Newsjournal of the Society for Industrial and Applied Mathematics sinews.siam.org

siam news

Volume 57/ Issue 7
September 2024

# Cataloging Biodiversity with Artificial Intelligence

*By Matthew R. Francis*

The International Union for Conservation of Nature[1] (IUCN) has assessed more than 163,000 species to identify those that are threatened with extinction. Of that list, roughly 22,000 species are considered "data deficient"[2] — which means that we simply do not know enough about them to discern whether they are in decline. To complicate matters, the IUCN list does not include every species on Earth because scientists have yet to catalog them all.

Unfortunately, time is of the essence for many organisms. Climate change, pollution, loss of habitat, and other human-imposed pressures have already led to widespread extinctions. Without knowing the number and population stability of certain species, we are missing vital information about the planet's overall health.

"If you look at biodiversity studies, there is a huge bias in where they are [conducted], who they're done by, and what species we study," computer scientist Tanya Berger-Wolf of The Ohio State University said. "This bias translates into a lack of understanding of drivers of biodiversity loss and whether policies work or don't work. And we're running out of time." Given the situation's severity, Berger-Wolf is collaborating with data scientists, ecologists, and biologists to develop new machine learning (ML) methods to fill these knowledge gaps.

However, the associated computer and data science challenges are immense. The identification of known species from visual or audio data is often difficult even for experts, particularly when species are closely related or bear a strong resemblance due to mimicry or simple coincidence (see Figure 1). Furthermore, many organisms—especially insects and plants—change their appearances significantly throughout their life cycles. "I'm not a field biologist, despite having just returned from the tropical rainforest in Panama," Berger-Wolf said. "I don't like bugs and all of that! So what can a computer scientist do with what is clearly an urgent need and a huge challenge in every possible way?"

Computers can only help fill the gaps in biodiversity knowledge if they are well programmed, which in turn requires large quantities of reliable data beyond current data sources. "That means including the millions of images, videos, sounds, and observations on social media and dedicated apps for nature," Berger-Wolf said. She added that these resources might contain vital information beyond the apps' intended use; for instance, a photo of a bird may also depict relevant plants in the background. "Data sources have exploded, but we need to develop methods that extract information from high-throughput observations," Berger-Wolf continued. "That's a very different approach than what we've done before."

The magnitude of this undertaking is similar to the creation of generative artificial intelligence (AI) models that draw training data from the entire internet (although the ethics of this practice[3] are admittedly

*See* **Biodiversity** *on page 2*

[1] https://www.iucnredlist.org
[2] https://www.iucnredlist.org/search?redListCategory=dd
[3] https://www.siam.org/publications/siam-news/articles/the-ethics-of-artificial-intelligence-generated-art

**Figure 1.** *Correctly identifying species can be difficult when multiple species resemble each other, either because they are closely related or one has evolved to mimic another — such as this Australian hoverfly that looks like a wasp. Figure courtesy of Fir0002/Flagstaffotos via Wikimedia Commons under the Creative Commons Attribution-NonCommercial license.*

# Where Algebra and Geometry Meet Systems Biology

*By Alicia Dickenstein*

Researchers often use mathematical models to investigate systems of interacting species across scientific fields such as chemistry, population biology, molecular biology, and epidemiology. Here, we will concentrate on algebra and geometry's beautiful relationship with systems biology in the context of biochemical reaction networks that use systems of ordinary polynomial differential equations. This modeling approach, which is based on the law of mass-action kinetics, is typically applicable when molecules are abundant and different chemical species are well mixed. Other techniques—including stochastic and partial differential equation models—find use under different sets of assumptions, while machine learning is the subject of current exploration.

The input information for a chemical reaction network is a finite set of $n$ species and a directed graph that represents a finite set of $r$ reactions among these species. Variables $(x_1,\ldots x_n)$ in the polynomial dynamical system correspond to the different species' concentrations; the solutions of the dynamical system are curves in the nonnegative orthant in $\mathbb{R}^n$ that describe the evolution of these concentrations over time. The polynomials have a combinatorial structure that is read from the digraph of reactions and involves $p$ parameters, which are frequently unknown and can vary across individuals and environmental conditions. Surveys in the literature provide exact definitions, further references, and precise formulations of important signaling pathways [5-7].

Classical applications attempt to build models that reproduce data. The primary objective of reaction network theory is to understand these models' dynamics across different parameter values and ascertain whether specific phenomena—like bistability or oscillations—can manifest under appropriate parameter choices. A particular type of parameter values are the *reaction rate constants* $\kappa=(k_1,\ldots,k_r)$ that are associated with each reaction. In the presence of mass-action kinetics, each reaction's velocity is proportional to the product of the reactant concentrations and the corresponding reaction rate constant. Because linear conservation relations appear naturally in most signaling pathways, the trajectories over an interval are constrained to linear varieties that are parallel to the *stoichiometric subspace* $S$. If $S$ is cut out by linear equations $\ell_1(x)=\ldots=\ell_s(x)=0$, a trajectory is then contained in the linear variety $\ell_1(x)=T_1,\ldots=\ell_s(x)=T_s$ for some choice $T=(T_1,\ldots,T_s)$ that is determined by the initial conditions. These parameter values are called the *total amounts*. In this setting, we must understand the system's *stoichiometrically compatible steady states* (SCSSs), which are the joint solutions of the system

$$f_1(\kappa,x)=\ldots=f_n(\kappa,x)=$$
$$\ell_1(x)-T_1=\ldots=\ell_s(x)-T_s=0,$$

where the differential equations are denoted by $\dot{x}_i=f_i(\kappa,x),\ i=1\ldots,n$. We are especially interested in the nonnegative or positive solutions of the algebraic varieties that these polynomials define. Trajectories do not need to converge, but if they do, the limit point is a steady state that is typically *stable*. *Unstable* steady states give rise to interesting dynamics.

We can partition the parameter space of the variables $(\kappa,T)$ into semialgebraic sets (defined by polynomial equalities and inequalities with real coefficients) such that the solutions' qualitative behavior is the same in each region. While many theoretical results and computational tools already exist, the high number of variables and parameters in even the smallest biochemical reaction networks forces us to understand the structure of the systems and pursue novel theoretical and computational results. For instance, the extracellular signal-regulated kinase (ERK) pathway is a cascade of cell protein phosphorylations that communicates a signal from a surface receptor through the membrane and eventually to the DNA in the cell's nucleus [13]. In a cascade, one layer's phosphorylated *substrate* acts as the next layer's *enzyme*.

*See* **Systems Biology** *on page 3*

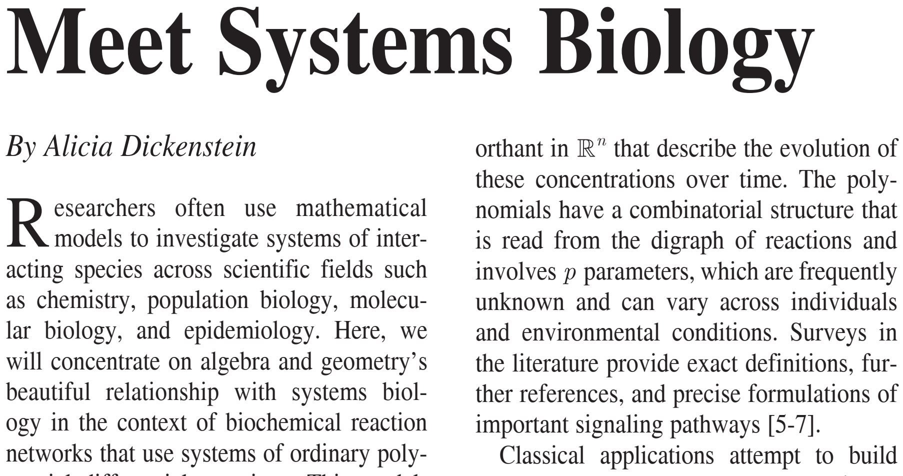


**Figure 1.** *The extracellular signal-regulated kinase (ERK) pathway: a cascade of protein reactions that communicates a signal from a receptor on the surface of a cell to its nucleus. Figure adapted from [10].*

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siam news
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## Biodiversity

*Continued from page 1*

questionable) [1]. However, the model output is very different and requires a type of synthetic analysis to recognize organisms across a wide variety of contexts and potentially identify previously undescribed species. This type of work extends far beyond the current capabilities of AI systems.

### A Fondness for Beetles

Data in the realm of modern population biology and ecology stems from a wide range of sources: field researchers, remote sensing, robotic drones, trail cameras, microphones, devices that attach directly to animals, and so forth. Regardless of provenance, the most prevalent information comes from photos and (to a lesser degree) videos. Visual data is often provided by non-scientists, as snapping a photo of a bird, bug, or flower does not require special equipment.

Some data sources are meant for sharing and identification. For example, the birdwatching community has hugely benefitted from the internet era; given the wide availability of cameras, apps, and online groups, the hobby is accessible to more people than ever. Other apps and citizen science organizations such as iNaturalist[4] are designed for broader species identification. Yet despite the growing popularity of nature apps, most users live in North America or Western Europe; in contrast, high-profile conservation work often focuses on "charismatic" animal species in more exotic areas — the lions, tigers, and bears of the world. Nothing is intrinsically wrong with these circumstances, but they do yield a glut of data for a relatively small subset of global biodiversity.

"We know a lot about very little," Berger-Wolf said, noting that scientific descriptions might not yet exist for the majority of species in multiple taxonomic groups. Although scientists estimate that beetles alone comprise roughly a quarter of all animal species, only about 400,000 beetle species have been categorized to date. As biologist J.B.S. Haldane was apparently fond of saying in one form or another, "God has an inordinate fondness for stars and beetles" [2].

Limiting one's efforts solely to animal species still leaves large blank spaces in the map of scientific knowledge. The catalogs of plants and fungi are even worse and invite additional challenges due to changes over time and the fact that these species spend large portions of their lives hidden from human eyes. For instance, a fungal colony can cover a huge area underground or inside of a tree, but people generally only notice when it sprouts mushrooms. Since many plants and fungi strongly resemble each other, teaching a computer to tell them apart is no small matter.

### When Biology Gives You Images, Make Imageomics

Berger-Wolf and her collaborators utilize a framework that was established by the Global Partnership on Artificial Intelligence[5] under the auspices of the Organisation for Economic Co-operation and Development:[6] an international think tank that pursues policies to combat climate change and environmental issues, among other causes. The ethical use of AI is part of their purview, including the application of ML to tackle biodiversity loss.

In a nod to genomics, Berger-Wolf and her colleagues proposed an umbrella category of study called *imageomics*.[7] Just as geneticists can extract crucial information from an organism's genome, imageomics integrates all available biologically relevant information from visual data to measure biodiversity; it even identifies new species and determines their relationships to other organisms [4].

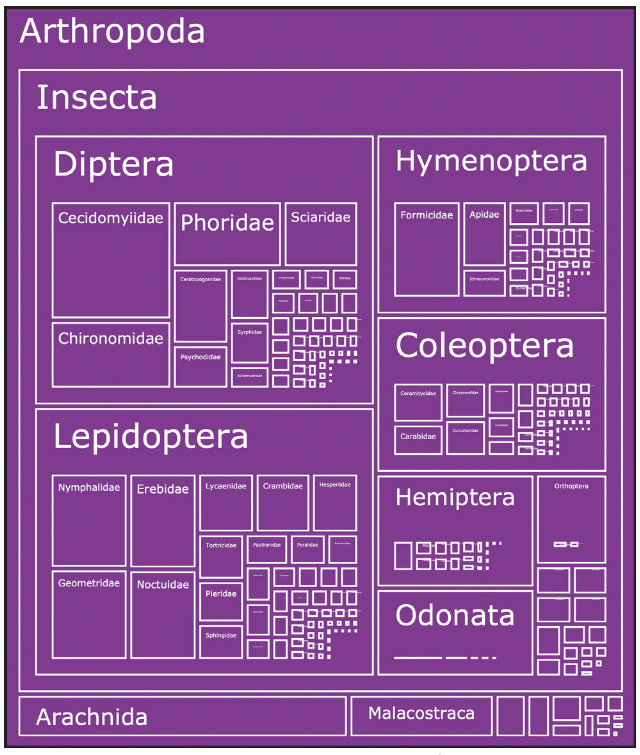


**Figure 2.** *Machine learning models that classify organisms must be trained on data that properly place them in scientific categories. The TreeOfLife-10M dataset allows users to file organisms in nesting categories from phyla down to species, shown here diagrammatically for arthropods: the group that contains insects, spiders, crabs, and other joint-legged invertebrates. Figure adapted from [3].*

To that end, the collaborators constructed a dataset called TreeOfLife-10M[8] that comprises more than 10 million images of over 450,000 species from iNaturalist, the Encyclopedia of Life,[9] and the Bioscan-1M catalog[10] of insect photos in labs. They curated the dataset to amass scientifically reliable images that pre-train their ML system, which they call BioCLIP [3]. As its name implies, this model's construction is based on the OpenAI image recognition project on Contrastive Language-Image Pre-training[11] (CLIP). Using 10 classification tests on biological images and a subset of images for rare and threatened species, Berger-Wolf's team compared BioCLIP's results to those of other image recognition ML models. They found that BioCLIP offered a substantial improvement over general-purpose models, with 16 to 17 percent more correct species identifications.

### From Cataloging to Synthesis

ML systems like BioCLIP use training data to output the most probable response based on an input; i.e., better-curated data and better-constructed goals have a higher likelihood of the response matching reality. Identifying known species in images—especially images that were taken precisely for this purpose—is a well-defined ML problem, even if it is not necessarily easy to solve.

Preliminary findings indicate that BioCLIP is able to sort images of organisms according to a broad hierarchy, separating them into scientific categories like phyla and families that document the relationships between species (see Figure 2). Given its narrow purpose, the model is also less energy intensive than large language models, which is another bonus for ecologically minded individuals.

Yet despite its success, BioCLIP is not the end goal for Berger-Wolf and her collaborators. The researchers wish to rethink how ML models work on a fundamental level so they can identify previously undescribed species and fit them within the tangled tree of life — the areas of biology known as *taxonomy* and *cladistics*. Doing so raises much broader, less-defined challenges than simple species identification. After all, human biologists are not always consistent in their own taxonomic efforts — particularly when species boundaries are blurry or organisms do not clearly fit into one genus or another.

"We definitely need something like the next generation of AI that can truly do synthesis," Berger-Wolf said, suggesting that AI techniques will need to combine different sources of information into something new. "This bumps up against the very frontier of computer science and machine learning today."

*Tanya Berger-Wolf gave an invited presentation[12] on this research at the 2024 SIAM International Conference on Data Mining,[13] which took place in Houston, Texas, this past April.*

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[4] https://www.inaturalist.org

[5] https://www.oecd.org/en/about/programmes/global-partnership-on-artificial-intelligence

[6] https://www.oecd.org

[7] https://imageomics.osu.edu

[8] https://huggingface.co/datasets/imageomics/TreeOfLife-10M

[9] https://www.eol.org

[10] https://biodiversitygenomics.net/projects/1m-insects

[11] https://openai.com/index/clip

[12] https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=80086

[13] https://www.siam.org/conferences-events/past-event-archive/sdm24

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## Systems Biology

*Continued from page 1*

Figure 1 (on page 1) corresponds to a reaction network with $n=22$ variables—eight substrates, four enzymes, and ten intermediate species—in 30 reactions with seven linearly independent conservation relations with $p=30+7$ parameters, which makes a total of 57 variables. This number is beyond the capability of current computer algebra systems. However, recent studies have used tools from algebra, geometry, and combinatorics to generate many new results about the associated family of polynomial dynamical systems.

