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An Algebraic Geometry Perspective on Topological Data Analysis

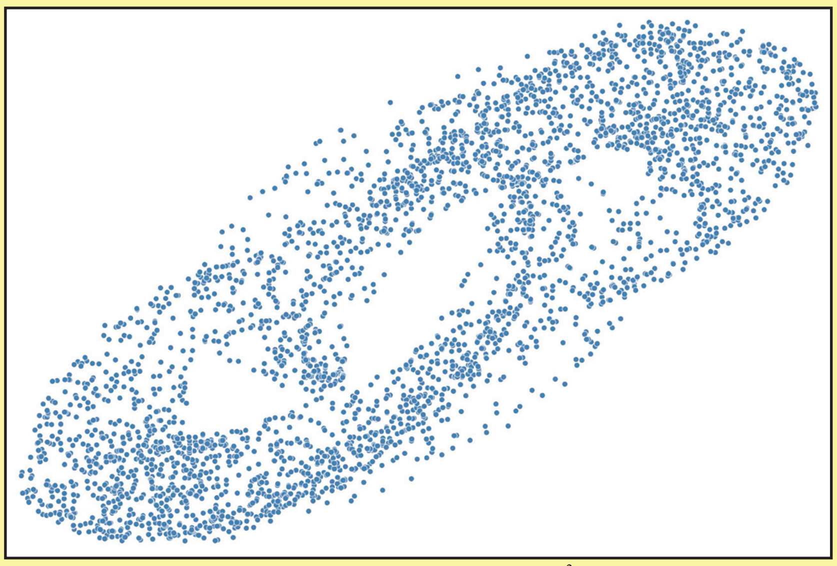

Figure 1. *A sample from the cyclooctane variety for $c^2 = 2$, projected on a two-dimensional space. Due to translational and rotational invariance, $x_1 = (0,0,0)$ and $x_2 = (c,0,0)$ are fixed and the last entry of x_3 is set equal to zero. Code and image courtesy of Paul Breiding and Sascha Timme.*

In an article on page 5, Paul Breiding offers an algebraic geometry perspective on topological data analysis (TDA). While algebra and algebraic geometry present obvious TDA applications, Breiding discusses some additional functions of algebraic geometry in TDA, such as applications of numerical algebraic geometry and enumerative algebraic geometry.

Topological Data Analysis of Collective Motion

By Henry Adams, Maria-Veronica Ciocanel, Chad M. Topaz, and Lori Ziegelmeier

Can you recover a shape via a finite number of points sampled from it? How do you numerically find periodic orbits in a complex dynamical system? Is it possible to determine whether a collection of environmental sensors not equipped with GPS covers a specified region? These types of questions inspired the birth of applied and computational topology in the early 2000s [3, 5, 6].

Topology studies shape and space, with particular focus on properties of objects that are left unchanged by transformations such as stretching, shrinking, bending, and warping. For much of the 20th century, it lived squarely in the realm of pure mathematics. Typical advancements involved building up fundamental tools, classifying topological spaces, and studying manifolds and the behavior of functions on them. However, in recent years topology has become increasingly quantitative, computable, and statistical. Now crystalized as a field of its own, *topological data analysis* (TDA) describes rich classes of topological spaces that have multiple scales of resolution, are randomly generated, or vary in time. TDA continues to grow into new application domains like mathematical biology, materials science, signal processing, computer vision, computational linguistics, and economics.

Persistent Homology

Persistent homology is the workhorse of TDA. *Homology* relates to partially classifying the topology of objects and potentially determining the topological equivalence of two objects. Calculating an object's *Betti numbers* can provide a great deal of homological information. The Betti number b_k calculates the number of holes with a k-dimensional boundary: b_0 counts the number of connected components, b_1 counts the number of flat holes, b_2 counts the number of trapped volumes, and so on.

