

# SIAM Activity Group Orthogonal Polynomials and Special Functions

## Charter Renewal Application

This CHARTER RENEWAL APPLICATION applies to the SIAM Activity Group on Orthogonal Polynomials and Special Functions (SIAG/OPSF). The SIAM Activity Group (SIAG - OPSF) to which this renewal applies was originally formed under the aegis of SIAM on July 15, 1990 by the SIAM Council and July 19, 1990 by the SIAM Board of Trustees with its initial operating period beginning January 1, 1990 and ending December 31, 1992. Its charter has been renewed by the Council and Board ten times thereafter.

This SIAG has 150 members, including 35 student members, as of December 31, 2020.

According to its Rules of Procedure, the objective(s) of the SIAG are:

- The purpose of the SIAG is to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.
- The activity group is concerned with the following topics and their applications: orthogonal polynomials (general systems, Askey scheme, asymptotic analysis, recurrence relations, numerical quadrature), harmonic analysis, approximation theory, representations of compact groups, integrable models, quantum mechanics, combinatorics, coding and design theory, representation theory, orthogonal polynomials in several variables, hypergeometric functions and their  $q$ -analogues and elliptic analogues, special functions in connection with Lie groups, tomography, quantum groups and other algebraic structures, optics, wave functions in crystals, classical special functions (for example, Bessel, gamma, beta, theta, spheroidal wave, etc.), nonlinear special functions (Painlevé equations), random matrices, solutions of ordinary and partial differential equations, statistical mechanics, integral transforms, number theory. Within the framework of SIAM, the SIAG will conduct activities that implement its purposes.

Its purposed functions were: The SIAG on Orthogonal Polynomials and Special Functions will organize activities in orthogonal polynomials. The SIAG is expected to:

1. Organize mini symposia at the SIAM Annual Meeting on years where there is no SIAG conference.
2. Organize a track of at least six mini symposia at the SIAM Annual Meeting at least once every five years. The VP for Programs and the VP at Large will coordinate the scheduling with the SIAG Chairs.
3. Dissemination of information about upcoming conferences and sponsoring special sessions at SIAM meetings. Also, the group will assist researchers in the use of symbolic computer calculations by publicizing available software for special functions. Another goal is to establish some working relationships with the various SIAM journals, especially the one on mathematical analysis, with the view of sporadically sponsoring some invited or contributed Articles.
4. With the approval of the SIAM Program Committee, the SIAG may organize special sessions at SIAM meetings, and conduct special one- or two-day meetings immediately before or after a regular SIAM meeting. Other SIAG meetings may be organized only with the approval of the SIAM President and Vice President for Programs.

5. Award the SIAG/OPSF Gábor Szegő prize every two years to an early-career researcher for outstanding research contributions, as determined by the prize committee, in the area of orthogonal polynomials and special functions. SIAG meetings, workshops, and conferences may be organized only with the approval of the SIAM President and the SIAM Vice President for Programs.

The SIAG has complemented SIAM's activities and supported its proposed functions. The answers to the questions below indicate how this was accomplished and what the officers propose as the future directions for the SIAG.

1. *List all current officers of the activity group (including advisory board, if relevant).*

Elected

- Chair: Peter Clarkson (University of Kent, UK)
- Vice chair: Luc Vinet (University of Montreal, Canada)
- Program director: Andrei Mart'inez Finkelshtein (Baylor University, Texas)
- Secretary: Teresa E. Pérez (University of Granada, Spain)

Appointed

- OP-SF NET Co-editors: Howard Cohl, Sarah Post
- OP-SF Talk moderators: Diego Dominici, Teresa E. Pérez, Bonita V. Saunders
- OP-SF Webmaster: Bonita V. Saunders

2. *How is the field covered by the activity group doing? Is it growing, is the focus shifting? What have been the significant advances over the last three years?*

The field of orthogonal polynomials and special functions continues to have a lot of ongoing activities in mathematics, statistics, computing, physics, and engineering, where special functions and, in particular, orthogonal polynomials, arise in new advances and are more than ever needed for various models and applications. Researchers in the fields of random matrices and integrable systems have also realized that special functions are very useful there. The Riemann-Hilbert approach to orthogonal polynomials and the steepest descent analysis of oscillatory Riemann-Hilbert problems have been used and extended by various members of the SIAG/OPSF. Multiple orthogonal polynomials have become an important new focus, with applications in (simultaneous) rational approximation, number theory, random matrix theory, and non-intersection random paths. The latter two are examples of determinantal point processes, and various important cases turn out to be using (multiple) orthogonal polynomials and special functions. Matrix-valued orthogonal polynomials and their use in representations of Lie groups are receiving renewed attention. The nonlinear special functions arising as a solution to the Painlevé equations turn out to play a remarkable role in the asymptotics of orthogonal polynomials and determinantal point processes. Another important development are the elliptic hypergeometric functions, which are under investigation by several members of SIAG/OPSF.