The blue and red curves in Figure 2 represent the zero sets of $f(\kappa,x)$ and $f(\kappa',x)$ for different values of the rate constants, and the green lines represent linear varieties that are associated with different values of the total amounts $T$. Small values of positive $T$ yield a single intersection with the blue curve. But as $T$ increases, we alternate between one and three positive steady states; large $T$ again has consistently only one value. If the curves were more blended, even more SCSSs could exist for some choices of parameters. On the other hand, only a single intersection point is present between the red curve and each green line for any value of $T$. If there are at least two points of intersection (steady states) in the positive orthant, then the system has the capacity for *multistationarity* and the corresponding vector of parameters $(\kappa,T)$ is multistationary.

The occurrence of multistationarity is necessary for *multistability*, i.e., the presence of at least two different stable SCSSs. Multistability is associated with cell differentiation and the manifestation of phenotypic differences under the same genetic or environmental conditions. Because steady states under inspection are usually implicit, the standard methods of stability study require further extensions.

In 1988, Bruce Clarke proposed an interesting point of view called *stoichiometric network analysis* that only accounts for the parameters $\kappa$ [3]. Even when it is impossible to parametrize the solutions of $f_i(\kappa,x)=0,\ i=1\dots,n$ for a fixed value of $\kappa$, Clarke observed that one *can* parametrize the values of $(\kappa,x)\in\mathbb{R}^{r+n}_{>0}$ that satisfy these parametric equations in terms of generators of an explicit positive cone that is read from the reactions. This method is algorithmic, though it may be difficult to compute for non-simplicial cones. Our approach uses algebro-geometric tools to understand the structure of these polynomials in biologically meaningful examples. One such class of underlying structures are the *Modifications of type Enzyme-Substrate or Swap with Intermediates* (MESSI) networks that encompass the most popular models in systems biology, including important enzymatic networks [14].

We prove general results that—under easily checkable combinatorial conditions on the directed graph that represents the network—allow us to preclude relevant boundary steady states, thus implying that the trajectories avoid the boundary of the positive orthant. These results also permit us to determine whether the system is conservative (and trajectories are defined for any $t\geq 0$); provide an explicit basis of conservation relations; guarantee the rationality of the steady state variety (e.g., the blue or red curve in Figure 2), which means that it can be explicitly parametrized (a very unusual property of general algebraic varieties); and—in many interesting cases—give explicit binomials that cut out the steady state variety and monomials in the concentrations that parametrize it. Using tools from the theory of toric varieties and Gale duality, we demonstrate decisions about multistationarity based on the oriented matroids that are associated with the system's exponents and coefficients [12]; we also state the first multivariate generalization of the beautiful Descartes' rule of signs, which René Descartes described in 1634 for univariate real polynomials.

Another algebro-geometric technique with biological applications is the *toric degeneration* for real solutions. Given a polynomial system with exponents (represented by the dots in Figure 3) and any lifting function $h$ that is defined on the set $A$ of exponents, the projection of the lifted points' lower hull defines a *regular subdivision* of $A$'s convex hull. If the signs of minors of the matrix of system coefficients that correspond to $q$ simplices in the subdivision satisfy certain conditions, we can find an open set in the full space of parameters $(\kappa,T)$ that guarantees the existence of at least $q$ SCSSs. It is surprising that these conditions—which are dictated by the combinatorics of the exponents—are generally interpretable in terms of the biochemistry. Further research has applied the basic theory to different enzymatic mechanisms with MESSI structures, even when the dimension is high [2].

In addition to efforts that utilize singular perturbation theory to investigate multistationarity [9], a recent approach relies on combinatorial tools [8] to comprehend the connectivity of regions wherein a given polynomial is negative in the positive orthant of the $\kappa$ variables, which is related to the regions of multistationarity. While many studies examine the occurrence and inheritance of oscillations, certain results demonstrate that understanding the oscillation capacity from the network's structure is not a straightforward process [1]. In 1972, Fritz Horn and Roy Jackson proposed an important open conjecture [11]—currently known as the *global attractor conjecture*—about the dynamics of particular chemical reaction networks [4]. It concerns *complex balanced networks*, which contain the more classical *detailed balanced networks*. In this case, there exists a single positive steady state with fixed total amounts that is a local attractor; Horn and Jackson conjectured that it is indeed a global attractor of the dynamics [11]. While the conjecture certainly seems true at first, it has thus far yielded only partial results. Ultimately, however, the union of algebra, geometry, dynamical systems, and biochemistry can undoubtedly drive new theoretical results in both the application domain and the realm of real algebraic geometry.

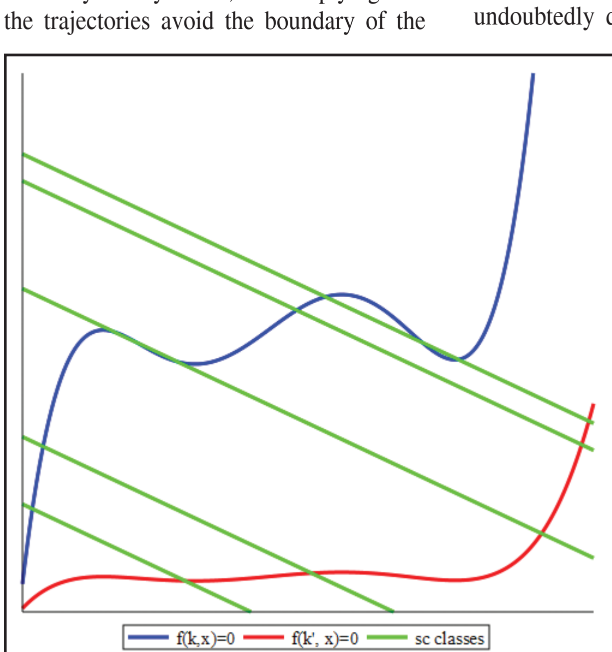


**Figure 2.** *Steady states for different parameter values. Figure courtesy of the author.*

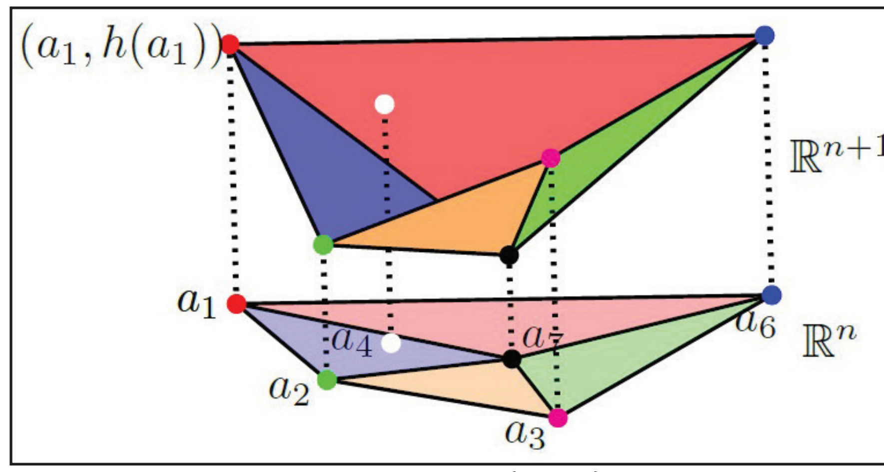


**Figure 3.** *A regular triangulation of the support $A=\{a_1,\dots,a_7\}$ for any polynomial of the form $\Sigma_i c_i x^{a_i}$, with all coefficients $c_i\neq 0$. The point $a_4$ does not belong to any simplex in this triangulation. Figure courtesy of the author.*

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# Film Screening and Discussion at AN24 Focus on the Journeys of Black Mathematicians

*By Jillian Kunze*

Despite continuing strides in equity, diversity, and inclusion (EDI), very few Black mathematicians are awarded Ph.D.s each year. According to the American Mathematical Society's Mathematical and Statistical Sciences Annual Survey,[1] less than three percent of individuals who received doctorates in the mathematical and statistical sciences from 2017 to 2018 (the last period with available data) identified as Black or African American [1]. In response to this stark disparity, the Simons Laufer Mathematical Sciences Institute[2] and ZALA Films[3] jointly commissioned a documentary titled *Journeys of Black Mathematicians: Forging Resilience*.[4] The film, which was created by George Csicsery, features interviews with Black mathematicians and current students who discuss their experiences in academia; affirm the importance of historically Black colleges and universities; and describe ongoing education, outreach, and community-building initiatives.

At the 2024 SIAM Annual Meeting,[5] which took place this July in Spokane, Wash., the Workshop Celebrating Diversity organized a screening and discussion[6] of *Journeys of Black Mathematicians*. Gonzaga University's Institute for Informatics and Applied Technology[7] sponsored the event, which began with a showing of the hour-long film. Ron Buckmire of Marist College—the SIAM Vice President for EDI—then chaired a conversation about the themes of the documentary with panelists Reginald McGee of Haverford College, Bonita Saunders of the National Institute of Standards and Technology (who was featured in the film), and Nathaniel Whitaker of the University of Massachusetts (UMass) Amherst.

Overall, the panelists reacted positively to the film. Saunders—who was born the year that the U.S. Supreme Court handed down its Brown v. Board of Education decision that declared the unconstitutionality of racial segregation in public schools, and who attended all-Black schools until eighth grade—enjoyed hearing the stories of Black mathematicians who had charted a path through the historically exclusive field. "I think that it's important to know the history of Black people and see what they went through to get where they are," she said. She was particularly impressed by the film's interview with Evelyn Boyd Granville — a mathematician who was involved in the early days of the Space Age, and who passed away last year[8] at the age of 99. Saunders recently published a tribute to Granville in *MAA Focus*—the news magazine of the Mathematical Association of America—and noted that Granville's story exemplified the impact of high-quality education for Black people throughout history [2].

Whitaker, who also attended segregated schools until ninth grade, thought that the film could be very impactful for Black students at predominantly non-Black institutions, as these students may not encounter many role models in their day-to-day lives who share their same backgrounds and experiences. "I think that this movie is a good means of showing [those role models] to students," he said. Young people may find it easier to envision their own paths when they

*See* **Black Mathematicians** *on page 5*

[1] https://www.ams.org/profession/data/annual-survey
[2] https://www.slmath.org
[3] https://www.zalafilms.com
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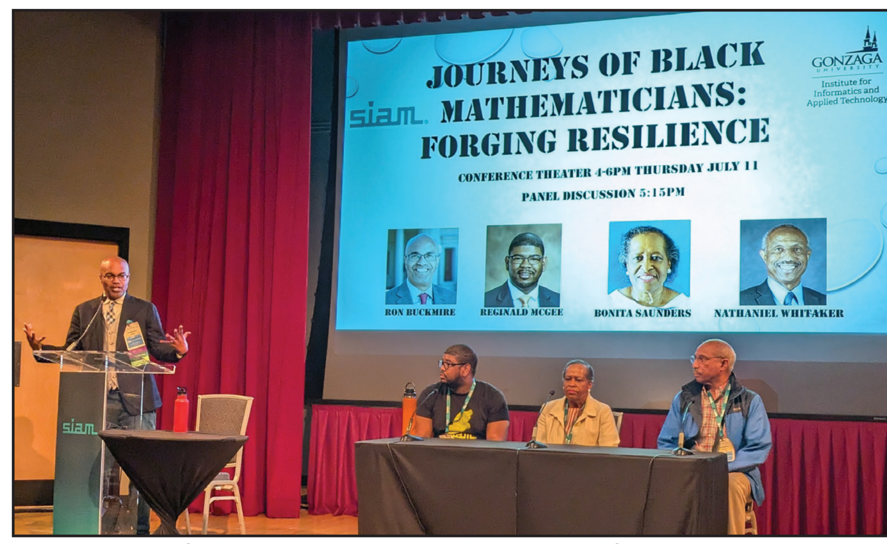


*During the 2024 SIAM Annual Meeting, which took place in Spokane, Wash., this July, the Workshop Celebrating Diversity hosted a screening and discussion of a 2024 documentary titled Journeys of Black Mathematicians: Forging Resilience. From left to right: moderator Ron Buckmire (Marist College) and panelists Reginald McGee (Haverford College), Bonita Saunders (National Institute of Standards and Technology), and Nathaniel Whitaker (University of Massachusetts Amherst). SIAM photo.*

# A Spin on Some Simple Geometry

MATHEMATICAL CURIOSITIES
*By Mark Levi*

Most middle schoolers probably know that rotating a rigid triangle does not change its area — a fact that is perhaps not mentioned because it seems so obvious and useless. High schoolers learn the Pythagorean theorem (at least, I hope they do) but are not told that the theorem is a consequence of the area's invariance.

Before we start rotating triangles and causing their sides to move, we need the simple observation in Figure 1: a moving segment of *constant length* sweeps the area at the rate

$$\underbrace{\frac{1}{2}\omega a^2}_{\text{rotation}} + \underbrace{va\sin\alpha}_{\text{translation}}. \qquad (1)$$

### The Theorem of Cosines

The *theorem of cosines* amounts to the statement $\frac{d}{dt}\text{area}=0$ when the triangle rotates on its vertex.[1] Rotating clockwise as in Figure 2, the area is gained by two of the sides and lost by side $AB$. For the sides $AC$ and $AB$ that rotate clockwise with $\omega=1$, the rates are $\frac{1}{2}a^2$ and $-\frac{1}{2}c^2$. For the remaining side $BC$, we substitute $v=\omega b=b$ into (1) and note that $\sin\alpha = \sin(\angle C-\pi/2) = -\cos\angle C$ in Figure 2. Therefore, $CB$ sweeps the area at the rate

[1] We think of the time as the angle of rotation, thus making the angular velocity $\omega=1$.

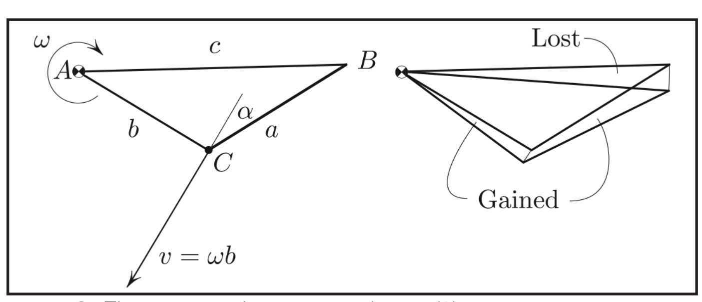


**Figure 2.** *The area remains constant, hence (2).*

$$\frac{1}{2}a^2 + ba\sin\alpha = \frac{1}{2}a^2 - ba\cos C.$$

To summarize, the rate of change of the area of $\Delta ABC$ is

$$\underbrace{\frac{1}{2}b^2}_{AC} + \underbrace{\frac{1}{2}a^2 - ba\cos C}_{CB} - \underbrace{\frac{1}{2}c^2}_{BA} = 0, \qquad (2)$$

where each underbrace represents the *rate* at which the area is swept by the corresponding side. We obtained the theorem of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C. \qquad (3)$$

Each underbraced term in (2) has a meaning that is lost in (3), but I still prefer the beauty of (3) — a shallow attitude, perhaps.