To understand the *persistent* part of persistent homology, we make a connection to data (see Figure 1, on page 4). Suppose we have N data points in $\mathbb{R}^m$. Furthermore, suppose N is large enough that it becomes necessary to summarize the

See **Topological Data Analysis** *on page 4*

Quantifying "Political Islands" with Persistent Homology

By Michelle Feng and Mason A. Porter

Recent political discourse in the U.S. has made much ado about a growing demographic and political divide between urban and rural communities. Moreover, cities and metropolitan areas have long been described as "islands of blue in a sea of red," prompting lively discussions of the potential implications of such political geography. Does something about urban living cause people to shift their political views? Are Democrats self-segregating into cities? Or is there another explanation? Additional concerns arise, as one may wonder whether the concentration of blue (Democratic) voters in dense urban areas makes it harder to avoid gerrymandering. How can we locate these political "islands" to better study them and identify trends in their formation?

These are just a few of the questions about America's political gap that pervade mainstream discussion. U.S. politics has become increasingly polarized[1] over the last several years, impacting policy debate and electoral behavior and in turn leading to partisan gridlock.

We use tools from topological data analysis (TDA) to examine the problem of large-scale identification of political islands. To explore this subject, we have proposed several methods to examine TDA of geospatial data and applied them to 2016 precinct-level election data[2] from the state of California [1].

When searching for political islands, we can imagine that we are looking for gaps in a "sea" of regions with similar electoral preferences. *Homology*, a tool from algebraic topology that characterizes topological spaces based on their "holes," is well-suited to finding these types of gaps [4]. A technique known as *persistent homology* (PH) enables us to locate these holes in data across a variety of scales. This is useful because of variations in physical size between urban and rural precincts. It also allows us to quantify the strength of the differences in opinion between precincts.

To apply PH [5], we need to transform our data into a suitable topological space. In our case, this space takes the form of *simplicial complexes* [4], which use simplices as building blocks and permit computational tractability for the study of PH. In our recent work, we developed two methods of doing so that yield different interpretations with respect to our original quest to pinpoint political islands [1].

We first introduced an adjacency method that utilizes the network of electoral precincts as its basic structure. Each precinct in this network is a node, with voting information attached to it; each network adjacency is an edge. We focused on the 2016 U.S. presidential election and considered the preference of voters for presidential candidates Hillary Clinton and Donald Trump. An edge exists between two nodes if the precincts that are associated with those nodes share a boundary. If three nodes are connected by all possible pairwise edges, we add a 2-simplex to the simplicial complex. Figure 1 illustrates this process. To exploit the power of PH, we construct a sequence of these simplicial complexes called a "filtered simplicial complex." We begin only with those precincts that possess the largest percentage of votes for Trump, and we continue to add precincts in order of decreasing preference. We then compute the PH of the entire sequence to track holes as they appear and disappear. More explicitly, we record the strength of preference at which a hole first forms, as well as the voting percentages at which we fill in all missing precincts. This allows us to determine the polarization strength between a voting island and its surrounding precincts. Holes that last longer indicate stronger polarization than those that rapidly appear and disappear.

Our second technique, based on level sets, tracks a voting island's physical size and uses a county map as its basic structure. For example, consider a map of all precincts in a county that voted for Trump. We triangulate this map by projecting it onto a triangular grid to form a simplicial complex. Any grid cells that are contained entirely within the map's boundaries become a 2-simplex. Using a level-set method for front propagation, we then evolve the map's boundaries outward until we fill all of the grid cells. By adding 2-simplices as we

See **Political Islands** *on page 3*

[1] https://www.pewresearch.org/topics/political-polarization/

[2] The data were provided by the *Los Angeles Times* data visualization team.