The NIST Handbook of Mathematical Functions (edited by F.W.J. Olver, D.W. Lozier, R.F. Boisvert, C.W. Clark), with an online companion: The Digital Library of Mathematical Functions (DLMF), <http://dlmf.nist.gov>, are an important source of information for our activity group and are widely used by other Mathematicians and scientists in different disciplines. Several members of the SIAG/OPSF were involved in the writing handbook and the DLMF, which is an ongoing project. The most recent update of the DLMF was March 15, 2022, version 1.1.5. As stated by Schneider, Miller, and Saunders<sup>1</sup>, "new material becomes relevant to applications and is added to the website." Currently, in the DLMF, revision of the chapters on orthogonal polynomials and Painlevé transcendents is underway, and a completely new chapter on orthogonal polynomials of several variables is in progress.

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<sup>1</sup>B.I. Schneider, B.R. Miller, and B.V. Saunders, NIST's Digital Library of Mathematical Functions, *Physics Today*, 71 (2018) 49–53

3. *How is the activity group doing? Is it remaining vibrant? Is the size of the SIAG stable or increasing? How is the SIAG keeping up with the changes in the field? How are the broader interests of SIAM reflected in the activities of the SIAG?*

Membership of the activity group has decreased by 17% from December 2015 to December 2020. The high number of members in 2015 is most likely the result of the fact that the International Symposium OPSFA13 in 2015 was organized by SIAM. We are now at a membership level that we had before 2015. The SIAG/OPSF Gábor Szegő Prize attracted a number of early career researchers. The list of people receiving our electronic newsletter OPSF-NET is much larger than the membership. The main reason that many people are not a member seems to be the high membership fee for SIAM and that a significant number of researchers in the field are based outside North America, primarily in Europe.

4. *Please list conferences/workshops the activity group has sponsored or co-sponsored over the past three years, and give a brief (one sentence or phrase) indication of the success or problems with each.*

The last two years have been very complicated by the Covid-19 pandemic situation. Most of the activities were postponed, held online, or cancelled. Evidently, events during this time have been altered by the global health situation and many travel restrictions. However, this is the time to start to resume the normal activities. The community of orthogonal polynomials wants to get back to the usual activity.

In every meeting, at least one member of the SIAG/OPSF was on the organizing committees of international conferences, workshops, special sessions, or mini-symposia:

- i) Our SIAG/OPSF is strongly involved in the biennial International Symposia on “Orthogonal Polynomials, Special Functions, and Applications” (OPSFA). The upcoming symposia (OPSFA16), which was originally due to take place in June 2021, was postponed by the pandemic and will instead be held June 13–17, 2022. Luc Vinet, the current Vice-Chair of SIAG/OPSF, is the chair of the organizing committees for OPSFA16. Initially, it was hoped that the meeting would be in person in Montreal (Canada), but it has since moved online. This symposia will be dedicated to the memory of Richard Askey, one of the promoters of the SIAM Activity Group on Orthogonal Polynomials and Special Functions. OPSFA16 has almost 200 registered participants, 13 plenary talks, 8 speakers for the Richard Askey Day, and 9 mini-symposia. We expect that it will be a complete success despite the virtual format.  
The last symposium (OPSFA15) was July 22–26, 2019, in Hagenburg, Austria, just before the pandemic. There were 200 registered participants, 9 plenary talks, and 158 regular talks organized into 12 minisymposia.
- ii) There is a steering committee for the OPSFA meetings, which consists of three local organizers of the last five meetings and a representative of the SIAG/OPSF (not necessarily the chair). This committee makes the selection of where the next OPSFA meeting will be. The place of the next OPSFA17 will be decided after OPSFA16 has concluded.
- iii) The steering committee for the OPSFA symposia also coordinates summer schools in Orthogonal Polynomials and Special Functions. The last summer school (OPSF-8) was held in Sousse, Tunisia, from June 25–29, 2018, with five lecture series and one guest lecture. This location attracted participants from Northern Africa. There were 68 participants, many from Tunisia and Algeria, but also a few from Europe.
- iv) The next summer school is due to be held in August 2022 at the Radboud University, Nijmegen, The Netherlands. The school was originally due to take place in August 2020 but has been postponed due to the Covid-19 pandemic. Initially, it was delayed to August 2021 and then again to August 2022. There will be five lecture courses during the week.