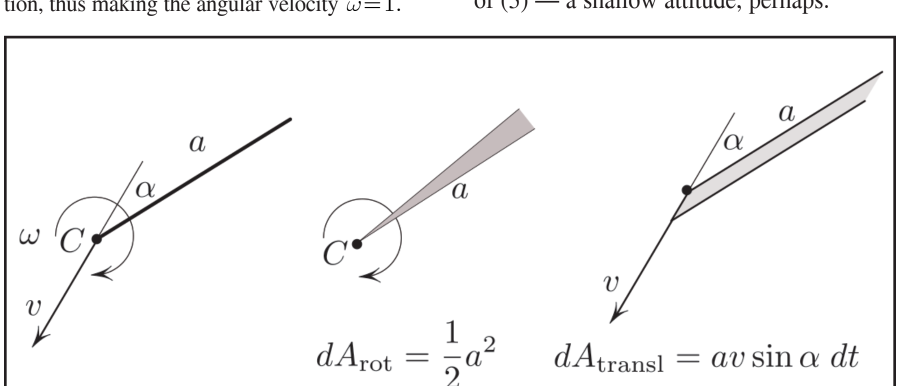


**Figure 1.** *Rigid motion's decomposition into pure rotation and pure translation, where $v$ is the speed of endpoint $C$. Rotation sweeps area $\frac{1}{2}\omega a^2$ per second; for translation, this rate is $va\sin\alpha$.*

### Milking the Invariance

By rotating the triangle around other points, we can (re)discover some identities. Here are two examples.

**Example 1.** When rotating about the orthocenter, Figure 3 gives

$$a^2+b^2+c^2 = a_1^2+b_1^2+c_1^2. \qquad (4)$$

The left side of the formula represents the rate at which the area is acquired, and the right side represents the rate of loss; equality amounts to the area's constancy. A quick alternative proof of (4) goes by application of the Pythagorean theorem, but this is of course not the point.

**Example 2.** Let us now rotate the triangle around its centroid, i.e., the point of intersection of the medians (see Figure 4). The constancy of the area translates to

$$am_a\cos\alpha + bm_b\cos\beta + cm_c\cos\gamma = 0, \qquad (5)$$

where $m_a$, $m_b$, and $m_c$ are the lengths of the medians. To justify (5), we note that the midpoint of side $a$ moves with speed (keeping $\omega=1$)

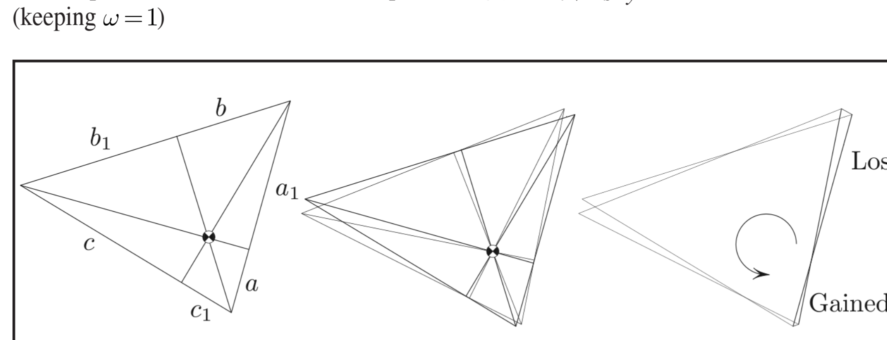


**Figure 3.** *The area remains constant, hence (4).*

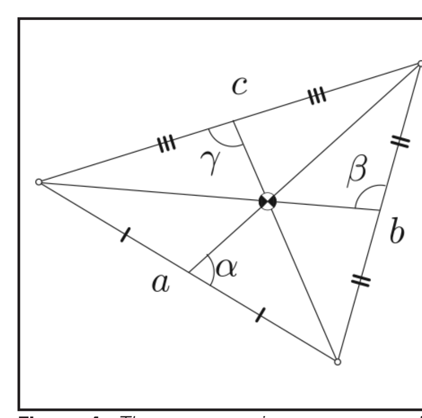


**Figure 4.** *The area remains constant under rotation, hence (5).*

$$m_a/3,$$

at the right angle to the median. Furthermore, rotation of the side around the *midpoint* contributes nothing to the change of area. The side thus sweeps the area at the rate

$$\frac{1}{3}m_a a\sin\left(\frac{\pi}{2}-\alpha\right) = \frac{1}{3}m_a a\cos\alpha;$$

adding the rates for all three sides yields (5).

One can further milk the area (or volume) invariance for other figures (or solids in $\mathbb{R}^3$).

*The figures in this article were provided by the author.*

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## Black Mathematicians

*Continued from page 4*

have concrete examples of other people like themselves who found success in mathematics or similar fields. “It was very inspiring,” McGee added. “At this point in my career, it helps me to become reinspired.”

SIAM and other professional organizations offer camaraderie and support to uplift underrepresented minorities. Panelists mentioned SIAM’s multiple opportunities for funding, networking, and visibility, including travel awards[9] and the MGB-SIAM Early Career Fellowship[10] (a joint program with Mathematically Gifted & Black[11]); other organizations such as the Math Alliance[12] provide similar benefits. Participation in conferences or special events can also help researchers connect with compatriots who are pursuing similar career goals. “Going to conferences through societies is an important way to be in the community and get organized,” McGee said. “Get out in fellowship and find community.”

Saunders, who joined SIAM as a graduate student, agreed that professional societies and meetings serve as useful avenues for junior scientists to explore different careers and receive encouraging pep talks. “Being in these societies impacts your career,” she said. “It gives you an opportunity to meet people like yourself, start collaborations, and see what others do to succeed.” Whitaker concurred with the value of these occasions in expanding one’s world. “One of the most important things for mathematics is being part of a community,” he said.

Next, audience members inquired as to how non-Black mathematicians can better serve as allies. McGee urged attendees to remember their Black colleagues for prize nominations and other openings to advance their careers. “Keep them in mind for opportunities that they may be overlooking,” he said. Meanwhile, Saunders focused on the need to foster healthy, assured workplace settings. “Make sure that you create an accepting environment so Black mathematicians feel comfortable talking to you,” she said. “Be open to listening to them and putting yourself in that person’s shoes.” And if someone makes a racist comment about another person behind their back, Saunders advised listeners to handle the situation as discretely as possible, since telling the subject about the event could be emotionally devastating. She also asked everyone to remain cognizant of how current news events or happenings within an organization may affect colleagues from different backgrounds. “Get to know other people who don’t look like you so that you know what’s going on in their lives,” she said.

*Journeys of Black Mathematicians* portrayed a generally positive, robust trajectory for EDI in the mathematical sciences, and attendees were curious whether this portrayal aligned with the panelists’ own experiences. Saunders admitted that the movie made it seem a bit too easy to inspire children to study mathematics, especially given her own efforts with tutoring. “I think that we need a lot more concentration on the elementary and middle school level,” she said. “If that elementary education isn’t solid, those kids can’t get to where you are.” Similarly, Whitaker helped to establish and run a Saturday morning program that encouraged kids to elevate their math skills — inspired by his observation that the academic pipeline did not produce many Black faculty candidates when he served as dean at UMass Amherst.

The panelists then commented on the ways in which people with different racial identities can contribute to EDI programs and initiatives. For instance, Whitaker recounted his experiences as a Ph.D. student at the University of California, Berkely, which at the time did not have any Black professors within the department but did enroll many Black students. It was thus necessary for other people to become involved and uplift these students. “If I had to depend on Black mentors, I wouldn’t have been able to because there were not that many,” Whitaker said.

Buckmire confirmed that there is no one correct solution or means of contribution, and programs and organizations that are not racially specific can also have an impact in tandem with more targeted initiatives. “Many flowers can bloom,” he said. “There’s lots of work to go around.” However, some efforts—such as serving as a role model for children from the same background—can only be taken on by people within a specific community.

Finally, the panelists addressed the difficulty of balancing the pressures of an early career with the goal of advancing EDI. All of the speakers agreed that in academic settings, it is important to prioritize the acquisition of tenure in order to earn the power and license to incite positive change. But while assuming extra responsibilities can certainly be worthwhile, there is no need to rush and fit everything into what is ultimately the first chapter of a long career. “When you’re early-career, find a balance that works for you,” McGee said. He also advised listeners to connect with a community of individuals in whom they can confide when circumstances become rough. In difficult times, Whitaker compares his struggles to those of his ancestors to maintain perspective. “That should inspire you to say, ‘I can tackle these things,’” he said.

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[2] Saunders, B.V. (2024). A tribute to mathematician Evelyn Boyd Granville: 1924-2023. *MAA Focus*, *44*(3), 20-24.


*Jillian Kunze is the associate editor of SIAM News.*

[9] https://www.siam.org/conferences-events/conference-support/travel-and-registration-support
[10] https://www.siam.org/programs-initiatives/programs/mgb-siam-early-career-fellowship
[11] https://mathematicallygiftedandblack.com
[12] https://www.mathalliance.org

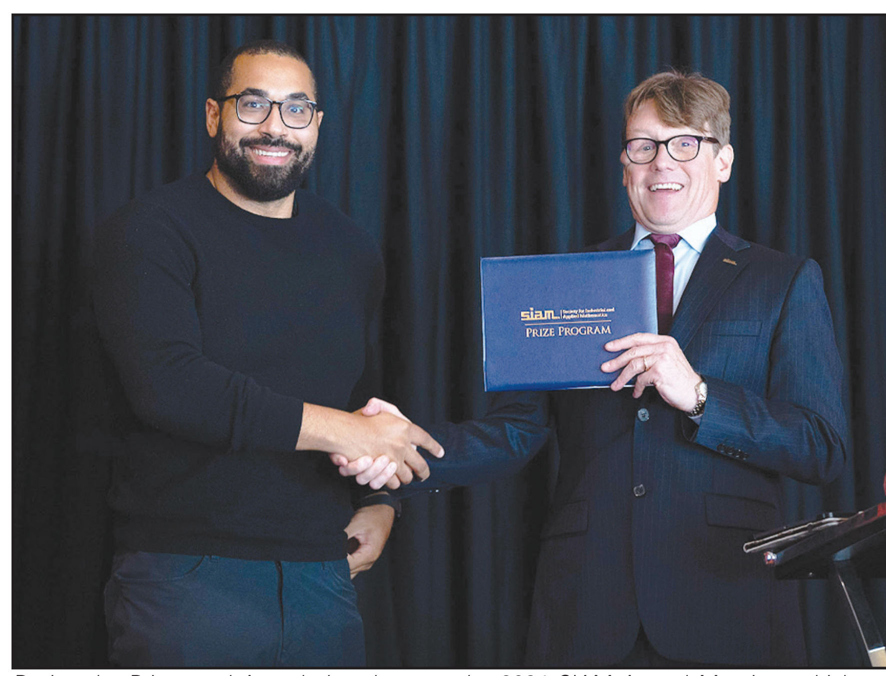
*From left to right: Ron Buckmire (Marist College), Reginald McGee (Haverford College), Bonita Saunders (National Institute of Standards and Technology), and Nathaniel Whitaker (University of Massachusetts Amherst) discuss notable themes of the documentary Journeys of Black Mathematicians: Forging Resilience during a session at the 2024 SIAM Annual Meeting, which was held in Spokane, Wash., this July. SIAM photo.*

# Photos from the 2024 SIAM Annual Meeting


*SIAM President Sven Leyffer (right) presents Jorge Nocedal of Northwestern University with the John von Neumann Prize: SIAM's highest honor and flagship lecture. Nocedal—who accepted the award during the Prizes and Awards Luncheon at the 2024 SIAM Annual Meeting, which was held in Spokane, Wash., in July—was acknowledged for his fundamental work in nonlinear optimization, both in deterministic and stochastic settings. He later delivered an associated prize lecture titled "Exploring the Mysteries of Deep Neural Network Optimization." SIAM photo.*


*SIAM's chief executive officer Suzanne Weekes (right) poses with Caoimhe Rooney of Astroscale and Mathematigals, who delivered the I. E. Block Community Lecture at the 2024 SIAM Annual Meeting in Spokane, Wash., this July. The lecture is named after I. Edward Block, SIAM's co-founder and first managing director; it is open to the public and meant to encourage societal appreciation of the excitement and vitality of science. Rooney's lecture—which was titled "Go Boldly Where No Math Has Gone Before"—highlighted mathematical contributions to some of the world's greatest challenges and addressed her own work in space exploration and sustainability. SIAM photo.*


*SIAM President Sven Leyffer (right) and Association for Women in Mathematics (AWM) President Talitha Washington (left) congratulate Sunčica Čanić of the University of California, Berkeley, who presented the AWM-SIAM Sonia Kovalevsky Lecture at the 2024 SIAM Annual Meeting in Spokane, Wash., this July. Čanić's prize citation praises her as "a highly influential applied mathematician working in the modeling, analysis, and computation of partial differential equations." She has made profound contributions across a wide spectrum of mathematics, including mathematical analysis of complex physical phenomena and the design and testing of new numerical methods. During her address, Čanić spoke about "Mathematics for Bioartificial Organ Design." SIAM photo.*




*During the Prizes and Awards Luncheon at the 2024 SIAM Annual Meeting, which took place this July in Spokane, Wash., SIAM President Sven Leyffer (right) congratulates John Urschel of the Massachusetts Institute of Technology for his receipt of the Richard C. DiPrima Prize. Urschel was recognized for his outstanding contributions to fundamental problems in applied linear algebra, as evidenced by his Ph.D. dissertation titled "Graphs, Principal Minors, and Eigenvalue Problems." SIAM photo.*


*Benedetto Piccoli of Rutgers University–Camden (left) accepts the W.T. and Idalia Reid Prize in Mathematics from SIAM President Sven Leyffer during the Prizes and Awards Luncheon at the 2024 SIAM Annual Meeting, which was held in Spokane, Wash., this July. The prize celebrated Piccoli's contributions to the fields of optimal control and conservation laws, with applications to vehicular traffic, robotics, and biology. He gave a corresponding talk during the meeting on "100 Years of Traffic Models: From Road Tolls to Autonomy." SIAM photo.*

*Members of the SIAM Fellows classes of 2023 and 2024 were recognized during the Fellows Reception at the 2024 SIAM Annual Meeting, which took place in Spokane, Wash., this July. From left to right: SIAM President Sven Leyffer, Nathaniel Whitaker of the University of Massachusetts Amherst (class of 2024), Daniela Calvetti of Case Western Reserve University (class of 2023), Ulrike Meier Yang of Lawrence Livermore National Laboratory (class of 2024), Michael Friedlander of the University of British Columbia (class of 2024), Hans De Sterck of the University of Waterloo (class of 2024), and Ron Buckmire of Marist College (class of 2023). SIAM photo.*

***View information about all 2024 SIAM major prize recipients at*** *https://www.siam.org/programs-initiatives/prizes-awards/major-prizes-lectures*

***A complete list of 2023 and 2024 SIAM Fellows is available at*** *https://www.siam.org/programs-initiatives/prizes-awards/fellows-program/fellows-directory*

# Numerical Linear Algebra at the Institute for Numerical Analysis

*By Claude Brezinski, Gérard Meurant, and Michela Redivo-Zaglia*

*A Journey Through the History of Numerical Linear Algebra*[1] *by Claude Brezinski, Gérard Meurant, and Michela Redivo-Zaglia was published by SIAM in 2022. This comprehensive book chronicles the development of numerical methods for linear algebra problems, describes the relationship between these methods and computing tools in various time periods, and features 78 biographies of important researchers in the field. The following excerpt is adapted from chapter 5, which is called "Iterative Methods for Linear Systems." It has been abridged and edited for clarity.*

Here, we review the history of the Institute for Numerical Analysis (INA) at the U.S. National Bureau of Standards (NBS)—now the National Institute of Standards and Technology (NIST)—and describe its important role in the early development of numerical linear algebra. NBS was founded in 1901 as a physical laboratory to provide standard weights and measures. Edward Uhler Condon served as NBS director from 1945 to 1951, while John Hamilton Curtiss joined in 1946 as an assistant to the director and took charge of statistical problems. That same year, Condon instructed Curtiss to assess the federal need for computers and the potential for a national computing center. This investigation had its source at the Office of Naval Research (ONR), as ONR's Mathematics Branch was formed in 1946 with Mina Rees as its head. It was suggested that NBS and ONR jointly establish a computing center to develop and use computers.

