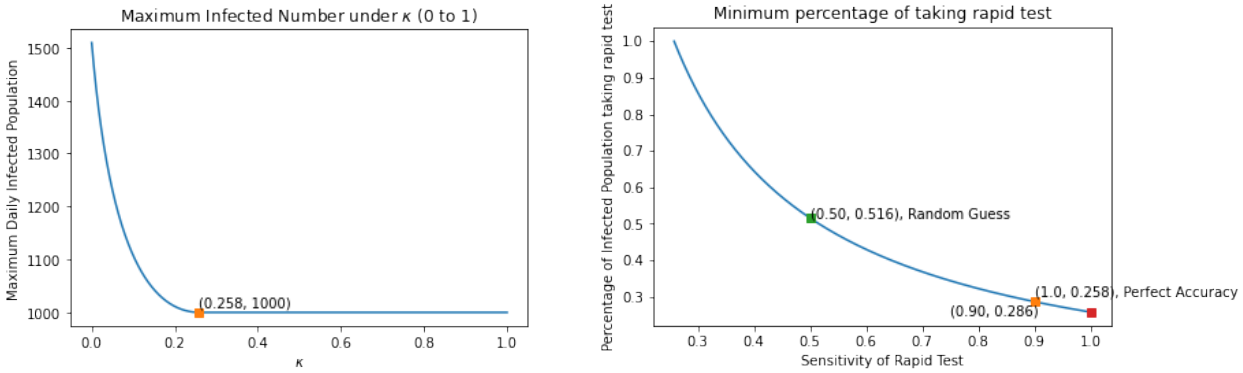

FIG. 3: Spread vs. Sensitivity

As defined, $\kappa = \eta\lambda$. Because the sensitivity of the rapid test is unknown and hard to control compared with the percentage of the population that takes the test, we need to know the minimum percentage of taking the test based on the sensitivity of the test. See the figure "Minimum percentage of taking the rapid test" for reference. From the graph, we shall see that if the sensitivity is only 0.5, it requires at least 0.26 of the population to take the test.

VI. SUMMARY

In this paper, the impact of the rapid test on Covid-19 is discussed. We developed a compartmental model (SEIQR) to describe the spread of the virus among the people of Canada. We came to the conclusion that rapid tests can have a remarkable impact in controlling the pandemic. It has the potential to significantly shorten the duration of the pandemic, flatten the peak, and reduce the number of casualties.

The sensitivity of different types of rapid tests may vary, but most of them have a sensitivity above 85% [5]. With such a value of sensitivity, our research shows that the minimum percentage of the population taking the rapid test per day should be about 30% if other assumptions are reasonable. This compliance should be readily achievable as the rapid test is easy to perform and less expensive compared with traditional tests.

We began our work in spring 2021 when the supply of vaccines in Canada was limited. We do not mean to suggest that the COVID pandemic could be overcome entirely through the use of rapid tests. We only would like to point out that rapid tests could have some positive benefit in fighting this pandemic. We also point out that in many parts of the world the supply of vaccines is still limited. Rapid tests (which are less expensive and easier to use than vaccines, and can be used at home without the assistance of health professionals) might continue to be useful in those countries.

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