- v) A mini-symposium on “Special Functions and Orthogonal Polynomials” was organized by members of our SIAG/OPSF during the Foundations of Computational Mathematics (FoCM 2020) meeting in Vancouver, Canada, July 15–24, 2020. However, the meeting was cancelled. The next FoCM meeting (FoCM 2023) is due to be held in Paris (France), June 12–21, 2023. Amongst the plenary speakers at FoCM 2023 is Walter Van Assche, the previous chair of SIAG/OPSF, and the mini-symposium “Special Functions and Orthogonal Polynomials” is being organized by Ana Loureiro, Paco Marcellan, a former chair of SIAG/OPSF, and Andrei Martínez Finkelshtein, the current Program Director of SIAG/OPSF.

The following meetings, programmed for 2020, were cancelled:

- i) VI Meeting of the Spanish Network on Orthogonal Polynomials and Approximation Theory (OrthoNet 2020), Universidad Politécnica de Madrid (Spain), March 26–27, 2020.
- ii) Workshop “Complex Approximations, Orthogonal Polynomials and Applications,” Sirius Mathematics Center, Sochi (Russia), April 5–11, 2020.
- iii) International Conference on Approximation Theory and Beyond, Vanderbilt University (USA), May 11–14, 2020.
- iv) Baylor Analysis Fest “From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory”, Baylor University (TX, USA), May 18–22, 2020.
- v) Conference Foundations of Computational Mathematics (FoCM’2020), Vancouver (Canada), June 15–24, 2020, especially the Workshop “Special Functions and Orthogonal Polynomials.”
- vi) VIII Iberoamerican Workshop on Orthogonal Polynomials and Applications (VIII EIBPOA), Universidad Nacional de Colombia, Bogotá (Colombia), June 23–26, 2020.
- vii) XXVI Conference on Differential Equations and Applications / XVI Conference on Applied Mathematics Aplicada (CEDYA + CMA 2020), University of Oviedo, Gijn (Spain), June 14–18, 2021.

Online meetings:

- i) 2020 SIAM Annual Meeting, jointly with the Canadian Applied and Industrial Mathematics Society (CAIMS), July 6–17, 2020. There was a mini-symposium, “Orthogonal Polynomials, Random Matrices and Asymptotic Methods,” on July 14, 2020, organized by SIAG/OPSF.
- ii) Conference “Functions and Operators, 10 years after” (F.H. Szafraniec’s fest), July 15–17, 2020.
- iii) 3rd Annual Meeting of the SIAM Texas-Louisiana Section, Virtual Zoom Meeting, Texas A&M University, October 16–18, 2020.
- iv) 2021 Joint Mathematics Meetings, American Mathematical Society, Cyberspace, January 6–9, 2021. In particular, Special Sessions on “The Legacy of Dick Askey” and on “Continued Fractions.”
- v) 8th European Congress of Mathematics, Portoroz (Slovenia), June 20–26, 2021. In particular, there was a mini-symposium on “Orthogonal Polynomials and Special Functions.”
- vi) 2021 SIAM Annual Meeting, Spokane, Washington (USA), July 19–23, 2021. There was no mini-symposium organized by SIAM/OPSF as OPSFA16 was due to take place in 2021, but in the event was postponed until 2022.
- vii) Mathematical Congress of the Americas, online, July 19–24, 2021, there was a session on “Special functions and orthogonal polynomials.”
- viii) VI Congreso Latinoamericano de Matemáticas (CLAM 2020), online, hosted by Montevideo (Uruguay), September 13–17, 2021, there was a session on “Special functions, orthogonal polynomials and approximation theory.”

- ix) 9th International Conference on Computational Methods and Function Theory (CMFT), Online, January 10–14, 2022.

Hybrid meetings:

- i) Workshop “Complex Approximations, Orthogonal Polynomials and Applications,” Sirius Mathematics Center, Sochi (Russia), June 6–12, 2021.
- ii) Baylor Analysis Fest, “From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory,” Baylor University (Texas, USA), May 23–27, 2022. Andrei Martínez Finkelshtein, the current Program Director of SIAG/OPSF, was one of the two main organizers of this meeting.