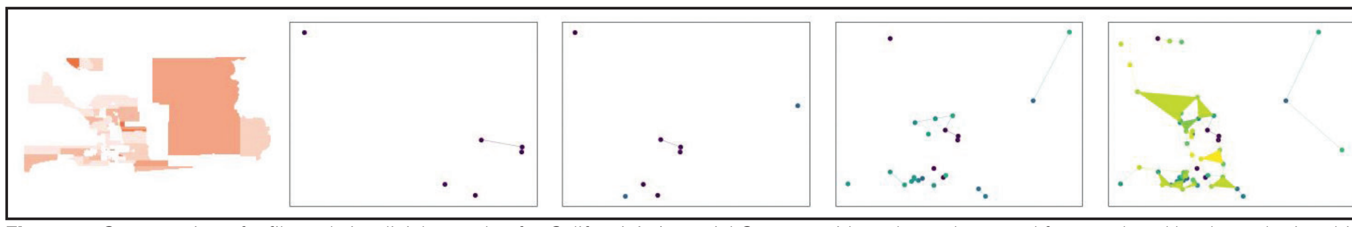

Figure 1. *Construction of a filtered simplicial complex for California's Imperial County, with nodes, edges, and faces colored by the order in which we add them to the filtration. The filtered simplicial complex is strictly increasing. We add only the darkest red precincts initially, and we add the lighter red precincts in order of preference, from strongest to weakest. Figure courtesy of [1].*

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6 **p4est: A Parallel Software Toolbox for Efficient Mesh Refinement and Partitioning**
Carsten Burstedde overviews the development of the p4est software library for adaptive mesh refinement, which adapts and partitions meshes in parallel. Various scientific applications and general numerical mathematics libraries—such as deal.II and PETSc—utilize the software toolbox as a mesh provider.

8 **The Threat of AI Comes from Inside the House**
Matthew Francis reviews *You Look Like a Thing and I Love You: How Artificial Intelligence Works and Why It's Making the World a Weirder Place* by Janelle Shane. Shane asserts that the real dangers of artificial intelligence (AI) emerge from its creators — while humans conceive and devise AI, they unreasonably expect computers to circumvent human failings.

9 **Advocating for Science as a Science Policy Fellowship Recipient**
Sheri Martinelli recounts her experiences as a SIAM Science Policy Fellow. She outlines activities that help participants understand advocacy and the government budget process, learn about funding agency goals and priorities, and meet with congressional staff. The experience allowed Martinelli to address her concerns about the unfavorable view of science within the government.

10 **The Mathematics Underlying Gun Violence**
Gun violence causes roughly 31,000 deaths and 78,000 injuries annually in the U.S. It has a disproportionately large effect on rural communities and young people, and introduces a higher likelihood of health conditions and risky social behaviors. Shelby Scott and Louis Gross present practical ideas for utilizing mathematical methods to study gun violence, enhance evaluation techniques, and improve data collection.

11 **Professional Opportunities and Announcements**

Obituary: Peter Deuflhard

By Christof Schütte and Ralf Kornhuber

On September 22, 2019, Peter Deuflhard passed away at the age of 75. The Berlin mathematics community lost one of its leaders — a highly respected colleague and overall wonderful person.

After earning a diploma in physics at the Technical University of Munich (TUM) and a doctorate in mathematics (with Roland Bulirsch on Newton methods) at the University of Cologne, Peter habilitated at TUM in 1977 with a thesis on multiple shooting techniques. At age 34, he was appointed as a full professor of numerical mathematics at the Ruprecht-Karls-Universität Heidelberg. He maintained this position until 1986, at which point he became chair of scientific computing at Freie Universität Berlin. Peter authored the pioneering green paper that led to the foundation of the Zuse Institute Berlin (ZIB)—the first German institute for scientific computing—in 1986. He served as president of ZIB for over 25 years, forging it into a role model for interdisciplinary mathematical research worldwide.

Peter Deuflhard, 1944-2019. Photo courtesy of Sandra Patzelt-Schütte.

In 2002, Peter co-founded MATHEON, the Berlin-based research center that established ongoing successful cooperation of all Berlin universities and mathematical research institutes. MATHEON operates under the motto "Mathematics for key technologies" and was the forerunner of MATH+, the Berlin Mathematics Research Center — a Cluster of Excellence. Peter was also a member of the Berlin-Brandenburg Academy of Sciences and Humanities.