Curtiss quickly realized that the necessary mathematics to exploit the new computers also had to be developed, and he and Rees were largely responsible for the formation of INA. In the fall of 1946, Curtiss traveled the West Coast to seek a home for the proposed INA. Although INA was ultimately established at the University of California, Los Angeles (UCLA), the Institute was originally housed at NBS in Washington, D.C.

INA's main goal was to plan and conduct a program of research in pure and applied mathematics that ultimately developed methods of analysis to enable the most efficient and general use of high-speed automatic digital computers. In response to some problems with the procurement of computers, a proposal in early 1948 recommended that Harry Douglas Huskey develop a small machine at INA; his efforts led to the Standards Western Automatic Computer (SWAC) [4]. The project began in January 1949, and the first Williams tube machine to be completed in the U.S. was dedicated on April 7, 1950. After a rather long period of debugging, SWAC finally became a reliable machine with many significant accomplishments (see Figure 1). When INA operations ceased in 1954, SWAC was transferred to UCLA and continued to function until 1967.

The first INA appointees were Olga Taussky, her husband John Todd, and Albert Cahn, Jr. The Institute's initial two projects were titled *Characteristic Roots of Matrices* and *Applications of Automatic Digital Computing Machines in Algebra and Number Theory*. The inaugural report about the activities of the National Applied Mathematics Laboratories (NAML)—of which Curtiss was named chief in July 1947—was published in January 1948; NAML was later renamed the Applied Mathematics Division (AMD). The new INA building at UCLA opened in April 1948; Gertrude Blanch organized the computational unit at that location. The months of April, May, and June 1948 were an organizational period during which scientific and computational staff were hired.

FROM THE SIAM BOOKSHELF

Douglas Rayner Hartree, a mathematical physicist from Cambridge, U.K., became INA's first director. He stayed only a short time, however, after which Curtiss served as acting chief until the fall of 1949. George Elmer Forsythe was INA's first regular member of research staff, Cornelius Lanczos was the second, and Wolfgang Richard Wasow was the third. Magnus Rudolph Hestenes, a professor at UCLA, was also heavily involved with INA. He assumed the titles of Assistant Director of Research and UCLA Liaison Officer in the mid-1950s.

In September 1949, John Barkley Rosser was appointed as INA's new director. Despite his brief one-year term, he had a great impact on INA. Rosser organized a weekly seminar that focused on iterative methods for the solution of linear equations and the computation of eigenvalues, while Forsythe sought to classify the various known methods for solving linear systems. Hestenes, William Karush, and Marvin Leonard Stein studied methods for computing eigenvalues, with a particular emphasis on gradient, power, and inverse power methods. Lanczos worked on Chebyshev polynomials and orthogonalization techniques for computing eigenvalues and solving linear systems.

After Rosser returned to Cornell University in September 1950, Fritz John—an expert in partial differential equations—became INA director for the subsequent year. From July to September 1950, staff included John, Blanch, Forsythe, Huskey, Karush, Lanczos, Wasow, Paul Erdős, Richard Feynman, Mark Kac, Edward James McShane, William Edmund Milne, Theodore Samuel Motzkin, and Otto Szász.

1951 was an important year for INA. That summer, Hestenes devised the conjugate gradient method for the solution of linear systems. Theoretically, this method delivers the solution in at most $n$ iterations for a matrix of order $n$. Hestenes offered three versions of his routine and later recounted the details in a NIST report with Todd [3]:

> "When [Eduard Ludwig] Stiefel arrived at INA from Switzerland, the librarian gave him a paper describing this routine. Shortly thereafter, Stiefel came to Hestenes' office with this paper in hand and said, "Look! This is my talk." It turned out that Stiefel had invented the same algorithm from a different point of view. He looked upon it as a relaxation routine, whereas Hestenes viewed it as a gradient routine on conjugate subspaces. The term 'conjugate gradient' was coined by Hestenes."

Stiefel accepted a joint appointment with INA and UCLA so that he and Hestenes could write an extensive paper about the conjugate gradient method. Stein, Robert Mayo Hayes, Urs Hochstrasser, and W. Wilson carried out the numerical experiments — although Forsythe, Karush, Rosser, and Lowell J. Paige also deserve credit for developing the conjugate gradient routine. In fact, Rosser and Stiefel presented the conjugate gradient method at the Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues in August 1951, which was a part of NBS' Semicentennial Celebration.

1953 saw the onset of problems at NBS that were linked to the so-called "battery additive scandal." Beginning in the early 1920s, NBS tested many commercial battery additives and never found any to be beneficial. The owner of a small company in California proclaimed that his product—a chemical additive called AD-X2—had a special composition and advantageous effect. The company lobbied at Congress and encouraged satisfied users to flood Congress and NBS with testimonial letters. NBS performed further tests, with the same results as before. Due to pressure on the federal administration, Allen Varley Astin (then-director of NBS) was asked to resign by the Assistant Secretary of Commerce for Domestic Affairs.

This situation generated unfortunate consequences for INA, as the Department of Defense (DoD) created new rules that complicated funding arrangements with ONR. Although Astin was reinstated as director in August 1953, four sections of NBS were transferred to DoD. One investigation committee praised AMD's activities but questioned the appropriateness of INA's inclusion within NBS. Eventually, NBS was forced to relinquish its sponsorship of INA.

On June 30, 1954, INA held a "Gala Party and Luncheon" in honor of its impending transfer to UCLA, the people leaving INA, and the individuals staying at UCLA. This marked the end of an Institute that produced many interesting results during its brief existence. It was a pity—and probably detrimental to U.S. research—that such a fascinating place was shuttered due to administrative problems.

The main sources of information about INA's history are the 1991 report by Hestenes and Todd [3] and the quarterly reports of NAML and AMD activities. Additional information about NBS [1] is also available in papers by Todd [8, 9], Gene Golub [2], and Dianne Prost O'Leary [5-7].

*Enjoy this passage? Visit the SIAM Bookstore*[2] *to learn more about A Journey Through the History of Numerical Linear Algebra*[3] *and browse other SIAM titles. Additionally, a book review*[4] *of this text by Volker Schulz of Trier University recently published in the June 2024 issue of SIAM Review (Vol. 66, Issue 2).*[5]

**Figure 1.** *Harry Douglas Huskey works at the Standards Western Automatic Computer (SWAC), which he designed and constructed for the Institute for Numerical Analysis at the U.S. National Bureau of Standards. Figure courtesy of the National Institute of Standards and Technology Digital Archives, Gaithersburg, MD, 20899.*

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*Claude Brezinski is a professor emeritus of numerical analysis at the University of Lille in France, where he was head of the Laboratory of Numerical Analysis and Optimization for 30 years. He is the founder and editor-in-chief of the journal Numerical Algorithms. Gérard Meurant is retired from the French Atomic Energy Commission, where he worked in applied mathematics from 1970 to 2008. He was research director at the time of his retirement. Michela Redivo-Zaglia is a retired professor of numerical analysis at the University of Padua in Italy, where she served as vice director of the Department of Mathematics for three years.*

[1] https://epubs.siam.org/doi/10.1137/1.9781611977233

[2] https://epubs.siam.org

[3] https://epubs.siam.org/doi/10.1137/1.9781611977233

[4] https://epubs.siam.org/doi/suppl/10.1137/1.9781611977233/suppl_file/sirev+book+review.pdf

[5] https://epubs.siam.org/toc/siread/66/2

# BLIS: Extending BLAS Functionality

By Devin Matthews, Robert van de Geijn, Maggie Myers, and Devangi Parikh

SOFTWARE AND PROGRAMMING

A wide range of computational mathematics applications build upon the Basic Linear Algebra Subprograms (BLAS) — a fact that underscores the importance of community software for dense linear algebra (DLA). The BLAS-like Library Instantiation Software[1] (BLIS) project[2] is both a widely used open-source BLAS implementation on central processing units (CPUs) and a toolbox/framework for the rapid instantiation of BLAS-like DLA functionality [5]. The recent 1.0 release of BLIS in May 2024 extended its portability to a wide variety of architectures, including x86-64 (Intel and AMD), ARM (ARM32, AArch64, and Ampere Altra), IBM Power Systems, and RISC-V (especially SiFive X280). In addition, this release significantly expanded BLIS's multithreading capabilities to support diverse end-user applications and enable scalability to hundreds of cores. BLIS 1.0 also incorporates proposed changes to BLAS that consistently handle exceptions [1], like the propagation of `NaN` and `Inf`. Ultimately, the updated software is an excellent open-source solution for all CPU BLAS needs. In fact, readers may already be using BLIS, as it is included in the AMD Optimizing CPU Libraries[3] and NVIDIA Performance Libraries;[4] it is even packaged in major Linux distributions.

But BLIS is so much more than simply a BLAS implementation. Have you ever needed to contort your code to fit the functionality of traditional BLAS? Has the performance, portability, maintainability, or readability of your code suffered as a result? The next major release—BLIS 2.0—will represent a fundamental shift in BLIS's seamless capability to enable the extension of BLAS to new high-performance operations. Users will be able to implement BLAS-like operations by remixing BLIS's "building blocks" with specified components. These features are already available for early evaluation ahead of the forthcoming release. In this article, we explore the nuances of BLIS 2.0 and explain its ability to enable high performance and novel capacities with low code complexity.

### Portable High Performance

High-performance implementations of the types of operations that BLAS supports require the careful orchestration of data movement between memory layers, amortizing overhead over useful computation. Goto's algorithm exploits this balance nearly optimally for the matrix-matrix multiplication `GEMM` operation within Level 3 BLAS. Figure 1 illustrates the implementation of this algorithm in BLIS. For $C := \alpha AB + \beta C$, the successive partitioning of operands allows for data resuse in caches. Along the way, submatrices of $A$ and $B$ are packed to improve data locality. When targeting a new CPU architecture, BLIS only requires a specialized *microkernel*; all other parts of the algorithm are implemented in ISO C99. The five loops are parallelizable with OpenMP directives or other threading mechanisms. Implementations of related matrix-matrix operations (e.g., other capabilities in Level 3 BLAS) employ a modified version of the same algorithm that takes advantage of special structures in one or more operands.

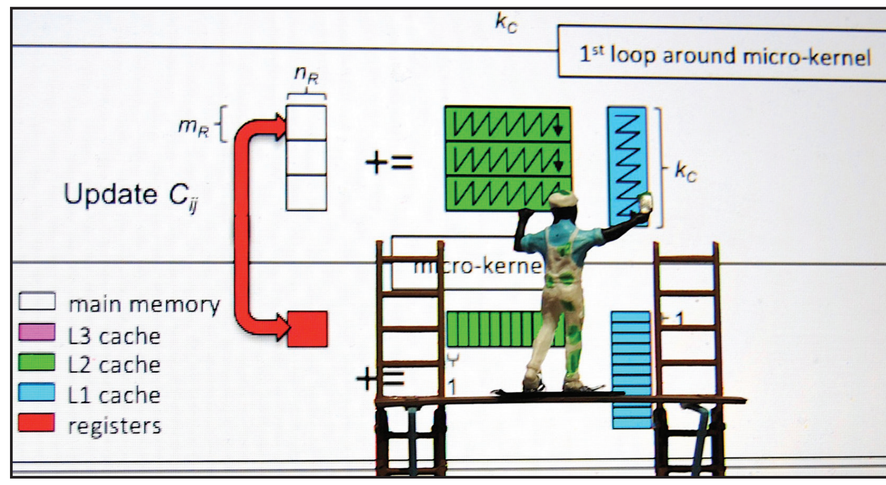
### BLIS Plugins and New Functionality

Modern DLA computations in scientific computing, machine learning, and data science often necessitate BLAS-like operations that—when implemented in terms of traditional BLAS—require extra data movement, workspace, or post-processing efforts from main memory. We can restructure and optimize this BLAS-adjacent overhead by organizing the algorithmic framework to enable the modular incorporation of additional or modified operations.

To support this process, a novel plugin mechanism in BLIS 2.0 lets users define new microkernels, packing routines, or other operations in their own code and easily incorporate these components—along with BLIS-supplied ones—into an innovative BLAS-like operation that retains high performance. A simple application programming interface (API) allows users to tweak existing operations like `GEMM` or design new algorithms from scratch via BLIS's control tree abstraction. For example, users can cast the following operations as BLIS plugins:

- "Sandwiched" matrix multiplication operations compute $C := AXB + C$, where $X$ might be a diagonal or tridiagonal matrix. BLIS can fuse multiplication by $X$ into packing, thereby reducing memory traffic and improving performance.
- The computation of $M = (X + \delta Y)(V + \epsilon W)$—followed by the result's addition to multiple distinct submatrices—is a critical building block for the practical high-performance implementation of Strassen's algorithm [2]. BLIS can incorporate the linear combination of matrices into the packing; $M$'s addition to multiple matrices is a post process in the microkernel.
- Calculation of the $k$-nearest neighbors for all points in a $d$-dimensional point set is an important primitive in clustering analysis. Previous research used a BLIS-like framework [6] to implement this operation and achieve impressive speedups.
- Tensor contractions arise in machine learning, quantum computing, quantum chemistry, data science, and other fields. A typical implementation requires the costly transformation of data into a `GEMM`-compatible ordering of elements.

See BLIS on page 10




BLAS-like Library Instantiation Software (BLIS): Building new functionality. Figure courtesy of Robert van de Geijn and Maggie Myers.