Standard meetings:

- i) Biennial Congress of the Spanish Royal Society of Mathematics (RSME), Universidad de Castilla-La Mancha (Ciudad Real, Spain), January 18–22, 2021. Celebrated on January 17–21, 2022. There was a Special Session on “Orthogonal Polynomials, Special Functions and Applications.”
- ii) International Conference on Orthogonal Polynomials Celebrating Francisco Marcellán’s 70th birthday, Cádiz (Spain), April 21–23, 2022. Francisco (Paco) Marcellán was a former chair of SIAG/OPSF.

5. *Please indicate the number of mini symposia directly organized by the activity group at the last two SIAM annual meetings. When did the SIAG last organize a track at an annual meeting or meet jointly with the SIAM Annual Meeting?*

At the 2020 SIAM Annual meeting, held jointly with the Canadian Applied and Industrial Mathematics Society, which was due to take place in Toronto, Canada, in July 2020, the SIAG/OPSF has organized a track. This included an invited speaker (Andrei Martínez Finkelshtein) and minisymposia on “Orthogonal Polynomials, Random Matrices and Asymptotic Methods,” “Symbolic Computation and Special Functions,” “Orthogonal Polynomials, Integrable Systems and Combinatorics,” and “Basic Hypergeometric Series and  $q$ -Orthogonal Polynomials.”

6. *Please indicate other activities sponsored by the activity group, to include newsletters, prizes and web sites. Have each of these been active and successful?*

- i) The most visible activity of our SIAG is the electronic newsletter OPSF-Net. There are six issues every year, containing information about conferences, workshops, and symposia in the field of Orthogonal Polynomials and Special Functions. The newsletter also has reports of recent meetings, obituaries, new books, and book reviews. There is also a list of papers on OPSF which appeared on arXiv.org during the last two months. The mailing list of OPSF-Net is much larger than the membership, but this is partly caused by multiple entries and entries of people no longer active.
- ii) The activity group has a website hosted at NIST (<https://math.nist.gov/opsf/>). The website contains a calendar, an archive of all the newsletters since 1990, a list of useful books, tools, obituaries, history, available positions, and links to useful places on the internet. Many of the links redirect to the SIAM webpage for SIAG/OPSF.
- iii) The SIAG/OPSF awards the Gábor Szegő Prize every two years to an early-career researcher for outstanding research contributions, as determined by the prize committee, in the area of orthogonal polynomials and special functions. The first prize was awarded in 2011, and the latest, the fifth prize, was awarded in 2019 to Thomas Bothner (King’s College, London, UK). The next prize, originally awarded in 2021 at the OPSFA16 symposium, which is due to take place in Montreal, Canada in July 2021, was postponed to the OPSFA16 in 2022 and has been obtained by Atul Dixit (Indian Institute of Technology Gandhinagar, India), who will be a plenary speaker at OPSFA16.

- iv) The scientific committee of OPSFA16 decided to dedicate the meeting to the memory of Richard Askey, who died on October 9, 2019, due to his enormous contribution to “Orthogonal Polynomials and Special Functions,” a long-standing member of SIAG/OPSF, and a fellow of both SIAM and AMS.
- v) Our activity group has strong connections with the Society for Special Functions & Their Applications in India, and in particular, they consult with us for their annual international conferences.
- vi) Some members of SIAG/OPSF are involved in the Ibero-American Workshops on “Orthogonal Polynomials and Applications.” The participants are mostly early career researchers interested in topics covered by our SIAG.
- vii) Several members of SIAG/OPSF are involved in an “Orthogonal Polynomials, Special Functions and Operator Theory and Applications” based in the UK and Leuven (Belgium), supported by a London Mathematical Society grant. In 2019 this group organized five one-day workshops in Canterbury, Brighton, London, Reading (all UK), and Leuven (Belgium). Due to the Covid-19 pandemic, there were no meetings in 2020, so regular virtual seminars started in June 2020, hosted by the International Centre for Mathematical Sciences in Edinburgh, Scotland.
- viii) During the pandemic, some members of the SIAG/OPSF have started to organize periodic international online seminars. Virtual seminars substituted the lack of in-person meetings, and now these are often hybrid. We think that online seminars are a good way to keep in touch with international researchers on orthogonal polynomials and special functions.