Based on his long and wide-ranging list of monographs, textbooks, and scientific papers, an unknowing contemporary might think that "Deuflhard" is the pseudonym for an entire interdisciplinary collaborative group of scientists from the fields of mathematics, physics, chemistry, medicine, engineering, and the humanities. Yet at his heart, Peter was a mathematician. In 2007, he received the Maxwell Prize of the International Congress for Industrial and Applied Mathematics (ICIAM), which is awarded to "a mathematician who has demonstrated originality in applied mathematics." ICIAM's prize committee published the following laudation about Peter:

> "Professor Peter Deuflhard's contributions to applied mathematics have a breadth, depth, and originality that is almost without parallel. His contributions to algorithm-oriented numerical analysis are fundamental and range from highly nonlinear algebraic systems through large-scale ordinary and partial differential equations to Markov chains. Within these fields, they cover direct and inverse problems, optimization aspects, and optimal control. Characteristic of his work is that he always lays a firm, often innovative mathematical basis on which he constructs highly efficient algorithms for hard real-life problems in science and technology. His style of research has revolutionized scientific computing, [and] a large number of highly-reputed scholars follow his tracks."

Peter supervised more than 30 doctoral students in mathematics, many of whom pursued academic careers at ZIB. He collaborated intensively with engineers, physicians, scientists, and practitioners in many different fields. He was also quintessential in the formation of modern scientific computing as a discipline that integrates a wide range of applied mathematicians, computer scientists, and other researchers who combine mathematics and computing technology to ultimately achieve a fundamental understanding of phenomena and processes.

The variety of application areas to which Peter contributed is stunning. They range from spacecraft mission design, chemical engineering, nano-optics, systems biology, and medicine to bioinformatics, molecular dynamics, drug design, and even the humanities. Peter never dropped a subject; he transformed it by identifying hidden structures and converting them into sources of possible future developments, perhaps in other fields. He thus built bridges between completely different areas. For example, Peter applied ideas from the numerics of partial differential equations to model-supported operation planning in head surgery. This proved to be an intriguing way to connect his results to the study of beautiful faces and contribute to research in the humanities.

Although Peter appreciated his international recognition, he gleaned even more enjoyment from his personal interactions with the many colleagues and students who benefitted both personally and professionally from his guidance.

We are so grateful for Peter's extraordinary contributions and mourn the loss of an outstanding member of the scientific community.

This obituary also appears in ICIAM Dianoia, the newsletter of the International Council for Industrial and Applied Mathematics.

Christof Schütte is a professor of scientific computing at Freie Universität Berlin and succeeded Peter Deuflhard as president of the Zuse Institute Berlin (ZIB). Ralf Kornhuber is a professor of numerical analysis and scientific computing at Freie Universität Berlin. He began his academic career as a member of Peter's group in the early days of ZIB.

Reflections on Big Data and Sensitivity of Results

LETTER TO THE EDITOR

Mathematicians are excited by the significant success of "big data," methods, and the recent *SIAM News* article about Julia[1] is no exception. But mathematicians have a responsibility to prove their results or define their validity, no matter how exciting. Many investigations of big data solve inverse problems using outputs of systems to define the inputs and the equations that define the system.

Inverse problems have been analyzed in some detail, and the reliability of results is a central subject in the analysis. The sensitivity of results to uncertainties is often large because of the inherent ill-posedness of most inverse problems. This sensitivity is crucial to determine the reliability and thus utility of results. The fact that Julia can help determine sensitivity is of great importance.

It is also important that workers on big data actually discuss issues of sensitivity and ill-posedness as they assess the reliability of their results. It is a sad fact that many papers on big data do not include the words "ill-posed" or "sensitivity," let alone confront the issues those words describe. The classical results of the theory of inverse problems cannot help solve the problems of big data unless they are used. Explicit discussion of the sensitivity of results and the ill-posedness characteristic of inverse problems is likely to lead to more reliable and useful results.

— Bob Eisenberg

Read "Scientific Machine Learning: How Julia Employs Differentiable Programming to Do it Best" by Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B. Shah, in the October 2019 issue of SIAM News.