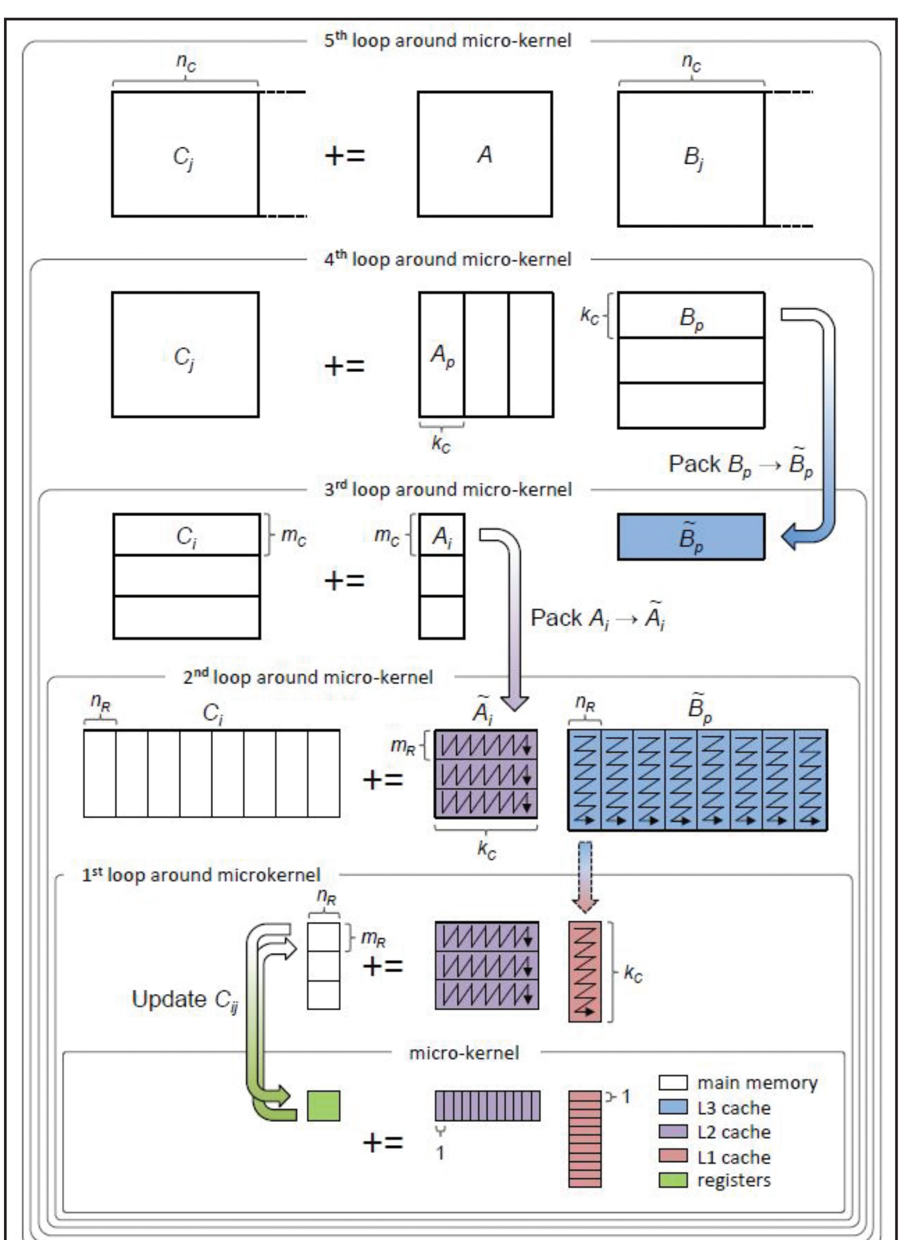



**Figure 1.** *The BLAS-like Library Instantiation Software (BLIS) refactoring of the GotoBLAS algorithm as five loops around the microkernel. Figure adapted from [4].*

[1] https://github.com/flame/blis
[2] https://www.siam.org/publications/siam-news/articles/blis-blas-and-so-much-more
[3] https://www.amd.com/en/developer/zen-software-studio/applications/spack/spack-aocl.html
[4] https://developer.nvidia.com/nvpl

# Research in Biomathematics: A Guide to Funding from the U.S. Army Research Office

*By Reinhard Laubenbacher and Luis Sordo Vieira*

In recent years, mathematics has become a key aspect of life sciences and biomedicine research. As these fields continue to amass increasingly more data, the workforce will need to expand considerably to keep pace with the demand for mathematical modelers and data scientists in academia, industry, and government agencies. Academia plays a vital role in the life sciences for two reasons: It significantly contributes to basic research in biology and biomedicine, and it is a critical gateway to the mathematical biology workforce. As such, all faculty members must have access to research funding and support for their students. This need is especially urgent for faculty at institutions that lack extensive research support and infrastructure, as they often cannot access the services of experienced sponsored program offices and do not have colleagues with active awards who can help with grant applications. Furthermore, it is quite difficult to navigate the application requirements at U.S. federal agencies—such as the National Institutes of Health (NIH) and National Science Foundation (NSF)—without expert guidance.

Other grant programs—including many initiatives from foundations or professional societies—are typically easier to parse, though they generally do not accept unsolicited applications. One such federal program is the Army Research Office's (ARO) Biomathematics Program[1] under the direction of Virginia Pasour. The process begins with a conversation about a possible project with the relevant program officer. If the proposed project fits the program's scope, then the officer encourages the researcher to apply. Applications are not confined by any specific or restrictive formatting guidelines. After reviewers provide their critiques, applicants can respond (much like journal submissions) and program officers work with them on their materials. This procedure is particularly helpful for inexperienced grant writers, who may stumble over grantsmanship issues despite the scientific soundness of their projects. Such protocols significantly improve the likelihood that faculty will secure funding for innovative projects and research endeavors — as does access to a mentor who has successfully obtained funding through the program in question.

With appropriate budgeting, funded projects can involve undergraduate students as research interns. Students who are exposed to biomathematics research are more inclined to consider a future career in this field, thereby expanding the workforce and providing the chance to welcome members of traditionally underrepresented communities. The existing structure of grant funding hence needs to (i) support faculty who conduct important biomathematics research and (ii) expand the participation of researchers from underrepresented groups by establishing a pipeline to graduate study in the field.

Thanks to financial support from ARO's Biomathematics Program and organizational support from SIAM, we organized a workshop[2] about ARO funding for biomathematics research at the 2024 SIAM Conference on the Life Sciences[3] (LS24), which took place in Portland, Ore., this past June. Approximately 50 participants attended the presentations, panel discussions, and breakout groups that comprised the three-hour session. The ARO grant allowed us to offer financial support for 10 participants — many of them from historically Black colleges and universities, minority-serving institutions, and primarily undergraduate institutions (PUIs).

As director of the Biomathematics Program, Pasour kicked off the workshop with a detailed description of the program, its scope, and its scientific focus. One key area of interest is *the fundamental laws of biology*, which encompasses research on theoretical biology questions that might elucidate general principles in biological systems that can be formulated in mathematical terms. Proposals in this realm are expected to be high risk/high reward and do not necessarily require extensive preliminary work. Another area of emphasis is *multiscale modeling in biology*, which addresses the mathematical challenges of this approach and its applications. Pasour also described the different stages of the application process, after which workshop participants asked extensive questions. A breakout session at the end of the event provided another opportunity for attendees to interact with Pasour.

An essential resource throughout the grant application process is a stable support structure that helps faculty navigate the necessary pre- and post-award administrative tasks. As such, Della Philman (assistant director of the Sponsored Programs Office in the Department of Medicine at the University of Florida, an R1 institution) and Rodney Granec (executive director of the Office of Sponsored Programs, Research and Outreach at the University of West Alabama, a PUI) introduced workshop participants to the wide range of grant support at institutions of higher education. Both presenters listed the types of available resources for principal investigators across various institutions and outlined effective strategies for identifying and accessing assistance and mentorship during the entirety of the grant process. Their presentation slides are available on the workshop's webpage.[4]

*Attendees of the workshop about Army Research Office funding for biomathematics research at the 2024 SIAM Conference on the Life Sciences—which was held in Portland, Ore., in June—listen to presentations about the elements that comprise a successful grant application. Photo courtesy of Luis Sordo Vieira.*

Anyone who is applying for funding should be aware of *grantsmanship*: the skill of presenting a research project to reviewers in such a way that they can appreciate the significance, innovation, and appropriateness of the technical approach. Applicants should frame the project as a scientific "story" that connects these considerations and allows reviewers—who might be reading as many as a dozen proposals for any given program—to easily understand the project's general approach and scientific context. Czerne Reid (a program director in the Department of Psychiatry and affiliate associate professor in the College of Journalism and Communications at the University of Florida), addressed this topic in the next workshop presentation. She taught attendees how to effectively present a scientific story for a general audience — whether in the form of a grant proposal, news article, or scientific manuscript.

To convey the firsthand experience of researchers who are looking for grant funding, Tongli Zhang (an associate professor in the Department of Pharmacology and Systems Physiology at the University of Cincinnati) spoke about his current grant from ARO's Biomathematics Program for systems pharmacology work. He advised the audience to always seek mentorship and guidance from as broad a spectrum of individuals as possible. Luis Sordo Vieira (an assistant professor in the Department of Medicine at the University of Florida and co-author of this article), reinforced this message. After earning a Ph.D. in pure mathematics, Sordo Vieira transitioned to computational biomedicine and added an experimental component to his research program; he recently received a career development award from the NIH. He also urged attendees to ask themselves three questions before requesting funds for a specific project: Should it be done? Can it be done? And will it be done?

After a brief panel discussion and question-and-answer session with the speakers, Suzanne Weekes (chief executive officer of SIAM) delivered the final talk of the workshop. She described SIAM's mission and overviewed its emphasis areas, activities, and programs that target education. At the event's conclusion, speakers and participants led several breakout groups and proffered themselves as mentors for any faculty members who were preparing to submit a grant proposal.

Following the workshop, a highly informative LS24 panel discussion[5]—which included Pasour as a panelist—featured program officers from the NSF, ARO, National Institute of Mental Health, and National Institute of General Medical Sciences. During the hour-long session, the officers outlined grant programs and other exciting happenings that focus broadly on the life sciences. For example, in addition to "regular" opportunities from NSF like the Division of Mathematical Sciences' Mathematical Biology Program,[6] new initiatives include two programs on digital twins[7] — one of which is joint with NIH.[8]

As the presenters emphasized, mentorship is of the utmost importance in the pursuit of funding opportunities. However, mentors are often difficult to find — especially for early-career researchers whose environments do not offer easy access to experienced faculty or extensive institutional resources. As a follow-up to the workshop, we intend to establish a "clearinghouse" for mentor/mentee contact information and have set up a mentoring sign-up feature on the workshop website that allows potential mentors to specify their expertise, interests, and the roles they are willing to undertake (from "one-time consultation" to "ongoing mentorship" and anything in between). This platform will permit researchers to describe their focus areas and mentoring needs, and we will provide advice to potential mentees and facilitate connections with interested parties. We encourage experienced researchers to sign up and help their early-career colleagues successfully establish their life sciences careers; this opportunity is open to anyone and is not limited to workshop participants.

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*Virginia Pasour, director of the Army Research Office's (ARO) Biomathematics Program, overviews the initiative and describes its scientific focus during a workshop about ARO funding at the 2024 SIAM Conference on the Life Sciences, which took place in June in Portland, Ore. Photo courtesy of Luis Sordo Vieira.*

[1] https://cftste.experience.crmforce.mil/arlext/s/baadatabaseentry/a3Ft0000002Y397EAC/opt0021

[2] https://systemsmedicine.pulmonary.medicine.ufl.edu/news-and-events/research-in-biomathematics-a-guide-to-funding-from-the-u-s-army-research-office

[3] https://www.siam.org/conferences-events/past-event-archive/ls24

[4] https://systemsmedicine.pulmonary.medicine.ufl.edu/news-and-events/research-in-biomathematics-a-guide-to-funding-from-the-u-s-army-research-office

[5] https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=80698

[6] https://new.nsf.gov/funding/opportunities/mathematical-biology

[7] https://new.nsf.gov/funding/opportunities/mathematical-foundations-digital-twins-math-dt/nsf24-559/solicitation

[8] https://new.nsf.gov/funding/opportunities/foundations-digital-twins-catalyzers-biomedical

# Career Opportunities Panel at AN24 Provides Comprehensive Look at the Job Market

*By Lina Sorg*

As students complete coursework, pursue internships, and work towards their degrees, they must make important decisions about their future careers. Specifically, individuals who are studying applied mathematics, computational science, data science, and other adjacent fields typically have to choose between employment in academia, industry, or the national laboratories.[1] Though these sectors do share some similarities, each one offers its own set of characteristics, advantages, and challenges.

During the 2024 SIAM Annual Meeting[2]—which took place in Spokane, Wash., this July—a panel[3] of applied mathematicians, computer scientists, and engineers at varying stages of professional development shared their insights about career prospects within these three settings. Jeffrey Hittinger of Lawrence Livermore National Laboratory (LLNL) chaired the session, which featured panelists Manuchehr Aminian of California State Polytechnic University, Pomona; Ron Buckmire of Marist College (SIAM's Vice President for Equity, Diversity, and Inclusion); Cosmin Ionita of MathWorks;[4] Shelby Lockhart of AMD;[5] Jason Torchinsky of Sandia National Laboratories; and Carol Woodward of LLNL (SIAM's President-Elect).

[1] https://www.energy.gov/national-laboratories

[2] https://www.siam.org/conferences-events/siam-conferences/an24

[3] https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=80855

**CAREERS IN MATHEMATICAL SCIENCES**

After summarizing their respective career trajectories, the panelists began by reflecting on their time as graduate students. For Torchinsky, graduate school was about finding someone that they wanted to work *with*, rather than a subject that they wanted to work *on*. Torchinsky encouraged students to maintain authority over their pursuits and ensure that the projects of potential advisors align with their own interests and research goals. Lockhart echoed this sentiment. "I wish I had known that it was *my* Ph.D.," she said, admitting that she switched advisors after two years in favor of a project for which she was better suited. "Advocate for yourself."

[4] https://www.mathworks.com

[5] https://www.amd.com

Aminian similarly recalled that upon starting graduate school, he did not yet have a sense of agency over his own career. He urged students to take advantage of travel grants from SIAM[6] and other societies in order to attend events with a greater sense of independence, network with junior and senior researchers, and explore trending topics. Buckmire seconded the importance of these funding opportunities. "There is a fair amount of money out there—generally supported by the National Science Foundation (NSF)—to expose grad students to lots of different experiences and help them make decisions about what paths they want to pursue," he said. "The Mathematical Sciences Institutes[7] are a great example of that."

Woodward's personal experiences speak to the value of internships and summer research programs. Prior to earning her undergraduate degree, she knew that she did not want to become a professor; instead, student positions at Oak Ridge National Laboratory and Pacific Northwest National Laboratory cemented her penchant for the national labs. Ionita was also certain that he wanted to utilize applied mathematics in a nonacademic setting. After defending his thesis, he applied for an available role within MathWorks' core math team. "It was a nice opportunity where I could continue doing math — not just random coding, but programming with a purpose," Ionita said of his now 10-year career at MathWorks.

Next, the panelists detailed the distinctions between the work environments in academia, industry, and national labs. Buckmire explained that the key "phase transition" in academia occurs with the acquisition of tenure, as faculty expend a lot of energy preparing for tenure and constructing their portfolios to meet rigorous standards. "Once you pass that phase transition and you're tenured, it can actually be a pretty difficult situation," he said. "You have this long expanse of being at an institution for as long as you want, and you can decide how to engage and what to engage in. It can take a while to figure out what the next frontiers will be."

[6] https://www.siam.org/conferences-events/conference-support/travel-and-registration-support

[7] https://mathinstitutes.org

Anyone who is seeking a position in academia should be passionate about teaching and mentoring, since these duties comprise a significant portion of an academic career. Additionally, the level of autonomy and flexibility in academia is much greater than in industry or the national laboratories, and there is often no one "right" way to do things. For instance, Buckmire's 30-year stint at Occidental College—he moved to Marist just last month after accepting a position as dean of the School of Computer Science and Mathematics—included a foray into higher education administration and two "tours of duty" as an NSF program director.

While much academic work is solo, the vast majority of industry and national lab projects are collaborative. "Everything we do is in teams," Woodward said. The interdisciplinary nature of labs also means that mathematics research is situated within the broader context of science and engineering, with the ultimate goal of delivering scientific knowledge to society. Woodward advised interested candidates to research each laboratory in advance. "There are different cultures at every lab," she said. "Different topics are emphasized based on the applications that are most active."

Woodward also noted the significance of coding, especially given the national labs' collective emphasis on efficiency and high-performance computing. "Impact happens through code," she said. "The ability to translate a new algorithm into efficient code is really important." Lockhart agreed, adding that coding is just as valuable in industry. She explained that employees must know how to build all types of code, evaluate performance on systems that may or may not yet exist, and connect their work to the organization's objectives. "Industry is driven by profit," Lockhart said. "You have to be able to tie it back to the company's mission, which is making money."

Communication is similarly critical in industry, especially since projects are meant to produce tangible outputs that benefit the customer. Aminian encouraged listeners to always consider the background and

*See* **Career Opportunities** *on page 12*

*A panel of researchers in academia, industry, and the national laboratories shared their respective employment experiences with a rapt audience at the 2024 SIAM Annual Meeting, which took place in Spokane, Wash., this July. From left to right: moderator Jeffrey Hittinger of Lawrence Livermore National Laboratory (LLNL) and panelists Cosmin Ionita of MathWorks; Shelby Lockhart of AMD; Jason Torchinsky of Sandia National Laboratories; Carol Woodward of LLNL; Ron Buckmire of Marist College; and Manuchehr Aminian of California State Polytechnic University, Pomona. SIAM photo.*

## BLIS

*Continued from page 8*

Practitioners can fuse this data movement into packing and microkernel post-processing to achieve near-peak performance. The TBLIS tensor contraction library[5] [3] leverages this fact and will employ a BLIS plugin in its next release.