7. *What activities are planned and proposed for the next period of the charter? Please describe scheduled and suggested future activities in detail.*

Due to the Covid-19 pandemic, the dates of all future activities are provisional at this juncture. However, it seems that we will resume our standard way of life very soon, though things will probably be different with some events being hybrid.

- The SIAG/OPSF will continue to support the organization of the OPSFA symposia; the next one is due to take place at the Centre de Recherches Mathématiques, Université de Montreal, Canada, in July 2022. This symposium will be dedicated to the memory of Richard Askey, a long-standing member of SIAG/OPSF, who died in 2019. One of the elected officers will be part of the steering committee of the future OPSFA symposia. These international symposia are an important tradition in our community, and we try to organize them in various locations around the world, with a list of plenary speakers containing specialists in the field, early career researchers, and diversity in geography and gender. The award ceremony for the Gábor Szegő Prize includes an invited lecture by the award winner.

We expect that the next OPSFA will be in 2024 in the place that the OPSFA Steering Committee will decide later this year.

- The SIAG/OPSF will also support the OPSF summer schools, which were organized annually from 2000 to 2004 and were revived recently with summer schools in 2016 (College Park, Maryland), 2017 (University of Kent, Canterbury, UK), and 2018 (Sousse, Tunisia). The next summer school is planned for 2021 at the Radboud University, Nijmegen, The Netherlands. This was originally due to take place in August 2020 but has been postponed due to the Covid-19 pandemic, provisionally to August 2021, and due to take place August 8–12, 2022.
- The SIAG/OPSF will get more involved with the organization of a track at the annual SIAM meetings, as illustrated by our involvement in the annual meeting in 2020. Our SIAG did not organize a mini-symposia at the SIAM annual meeting in 2021 or 2022 due to OPSFA16.

- The organization by members of the SIAG/OPPSFA activity group of mini-symposia about orthogonal polynomials, special functions, and applications in several international meetings.
- The continuation of the hybrid periodic seminars started during the pandemic.

8. *How can SIAM help the activity group achieve its goals?*

We note that “Orthogonal Polynomials and Special Functions” is not listed as one of the Research Areas of SIAM on its website.

The SIAM journal which is closest to our activity group is *SIAM J. Math. Anal.* (SIMA). We regret that special functions do not appear in the journal description and that there is a lot of emphasis that every paper should relate to a model for natural phenomena in such areas as fluid mechanics, materials science, quantum mechanics, biomathematics, mathematical physics, or to the computational analysis of such phenomena. This makes the distinction with *SIAM J. Appl. Math.* (SIAP) very small. We would like to see the topic of Special Functions in the journal description and to have more SIAG/OPPSF members on the editorial board of SIMA.

One of the main obstacles for people to attend the SIAM annual meetings is the high registration fee, even as a speaker in a mini-symposium, in addition to the cost of international travel for many members of our SIAG. To support young researchers interested in our field, it would help to have a reduced registration, in particular for early career researchers who give a talk in a mini-symposium. What we found very useful in the past is that SIAM offers travel grants to early career researchers enabling them to attend our workshops and conferences. We would very much like SIAM to continue this.

Several members of SIAM who have research interests in orthogonal polynomials and special functions are Outreach members. For them, the SIAM membership fee and the SIAG/OPPSF membership fee are the same, currently \$15. It would be helpful if Outreach members of SIAM could join a SIAG with a reduced fee, say \$5. This concern has been raised previously and discussed with Ron Buckmire, who heartily endorsed the proposal, Tim Fest, and Carol Woodward, though to date, there has been no change.

9. *How can the activity group help SIAM in its general role of promoting Orthogonal Polynomials and Special Functions?*

Our activity group should get more involved in contributing news items for the monthly SIAM News. We will more actively look for news items related to OPSF and encourage SIAG/OPPSF members to write a text for a general audience (SIAM News readership).

In our newsletter (OPPSF-Net), we should emphasize the links between our SIAG and other SIAGs that have some intersection with our research field, e.g., Discrete Mathematics (SIAG-DM), Dynamical Systems (SIAG-DS), Linear Algebra (SIAG-LA) and Nonlinear Waves & Coherent Structures (SIAG-NWCS).

This SIAG requests that the SIAM Council and Board of Trustees renew its charter for a three-year operating period beginning January 1, 2023.

Signed



Peter Clarkson, SIAG/OPSF chair June 11, 2022