BLIS 2.0 utilizes user-defined plugins to provide support for these and many other extended linear algebra operations. It leverages the core BLIS framework and its performance portability, built-in threading layer, and other features to help users easily and rapidly instantiate high-performance algorithms.

### Mixing and Matching Precisions

BLIS 2.0 enables BLAS and BLAS-like operations where one or more operands have different precisions (single, double, etc.) or domains (real or complex). For example, $C := \alpha AB + \beta C$ may involve double-precision complex matrices $C$ and $A$ and single-precision real matrix $B$, for which the computation should be performed in double-precision arithmetic. The implementation of mixed-precision/mixed-domain capabilities for all Level 3 operations (except `TRSM`) builds upon the same mechanisms that support plugins. Other planned changes will allow users to register new data types and offer built-in support for low-precision computations, such as bfloat16.

[5] https://github.com/devinamatthews/tblis

The following key insights tame the exponential growth in combinations within the implementation: (i) Users can incorporate conversions between precisions into the packing that already transpires in Goto's algorithm, (ii) post-processing can take place within the microkernel, and (iii) users can pack matrix data into appropriate formats that leverage real-domain computation to support the mixing of domains. This three-pronged approach only requires a number of kernels that scales quadratically with the number of data types. As with user-defined plugins, combining the appropriate components permits a small number of explicit kernels to enable a wide range of functionality without changes to the native BLIS interface, wherein an operand's precision and domain is specified in an object. This flexibility highlights the advantages of APIs beyond the traditional BLAS interface.

### Get Involved With BLIS

A vibrant developer and user community greatly bolsters the continued growth of BLIS. The BLIS GitHub page[6] features BLIS releases, documentation, performance data, and examples of further reading. Submit an issue or a pull request, ask a question, or simply give us a "star." Then, join the more than 200 members of the BLIS Discord server[7] who are eager to discuss BLIS, linear algebra, and scientific computing. Finally, consider attending our upcoming "BLIS Retreat"[8] in late September—an annual event that originated in 2014—for the opportunity to mingle and interact with contributors from academia, industry, and the broader community.

[6] https://github.com/flame/blis

[7] https://github.com/flame/blis/blob/master/docs/Discord.md

[8] https://www.cs.utexas.edu/~flame/BLISRetreat2024/BLISRetreat2024.html

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*Devin Matthews is an associate professor of chemistry at Southern Methodist University who specializes in high-performance computing in the context of molecular structures and spectroscopy. Robert van de Geijn is a faculty member at The University of Texas at Austin who is affiliated with the Department of Computer Science and the Oden Institute for Computational Engineering and Sciences. He is a leading expert in the areas of high-performance computing, linear algebra libraries, and parallel processing. Maggie Myers is a Senior Research Fellow at UT Austin's Oden Institute with extensive experience in the development of educational materials for K-12, college, and graduate-level courses. Devangi Parikh is an assistant professor of instruction in the Department of Computer Science at UT Austin.*

# The Curious History of Vectors and Tensors

BOOK REVIEW
By Ernest Davis

**Vector: A Surprising Story of Space, Time, and Mathematical Transformation.** *By Robyn Arianrhod. The University of Chicago Press, Chicago, IL, May 2024. 376 pages, $28.00.*

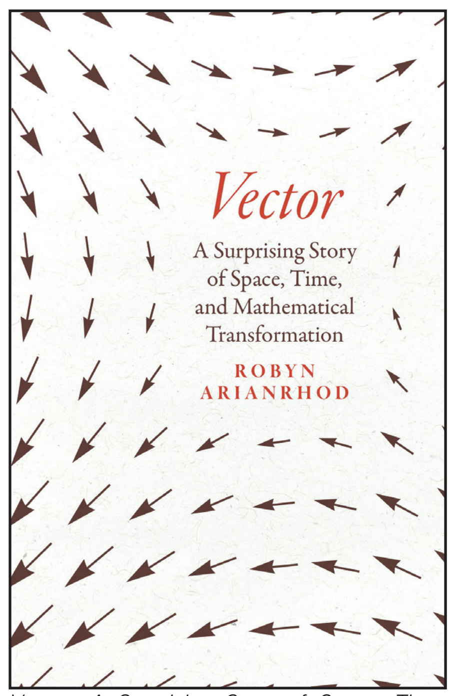
In the history of science and mathematics, a concept that currently seems straightforward and unproblematic often evolved in a way that appears weirdly indirect and convoluted in hindsight. Sometimes such ideas were met with a hostility that now seems incomprehensibly wrongheaded and pointless. One noteworthy example is the emergence of the concept of vectors in the 19th century, which is the subject of the first half of Robyn Arianrhod's new book, *Vector: A Surprising Story of Space, Time, and Mathematical Transformation*.

Arianrhod's account begins with various precursors to vectors, including work by Isaac Newton and Thomas Harriot (a favorite subject for Arianrhod, who authored his biography in 2019) [1]. From our point of view, velocities, accelerations, and forces in Newtonian mechanics are all vectors; if multiple forces act on an object, then the net force is the vector sum of the individual forces. Although Newton and his successors performed these calculations correctly and drew geometric diagrams of parallelograms that we now interpret as showing the vector sum of two forces, they had no concept of a vector as such. Over the ensuing 150 years, many foundational theorems of vector analysis—the divergence theorem, Stokes' theorem, the finite Cauchy-Schwarz inequality, and so on—were proved *avant la lettre*, before the concept of a vector was formulated. Interestingly, the same is true of matrices; a large mathematical literature on both determinants and eigenvalues existed well before the concept of a matrix gradually emerged during the 19th century [2].

Out of necessity, all of these developments were expressed in purely coordinate notation. The idea of a vector as a mathematical object in its own right first appeared as part of William Rowan Hamilton's theory of quaternions. Hamilton spent many years looking for a mathematical object that could express three-dimensional (3D) rotations in the same way that complex numbers express two-dimensional rotations. In 1843, he finally solved the problem by constructing the quaternion: an expression of the form $a+b\mathbf{i}+c\mathbf{j}+d\mathbf{k}$, where $a$, $b$, $c$, and $d$ are real numbers and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are separate square roots of $-1$ that correspond to three orthogonal directions in space. Hamilton demonstrated that quaternions can be added, subtracted, multiplied, and divided; he also showed that a 3D rotation can be represented as a quaternion in such a way that the composition of two rotations corresponds to the product of the quaternions.

Hamilton called component $a$ the "scalar" part of $q$ and the sum $b\mathbf{i}+c\mathbf{j}+d\mathbf{k}$ the "vector" part of $q$. He noted that if $u$ and $v$ are quaternions with scalar part 0, then the two components of the product $w=u\cdot v$ have meaningful properties: The scalar part of $w$ is the negative of what we now call the dot product $\vec{u}\cdot\vec{v}$, and the vector part is what we now call the cross product $\vec{u}\times\vec{v}$.

Arianrhod tells the story of Hamilton's discovery at length and with great gusto. After a chapter that describes the independent, lesser-known development of vectors by Hermann Grassmann, her narrative reaches its central episode: the story of how James Clerk Maxwell—in collaboration with Peter Guthrie Tait—realized that the vector part of Hamilton's quaternions provided exactly the mathematical tool that he needed in his brilliant unification of electromagnetic theory. Maxwell, Tait, and later Oliver Heaviside formulated the algebra and calculus of vectors—including vector fields, the vector differential operator $\vec{\nabla}$, the divergence (Maxwell called it the "convergence," with the opposite sign), the gradient, the curl, and the Laplacian—much like how we think about these concepts today and teach them in every third-semester calculus course.




*Vector: A Surprising Story of Space, Time, and Mathematical Transformation. By Robyn Arianrhod. Courtesy of the University of Chicago Press.*

However, Maxwell's formulation did not meet immediate acceptance. For the next 40 years, a bitter three-sided debate raged about the correct way to think about and represent this type of mathematics and physics. First, there were the vector enthusiasts, notably Maxwell (until his untimely death), Heaviside, William Kingdon Clifford, and Josiah Willard Gibbs. Second, there were those who preferred Hamilton's original quaternions, including Tait and Alexander McAulay. Finally, there were those—especially William Thomson, who would later become Lord Kelvin—who scorned both vectors and quaternions as useless, instead insisting that everything be fully written in terms of their Cartesian components. These key players flung nasty insults back and forth for decades.

The second half of *Vector* details the development of tensor calculus and its central role in Albert Einstein's general theory of relativity. This historical thread is comparatively straightforward. The backstory to tensor theory involves Carl Friedrich Gauss and Bernhard Riemann's creation of the theory of the intrinsic curvature of manifolds, as well as Augustin-Louis Cauchy's study of stress in continuum mechanics. Gregorio Ricci-Curbastro—who was primarily responsible for the actual theory of tensors—worked with Tullio Levi-Civita to compose a classic exposition of the theory in their 1900 textbook, *Méthodes de Calcul Différentiel Absolu et Leurs Applications*.

In 1912, Einstein was looking for an existing mathematical theory to use for his own theory of spacetime and gravity that would accommodate his intuitions. His friend Marcel Grossmann searched the mathematical literature and told Einstein that Riemannian geometry and the theory of tensors might have what he needed. After years of intense study and labor, Einstein finally found success; in 1915, he published his theory of general relativity in the language of tensor analysis.

Arianrhod's book is deeply researched. Her 320 pages of text are supplemented with nearly 60 pages of endnotes that provide additional mathematical and historical detail. She also supplies a timeline, which is an invaluable apparatus for a complex narrative such as this. Her account of the history of the "vector wars" and her capsule biographies of Hamilton, Maxwell, Tait, Grassmann, Ricci-Curbastro, and Levi-Civita are vivid and fascinating. However, Arianrhod's explanation of Einstein's work on general relativity is less satisfying. I instead recommend the relevant chapters of Abraham Pais' classic biography, *Subtle is the Lord: The Science and the Life of Albert Einstein*, which offers a much richer, clearer, and more vivid account of Einstein's thoughts on gravity and the physical intuitions that guided him [3].

Regrettably, *Vector* suffers from a number of flaws that significantly diminish its value — the most important being that the mathematical exposition is often unclear. The content on quaternions and vectors in the first part of the book is tedious for readers who know the material and unenlightening for those who do not, and the sections about tensors and general relativity are frustrating for readers who know the material and impenetrable for those who do not. Of course, these latter subjects are inherently challenging. The explanation of how objects that move in a gravitational field can follow a geodesic through curved Minkowski spacetime—and the expression of this theory via Einstein's tensor equations—is considerably more difficult than most topics in popular math books. However, readers are unlikely to gain a deeper understanding of the mathematics based on *Vector*'s description.

Part of this issue is that Arianrhod's perception of the target reader is extremely inconsistent. At the beginning of the text, she writes that readers may vaguely remember vectors from high school. But by the end of the book, she is writing sentences like, "It turns out that all you have to do to change a Lorentz-covariant equation to a generally covariant one is to replace partial derivatives by invariant derivatives ... this definition means you simply have to change all the commas in the above equations to semicolons." Anyone who wishes to progress from a nebulous grasp of vectors to a detailed understanding of tensor calculus will need to spend serious time and effort with textbooks or lectures on differential geometry, tensor theory, and general relativity. *Vector* will not help much with their studies. Admittedly, this type of inconsistency is somewhat common in popular science and math books, but *Vector* is extreme.

Another flaw, which is less important but still annoying, is Arianrhod's tendency to wander into irrelevant tangents that often pertain to the pros and cons of 21st century technology. During her discussion about Arthur Cayley's study of invariants, she mentions that Cayley lamented the absence of a rail connection between Lincoln and Cambridge in a letter to George Boole from the 1840s. This digression leads to a lengthy riff about technology, climate change, and Henry David Thoreau. Arianrhod later interrupts her description of tensor theory with a long aside about the use of tensors in current computer technology and quantum physics, which is unhelpful to readers who are struggling to understand tensor theory.

Arianrhod very much overstates the importance of quaternions in current mathematics and computer science. While the theory is more widely used in the present day than it was 40 years ago, it is still quite niche; few math departments offer a mathematics course that spends much time on the subject. Many current mathematicians have presumably heard the story of Hamilton carving his multiplication formula into Broom Bridge; they are likewise aware that quaternions are somehow related to 3D rotations and that their space is a flagship example of a skew field. However, I would guess that only a small fraction knows much more about them.

That being said, *Vector* is a valuable contribution to the history of math. Readers who skip the digressions and do not attempt to comprehend the technical materials with which they are unfamiliar will learn a lot about these remarkable and important episodes in 19th-century mathematics.

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## SIAM's Virtual Resume-building Workshop

SIAM's upcoming virtual resume-building workshop on **October 3rd** is the perfect opportunity for postdoctoral researchers and junior scientists to create strong job application portfolios and prepare for the current job market.

Each participant will sign up for a time slot and be matched with a mentor who will provide personalized feedback and one-on-one coaching to strengthen resumes, CVs, and cover letters. After completing the workshop, attendees will be ready to showcase their portfolios at recruitment events — including the **Fall 2024 Career Fair on October 23rd** at the 2024 SIAM Conference on Mathematics of Data Science in Atlanta, Ga.

Registration is required for the resume-building workshop. Monitor SIAM Engage or the *SIAM News* announcement page[1] for more information.

[1] https://www.siam.org/publications/siam-news/articles/announcing-siams-virtual-resume-building-workshop

# SIAM-IMA Student Chapters Hold Applied Mathematics Postgraduate Conference at the University of Warwick

*By Phurinut Sriawad and Rachel Seibel*

In early July, the SIAM-IMA Student Chapters at the University of Warwick[1] and the University of Oxford[2] collaborated to jointly host the 2024 Applied Mathematics Postgraduate (AMP) Conference[3] at the University of Warwick. The three-day event served as an engaging platform for early-career mathematicians to showcase their research and hone their communication skills. It attracted more than 50 postgraduate students with diverse backgrounds from over 20 distinct universities.

The theme of this year's AMP Conference was "Effective Research Communication," and the meeting focused on the art of conveying mathematical material to different audiences — namely policymakers, industry, and the public. The conference showcased a wide array of topics—including mathematical biology, stochastic analysis, network analysis, fluid dynamics, and machine learning—all of which illustrated the extensive scope of applied mathematics research in the U.K. and beyond. Participants presented their work in a supportive, inclusive environment via a series of contributed talks and a lively poster session.

The first day of the event featured a workshop that was led by Kat Phillips, a postgraduate student at the University of Bath and an incoming postdoctoral researcher at the University of Warwick. The workshop equipped attendees with the necessary tools to communicate their technical research to non-specialist audiences — a crucial skill for engaging with the general public, decision-makers, and industry professionals. This type of dialogue directly influences society by broadening the impact of mathematical research beyond academic circles.

On the second and third day of festivities, Thomas Finnie of the U.K. Health Security Agency[4] and Michael Pearce of Neochromosome[5] delivered insightful presentations about communication efforts with policymakers and industry representatives. Finnie spoke about "Making Maths Matter: How Modelling Can Shape Action for Better Public Health Outcomes," while Pearce answered the question of "With No Biology Degree or Wet Lab Experience, What Does a Mathematician Really Do in a Bioinformatics Team?" These inspiring talks underscored the importance of clear, impactful discourse to advance both scientific understanding and practical applications.

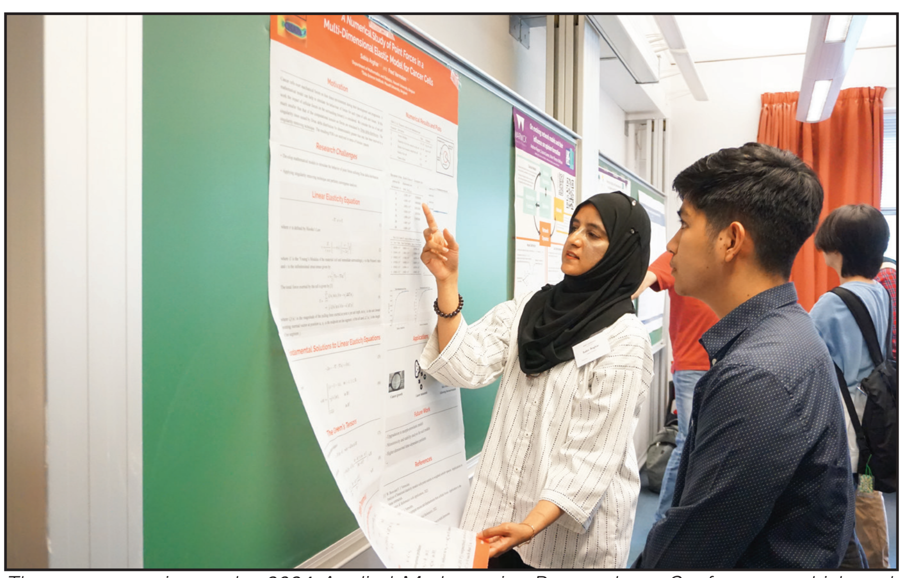
A key success of the AMP Conference is the commitment between two SIAM student chapter committees to provide an affordable, accessible opportunity for postgraduate students. Thanks to funding from the London Mathematical Society's[6] Postgraduate Research Conference Grants scheme[7] and support from the University of Warwick's Mathematics Research Centre,[8] attendees were not charged a registration fee and received complimentary lodging and meals. This generous accommodation offered an invaluable opportunity for those who otherwise might not have been able to attend.

By assembling a diverse group of postgraduate students and providing a platform for the exchange of knowledge and ideas with seasoned experts, the 2024 AMP Conference emphasized the value of collaborative learning and professional growth in applied mathematics and other related fields. We look forward to future events from the SIAM-IMA Student Chapters at the University of Warwick and the University of Oxford and hope to continue the conversation.

*Phurinut Sriawad and Rachel Seibel are Ph.D. students in the Mathematics for Real-World Systems Centre for Doctoral Training at the University of Warwick. Sriawad is the social media officer for the University of Warwick SIAM-IMA Student Chapter, and Seibel is vice president of the University of Warwick SIAM-IMA Student Chapter.*

[1] https://warwick.ac.uk/fac/sci/maths/currentstudents/currentpostgraduates/siamstudentchapter
[2] https://people.maths.ox.ac.uk/siamsc/index.html
[3] https://warwick.ac.uk/fac/sci/maths/research/events/2023-2024/amp24
[4] https://www.gov.uk/government/organisations/uk-health-security-agency
[5] https://neochromosome.com
[6] https://www.lms.ac.uk
[7] https://www.lms.ac.uk/grants/postgraduate-research-conference-grantsscheme-8
[8] https://warwick.ac.uk/fac/sci/maths/research/mrc

*Attendees of the 2024 Applied Mathematics Postgraduate Conference—which was hosted at the University of Warwick this July by the SIAM-IMA Student Chapters of the University of Warwick and the University of Oxford—gather for a group photo. Image courtesy of Yi Ting Loo.*



*The poster session at the 2024 Applied Mathematics Postgraduate Conference, which took place this July at the University of Warwick, provided an opportunity for early-careers researchers to showcase their work and connect with colleagues. Image courtesy of Yi Ting Loo.*

## Career Opportunities

*Continued from page 10*

perspectives of the people with whom they are communicating, as many exchanges will involve customers or recipients who may be literate in science but not in applied mathematics specifically. Ionita affirmed the value of this advice. "At MathWorks, you have to encapsulate deep technical concepts into something that's easy to use for people who don't necessarily know what they're using," he said. "You need to eliminate as many parameters as possible and still make it work for a ton of people."

Conversation then turned to strategies for self-promotion when applying for positions as a junior scientist. Lockhart reminded the audience that all types of experiences—even unofficial side projects—are worthy of mention because they demonstrate initiative, a willingness to learn, and the ability to master new concepts. "If you didn't do your thesis on optimization but have done some software optimization, make sure that it's visible so people know what you know," she said. "In industry, self-learning is just as credited as traditional learning." Aminian suggested that early-career researchers maintain an active presence on GitHub, list previous projects on their personal websites, demonstrate proficiency in Linux-like environments, and use their code to generate figures for their portfolios. Ionita likewise encouraged job seekers to readily share their code, despite any imperfections. "People won't have the expectation of grad school code being bulletproof," he said. "It's more important to get it out there and show it."

Torchinsky advised anyone who is looking for an internship, postdoctoral placement, or permanent position to promote themselves whenever possible. While in the market for employment as a postdoc, they always ended their conference presentations with slides that advertised their availability. As a result, prospective employers often approached Torchinsky after the talk to ask for their CV or request an interview, since they had just seen a concrete example of their work in action. "There are individuals in this room who are looking to hire people," Torchinsky said, noting that SIAM conferences[8] are great places to make connections.

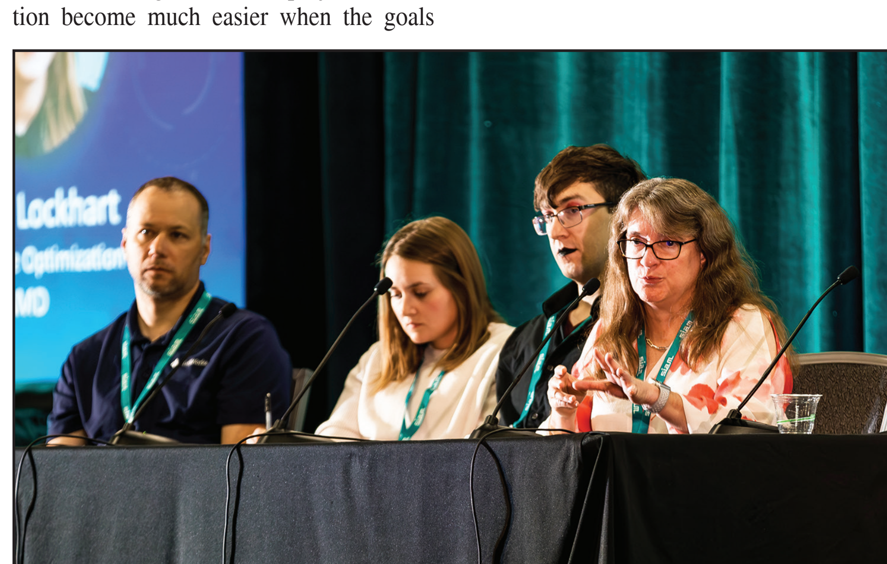
When asked how to best prepare for management and leadership positions, Woodward stated that LLNL—like many other organizations—had provided leadership training in the form of classes. One lesson in particular stuck with her: the idea that everyone is a leader in some capacity, regardless of their title. "We all have autonomy over our careers and organizations to some degree," Woodward said. "I took a lot of that to heart." It is also important to acknowledge the existence of learning curve for all new responsibilities. Whenever Woodward accepts a new leadership role, she connects with experienced individuals who can mentor her in the duties of that position. As SIAM's current President-Elect, she has been talking to past SIAM presidents about difficulties that may arise and how to best address them. "Every leadership role has challenges that require consideration of issues at a level I wasn't thinking before," she said. "It doesn't mean that I *can't* think about it, it just means that I haven't been exposed to what those issues are."

The panel session concluded with a brief discussion about effective time management when balancing multiple projects. Lockhart maintains a personal priority queue, rotates through deadline-driven items at the top of her list, and designates certain time blocks for specific responsibilities. Buckmire affirmed the benefits of this strategy but urged people in team-oriented settings to also pay attention to the bandwidth of their collaborators in order to fairly distribute the workload and prioritize tasks based on availability.

Time management and project allocation become much easier when the goals of researchers and their employers closely align. Woodward admitted that while she has always struggled somewhat to prioritize simultaneous tasks, her positive relationship with LLNL simplifies the process. "You should be picking the things more often that are most important to the organization because that's what's going to keep you in the job," she said. "Every day I come in and have to make choices. But I love the national lab environment and I get to work on really challenging problems on which I feel that I can make some progress."

*Lina Sorg is the managing editor of SIAM News.*

[8] https://www.siam.org/conferences-events




*During the 2024 SIAM Annual Meeting, which was held this July in Spokane, Wash., a panel of researchers at various stages of their careers discussed their employment journeys in academia, industry, and the national laboratories. From left to right: panelists Cosmin Ionita of MathWorks, Shelby Lockhart of AMD, Jason Torchinsky of Sandia National Laboratories, and Carol Woodward of Lawrence Livermore National Laboratory. SIAM photo.*

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# SIAM | PROGRAMS AND AWARDS

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## Nominate a Colleague for 2025 Major Awards

***siam.org/deadline-calendar***

The SIAM Prize Program provides an opportunity to honor members of our community — from lifetime contributions to student and early career recognition. Being nominated recognizes and rewards outstanding accomplishments, brings honor and prestige to your place of work, and demonstrates the importance of your field to students, research funders, and the scientific community at large.

**2025 Major Awards**

- AWM-SIAM Sonia Kovalevsky Lecture*
- George Pólya Prize in Mathematical Exposition
- Germund Dahlquist Prize
- Jerald L. Ericksen Prize (first time being awarded)
- Ralph E. Kleinman Prize
- SIAM Industry Prize (first time being awarded)
- W. T. & Idalia Reid Prize

**The deadline for nominations is October 15, 2024.**
**Submit your nominations at *go.siam.org/prizes-nominate*.**

*Open dates and deadlines may vary. Contact *prizeadmin@siam.org* with questions.

## Applications are Being Accepted for the 2025 Class of MGB-SIAM Early Career Fellows

***siam.org/msec-fellowship***

The MGB-SIAM Early Career Fellowship aims to recognize the achievements of early-career applied mathematicians and provide support for professional activities and career development — particularly for those who belong to racial and ethnic groups that are historically excluded from the mathematical sciences in the United States.

SIAM encourages all qualified individuals to apply. The MGB-SIAM Early Career Fellowship is a joint program of Mathematically Gifted & Black (MGB) and SIAM.

**Fellows receive:**

- Complimentary SIAM membership for the duration of the fellowship
- Complimentary registration to SIAM Annual Meetings and one additional SIAM Activity Group conference
- Travel support to attend SIAM conferences
- Mentoring and professional development opportunities
- Leadership and conference organization experience

**The deadline to apply is November 1, 2024.**

## SIAM Postdoctoral Support Program Seeks Applicants

***siam.org/postdoctoral-support***

The SIAM Postdoctoral Support Program provides financial support for postdoctoral scholars to receive mentoring and collaboration opportunities that enable successful career advancement. Up to $15,000 in financial support is provided for postdoctoral researchers to work with a mentor from a different institution. The goal is to foster direct research experience and professional development. Up to four postdoc/mentor pairs are selected annually. **Applicant teams design and describe a mentoring, research, and collaboration plan that works best for them.**

The application will remain open until annual funding is expended, but the **priority deadline for applications is November 1, 2024.**

## Apply Now for the SIAM Science Policy Fellowship Program

***siam.org/science-policy-fellowship***

The SIAM Science Policy Fellowship Program develops postdoctoral fellows and early career researchers into strong advocates for U.S. federal support in applied mathematics and computational science. The program enables participants to gain in-depth knowledge of the policy processes that determine science funding and policy decisions while still pursuing their research and teaching.

The SIAM Science Policy Fellowship Program is open to SIAM members currently working and living in the United States. SIAM encourages all qualified individuals to apply.

**Three to five fellowship recipients will be selected each year to serve two-year terms that include:**

- In-person and remote training
- Attending biannual SIAM Committee on Science Policy meetings
- Interfacing with federal officials, congressional staff, and policy-makers
- Participating in an advocacy day on Capitol Hill in Washington, D.C.

**The deadline to apply is November 1, 2024.**

## Nominate a Colleague for 2025 SIAM Activity Group Prizes

***siam.org/deadline-calendar***

SIAM is accepting nominations for activity group prizes that will be awarded at 2025 conferences. These prizes identify outstanding contributions to specific fields by recognizing published papers and individuals in various stages of their careers.

Every nomination counts! Visit the website for full eligibility requirements and required materials.

**2025 SIAM Activity Group Prizes**

- J. D. Crawford Prize
- Jürgen Moser Lecture
- SIAG/ACDA Early Career Prize
- SIAG/AG Agnes Szanto Early Career Prize
- SIAG/APDE Best Paper Prize
- SIAG/APDE Early Career Prize
- SIAG/CST Best SICON Paper Prize
- SIAG/CST Prize
- SIAG/FME Conference Paper Prize*
- SIAG/FME Early Career Prize
- SIAG/GD Early Career Prize
- SIAG/GS Career Prize
- SIAG/GS Early Career Prize

**The deadline for nominations is October 15, 2024.**
**Submit your nominations at *go.siam.org/prizes-nominate*.**

*Open dates and deadlines may vary. Contact *prizeadmin@siam.org* with questions.

siam Fellow

**Nominations Close October 16 for the SIAM Fellows Class of 2025! Make your nomination at *siam.org/nominate-fellows*.**

FOR MORE INFORMATION: ***siam.org/programs*** AND ***siam.org/prizes-recognition***

# SIAM | MEMBERSHIP

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# Renew your SIAM membership for 2025 and remain a part of our international and interdisciplinary community. Go to *my.siam.org* to renew today.

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- SIAM Presents, a collection of talks and tutorials from SIAM conferences and virtual webinars
- Science policy electronic mailing list — keeps you informed on policy and funding opportunities

### ADVANCE YOUR CAREER

- Access to career information through the SIAM career webpages, SIAM Engage community site, job board, and conferences
- Recognition through prizes, awards, and the SIAM Fellows Program
- The Postdoctoral Support Program helps recent Ph.D.s advance their research agenda by funding travel to work with a new mentor


- Career Fairs (in-person and virtual) connect you to recruiters who are looking for applied math, computational, and data science skills
- Careers in the Mathematical Sciences in *SIAM News*, an ongoing series of career-related articles by members of the SIAM community
- Discounted memberships for those early in their careers

### NETWORKING OPPORTUNITIES

- 23 SIAM Activity Groups provide opportunities to network with professionals with similar research interests and enhance your visibility within these specialized communities
- Conferences connect you with other professionals who share similar research interests, with discounted registration for members
- SIAM Section activities allow you to meet peers in your geographic region
- Committees, activity groups, and chapters provide volunteer and leadership opportunities

**Nominate two students for free membership!**
*siam.org/Forms/Nominate-a-Student*

### SIAM JOURNALS AND BOOKS

- Free subscription to print and electronic versions of *SIAM Review*, SIAM's flagship journal that contains articles of broad interest to the community
- 18 peer-reviewed journals as well as an online journal archive (Locus), both available to members at a discounted rate
- Access to quality research via print or electronic format with 500+ books at member discounts of at least 30%
- Price break for students on books adopted by a SIAM member for classroom use

### RESOURCES FOR STUDENTS

- $250,000 in funding for SIAM Student Travel Awards
- $45,000+ in funding for SIAM Student Chapters
- Free student memberships for those who attend a member institution, are in a SIAM Student Chapter, or are nominated by a SIAM member
- Discounts on membership and conference registrations
- SIAM-Simons Undergraduate Summer Research Program, providing mentored research for 10 students per year
- Graduate Student Math Modeling Camp for graduate students and advanced undergraduates, feeding into Mathematical Problems in Industry Workshop, where teams of students and faculty work on real-world problems in collaboration with industry partners
- Gene Golub SIAM Summer School (G2S3), a free graduate-level workshop
- Student Days events at the SIAM Annual Meeting designed to foster community and support student attendees
- *SIAM Undergraduate Research Online* (SIURO), a publication showcasing student research
- Visiting Lecturer Program (VLP), a roster of experienced and inspirational applied mathematicians and computational scientists working in industry, government, and academia, available to speak on topics that are of interest to developing professional mathematicians

### PUBLIC AWARENESS

- SIAM promotes understanding of the value of mathematics both in daily life and in the advanced sciences
- SIAM speaks on behalf of its members to key congressional representatives and organizations in Washington, D.C., to promote important research funding and the development of science policy
- Community outreach programs—including MathWorks Math Modeling Challenge, a high school math modeling competition organized by SIAM—advance the application of mathematics and computational science

## What's New at SIAM?

- The first full in-person SIAM Conference on Mathematics of Data Science (MDS24) will be held October 21–25, 2024 in Atlanta, Georgia
- Individuals wanting to transition their career in AI and learn how AI can benefit companies and pioneer advancements in the field can take SIAM's course, "From Machine Learning to Large Language Models—An Introduction," being held in-person October 20, 2024 in Atlanta, Georgia, just prior to MDS24
- SIAM Student Days took place at AN24 in Spokane, Washington. With 46 chapters sending representatives from around the world, chapter leaders networked, attended student-oriented sessions, and got to meet SIAM VIPs!
- Five new student chapters have been established so far in 2024. Consider starting or re-starting a chapter at your academic institution!
- The new Northern and Central California Section of SIAM will hold its inaugural meeting October 9–11, 2024 at University of California, Merced, California.
- This year's career development opportunities will include a career fair happening in-person at the SIAM Conference on Mathematics of Data Science (MDS24) on Wednesday, October 23, 2024 in Atlanta, Georgia
- The SIAM Activity Group on Equity, Diversity, and Inclusion will hold its first track July 28–August 1, at the 2025 SIAM Annual Meeting in Montreal, Quebec

## Take Advantage of Special Dues Rates

If you are a student, an early career member, unemployed, retired, or a member of a mathematical society with which SIAM has a reciprocity agreement, you qualify for reduced membership rates. Go to *siam.org/membership/individual* or contact customer service.

## Renew Your SIAM Membership at *my.siam.org*

Consider checking the "auto-renew" box to have your membership renew automatically at the end of 2024.

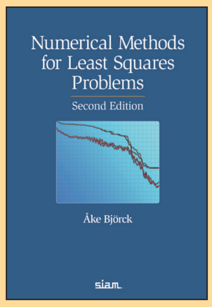
FOR MORE INFORMATION ON SIAM MEMBERSHIP: *siam.org/membership*

# SIAM | BOOKS

Quality and Value in Mathematical Science Literature

# New

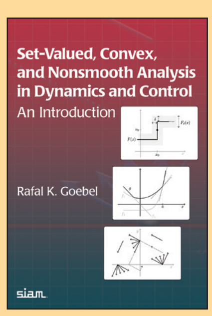
## Numerical Methods for Least Squares Problems
### *Second Edition*
Åke Björck




The first edition was the leading reference on the topic for many years. The updated second edition provides an in-depth and up-to-date treatment of direct and iterative methods for solving different types of least squares problems and for computing the singular value decomposition. It covers generalized, constrained, and nonlinear least squares problems as well as partial least squares and regularization methods for discrete ill-posed problems. The bibliography of over 1,100 historical and recent references provides a comprehensive survey of past and present research in the field.

*2024 / xiv + 494 pages / Softcover / 978-1-61197-794-3 List $89.00 / SIAM Member $62.30 / OT196*

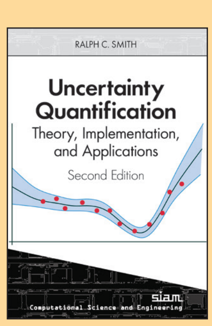
## Set-Valued, Convex, and Nonsmooth Analysis in Dynamics and Control
### *An Introduction*
Rafal K. Goebel




Set-valued analysis, convex analysis, and nonsmooth analysis are relatively modern branches of mathematical analysis that have become increasingly relevant in current control theory and control engineering literature. This book serves as a broad introduction to analytical tools in these fields and to their applications in dynamical and control systems and is the first to cover these topics with this scope and at this level. Both continuous-time and discrete-time mutlivalued dynamics, modeled by differential and difference inclusions, are considered.

*2024 / xiv + 221 pages / Softcover / 978-1-61197-797-4 List $72.00 / SIAM Member $50.40 / OT197*

## Uncertainty Quantification
### *Theory, Implementation, and Applications, Second Edition*
Ralph C. Smith

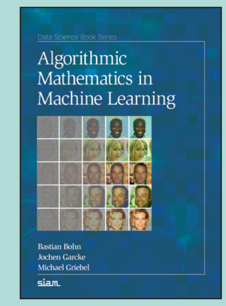



Uncertainty quantification serves a fundamental role when establishing the predictive capabilities of simulation models. This book provides a comprehensive and unified treatment of the mathematical, statistical, and computational theory and methods employed to quantify uncertainties associated with models from a wide range of applications.

*2024 / xxiv + 541 pages / Hardcover / 978-1-61197-783-7 List $89.00 / SIAM Member $62.30 / CS30*

# More Great SIAM Titles

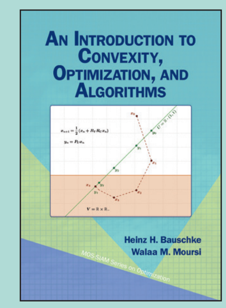
## Algorithmic Mathematics in Machine Learning
Bastian Bohn, Jochen Garcke, and Michael Griebel




This unique book explores several well-known machine learning and data analysis algorithms from a mathematical and programming perspective. The authors present machine learning methods, review the underlying mathematics, and provide programming exercises to deepen the reader's understanding. They provide new terminology and background information on mathematical concepts, as well as exercises, in "info-boxes" throughout the text. Application areas are accompanied by exercises that explore the unique characteristics of real-world data sets.

*2024 / xii + 225 pages / Softcover / 978-1-61197-787-5 / List $64.00 / SIAM Member $44.80 / DI03*

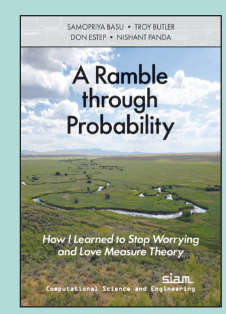
## An Introduction to Convexity, Optimization, and Algorithms
Heinz H. Bauschke and Walaa M. Moursi




This concise, self-contained volume introduces convex analysis and optimization algorithms, with an emphasis on bridging the two areas. It explores cutting-edge algorithms that have applications in machine learning, signal processing, image reconstruction, and other fields. It contains algorithms illustrated by Julia examples and more than 200 exercises. Clear explanations and step-by-step algorithmic descriptions facilitate self-study for individuals looking to enhance their expertise in convex analysis and optimization.

*2024 / xii + 186 pages / Softcover / 978-1-61197-779-0 / List $60.00 / SIAM Member $42.00 / MO34*

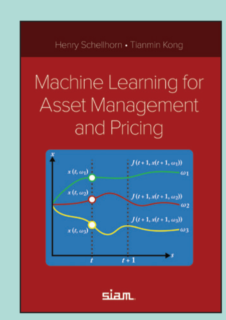
## A Ramble through Probability
### *How I Learned to Stop Worrying and Love Measure Theory*
Samopriya Basu, Troy Butler, Don Estep, and Nishant Panda




Measure theory and measure-theoretic probability are fascinating subjects. Proofs describing profound ways to reason lead to results that are frequently startling, beautiful, and useful. Measure theory and probability also play roles in the development of pure and applied mathematics, statistics, engineering, physics, and finance. This book traces an eclectic path through the fundamentals of the topic to make the material accessible to a broad range of students. It brings together the key elements and applications in a unified presentation aimed at developing intuition; contains an extensive collection of examples that illustrate, explain, and apply the theories; and is supplemented with videos containing commentary and explanations of select proofs on an ancillary website.

*2024 / xvi + 611 pages / Softcover / 978-1-61197-781-3/ List $94.00 / SIAM Member $64.80 / CS29*

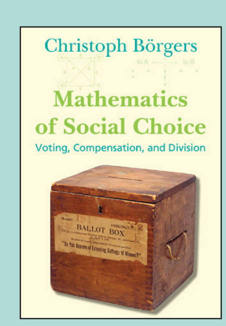
## Machine Learning for Asset Management and Pricing
Henry Schellhorn and Tianmin Kong




This textbook covers the latest advances in machine-learning methods for asset management and asset pricing. Recent research in deep learning applied to finance shows that some of the techniques used by asset managers (usually kept confidential) result in better investments than the more standard techniques. Cutting-edge material is integrated with mainstream finance theory and statistical methods to provide a coherent narrative. Coverage includes an original machine learning method for strategic asset allocation; the no-arbitrage theory applied to a wide portfolio of assets as well as other asset management methods; and neural networks and other advanced techniques.

*2024 / xxiv + 242 pages / Softcover / 978-1-61197-789-9 / List $74.00 / SIAM Member $51.80 / OT195*

## Mathematics of Social Choice
### *Voting, Compensation, and Division*
Christoph Börgers




How do you select a winner from a field of candidates? How do you rank a field of candidates? How do you share a divisible resource like a cake, or an indivisible one like a pet or a house? These are the questions addressed in this fun and accessible book that takes an entertaining look at the choices made by groups of people with different preferences, needs, and interests. Divided into three parts, the text first examines voting methods for selecting or ranking candidates. A brief second part addresses compensation problems wherein an indivisible item must be assigned to one of several people who are equally entitled to ownership of the item, with monetary compensation paid to the others. The third part discusses the problem of sharing a divisible resource among several people.

*2010 / xi + 222 pages / Softcover / 978-0-898716-95-5 / List $47.00 / SIAM Member $32.90 / OT119*

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# SIAM | CONFERENCES

A Place to Network and Exchange Ideas

## Upcoming Deadlines

### SIAM Conference on Mathematics of Data Science (MDS24)

**October 21–25, 2024 | Atlanta, Georgia, U.S.**
*go.siam.org/mds24* | #SIAMMDS24

**ORGANIZING COMMITTEE CO-CHAIRS**
Eric Chi, *Rice University, U.S.*
David Gleich, *Purdue University, U.S.*
Rachel Ward, *University of Texas at Austin, U.S.*

**EARLY REGISTRATION RATE DEADLINE**
September 18, 2024

**HOTEL RESERVATION DEADLINE**
September 18, 2024

### SIAM Conference on Computational Science and Engineering (CSE25)

**March 3–7, 2025 | Fort Worth, Texas, U.S.**
*go.siam.org/cse25* | #SIAMCSE25

**ORGANIZING COMMITTEE CO-CHAIRS**
David Bindel, *Cornell University, U.S.*
Elizabeth Cherry, *Georgia Institute of Technology, U.S.*
Judith Hill, *Lawrence Livermore National Laboratory, U.S.*

**SUBMISSION DEADLINE**
September 3, 2024: Contributed Lecture, Poster, Miniposterium and Minisymposium Presentation Abstracts

### SIAM International Conference on Data Mining (SDM25)

**May 1–3, 2025 | Alexandria, Virginia, U.S.**
*go.siam.org/sdm25* | #SIAMSDM25

**GENERAL CO-CHAIRS**
Matteo Riondato, *Amherst College, U.S.*
Vagelis Papalexakis, *University of California, Riverside, U.S.*

**SUBMISSION DEADLINES**
September 27, 2024: Abstract Submission
October 4, 2024: Paper & Blue Sky Idea Submission
October 11, 2024: Workshop Proposals
October 11, 2024: Tutorial Proposals

### SIAM Conference on Applications of Dynamical Systems (DS25)

**May 11–15, 2025 | Denver, Colorado, U.S.**
*go.siam.org/ds25* | #SIAMDS25

**ORGANIZING COMMITTEE CO-CHAIRS**
Ryan Goh, *Boston University, U.S.*
Alexandria Volkening, *Purdue University, U.S.*

**SUBMISSION DEADLINES**
October 14, 2024: Minisymposium Proposal Submissions
November 11, 2024: Contributed Lecture, Poster, and Minisymposium Presentation Abstracts

### SIAM Conference on Applied Algebraic Geometry (AG25)

**July 7–11, 2025 | Madison, Wisconsin, U.S.**
*go.siam.org/ag25* | #SIAMAG25

**ORGANIZING COMMITTEE CO-CHAIRS**
Carina Curto, *Brown University, U.S.*
Sonja Petrovic, *Illinois Institute of Technology, U.S.*

**SUBMISSION DEADLINES**
December 6, 2024: Minisymposium Proposal Submissions
January 6, 2025: Contributed Lecture, Poster, and Minisymposium Presentation Abstracts

Information is current as of August 8, 2024. Visit *siam.org/conferences* for the most up-to-date information.

### Students and Early Career Researchers: Apply Now for Conference Support

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## Upcoming SIAM Events

**GLSIAM Meeting 2024**
October 12, 2024
Hammond, Indiana, U.S.

**SIAM Conference on Mathematics of Data Science**
October 21–25, 2024
Atlanta, Georgia, U.S.
Sponsored by the SIAM Activity Group on Data Science

**Bulgarian Section of SIAM Annual Meeting 2024**
December 9–11, 2024
Sofia, Bulgaria

**ACM-SIAM Symposium on Discrete Algorithms**
January 12–15, 2025
New Orleans, Louisiana, U.S.
Sponsored by the SIAM Activity Group on Discrete Mathematics and the ACM Special Interest Group on Algorithms and Computation Theory

**SIAM Symposium on Algorithm Engineering and Experiments**
January 12–13, 2025
New Orleans, Louisiana, U.S.

**SIAM Symposium on Simplicity in Algorithms**
January 13–14, 2025
New Orleans, Louisiana, U.S.

**SIAM Conference on Computational Science and Engineering**
March 3–7, 2025
Fort Worth, Texas, U.S.
Sponsored by the SIAM Activity Group on Computational Science and Engineering

**SIAM International Conference on Data Mining**
May 1–3, 2025
Alexandria, Virginia, U.S.
Sponsored by the SIAM Activity Group on Data Science

**SIAM Conference on Applications of Dynamical Systems**
May 11–15, 2025
Denver, Colorado, U.S.
Sponsored by the SIAM Activity Group on Dynamical Systems

**SIAM Conference on Applied Algebraic Geometry**
July 7–11. 2025
Madison, Wisconsin, U.S.
Sponsored by the SIAM Activity Group on Algebraic Geometry

**The Third Joint SIAM/CAIMS Annual Meetings**
July 28–August 1, 2025
Montreal, Quebec, Canada

**SIAM Conference on Control and Its Applications**
July 28–30. 2025
Montreal, Quebec, Canada
Sponsored by the SIAM Activity Group on Control and Systems Theory

**SIAM Conference on Computational Geometric Design**
July 28–30, 2025
Montreal, Quebec, Canada
Sponsored by the SIAM Activity Group on Geometric Design

**SIAM Conference on Applied and Computational Discrete Algorithms**
July 30–August 1, 2025
Montreal, Quebec, Canada
Sponsored by the SIAM Activity Group on Applied & Computational Discrete Algorithms

FOR MORE INFORMATION ON SIAM CONFERENCES: *siam.org/conferences*