[1] https://sinews.siam.org/Details-Page/scientific-machine-learning-how-julia-employs-differentiable-programming-to-do-it-best

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SIAM Announces New 2020 Leadership

SIAM is pleased to announce our new President-Elect, Vice President-at-Large, Secretary, and members of the Board of Trustees and Council. We congratulate these 10 esteemed individuals and thank all of the excellent candidates for their willingness to serve the SIAM community in this capacity.

President-Elect

The President-Elect will serve in this position for one year, as President for the subsequent two years, and as Past-President for the final year. This totals a four-year term.

Susanne C. Brenner.

Susanne C. Brenner, a professor in the Department of Mathematics and Center for Computation & Technology at Louisiana State University, will use her position to solicit input from SIAM membership and work with the outstanding officers and SIAM staff to address challenges and create opportunities. Brenner will strive for best practices in all endeavors towards the SIAM mission of building cooperation between mathematics and the worlds of science and technology.

Vice President-at-Large

The Vice President-at-Large will serve a two-year term and can serve up to three consecutive terms.

Carol S. Woodward, a project leader in the Center for Applied Scientific Computing at Lawrence Livermore National Laboratory, was re-elected as Vice President-at-Large and will continue to serve on the Council. She will direct her efforts towards helping SIAM tackle challenges by supporting SIAM Activity Groups in developing and maintaining their research communities. Woodward will also encourage stimulated recognition of numerous research fields via the various SIAM Prize and Fellows Programs.

Secretary

The Secretary will serve a two-year term and can serve up to three consecutive terms.

Susan E. Minkoff, a professor in the Department of Mathematical Sciences and affiliated professor in the Departments of Geosciences and Science/Mathematics Education at the University of Texas at Dallas, will ensure that the committees she chairs are impactful and as active as possible. She aims to hear from everyone on the committees and inspire members to work towards implementing ideas about which they are passionate.

Carol S. Woodward.

Board of Trustees

Three SIAM members were elected to the Board for three-year terms. The Board consists of nine elected trustees, up to two appointed trustees, the President, and the Treasurer. Board members can serve a total of three consecutive terms.

Margot Gerritsen, senior associate dean of the School of Earth, Energy and Environmental Sciences at Stanford University, was re-elected to the Board. She will concentrate on ethics, fairness, transparency, and accountability—particularly in relation to computational math and data science—while retaining her main focus on education and diversity.

Susan E. Minkoff.

Randall J. LeVeque, emeritus professor of applied mathematics at the University of Washington, was also re-elected to the Board. He wishes to foster the future of scholarly publication and enhance the impact of the rapidly-changing landscape of SIAM books and journals.

Bonita V. Saunders, a research mathematician in the Applied and Computational Mathematics Division at the National Institute of Standards and Technology, will use her position to help SIAM expand and continue its legacy by supporting wise decision-making in the control and use of SIAM funds and investments.

Council

Four SIAM members were elected to the Council for three-year terms. The Council consists of 12 elected members, the SIAM officers, and the chair of the Board. Council members can serve up to two consecutive terms.

Heike Faßbender, managing director at the Technische Universität Braunschweig's Institute of Computational Mathematics, will use her position to strengthen the "I" in SIAM (because industry seems more relevant today than ever before) to open new channels of communication between researchers and potential users of mathematical tools.

Helen Moore, director of applied mathematics at Applied BioMath, was re-elected to the Council. She hopes to proactively address challenges such as decisions about ethical issues, open-access journal policies, and declining journal subscriptions, and focus on emerging platforms for communicating research results.

Valeria Simoncini, a professor of numerical analysis at Università di Bologna, will foster the applied mathematics community's visibility through publications, conferences, and social media. She will also strongly pursue recent activities in innovative application areas like data science, where new mathematics is being created at incredible speed.

Suzanne L. Weekes, Associate Dean of Undergraduate Studies, ad interim at Worcester Polytechnic Institute, intends to strengthen efforts towards broadening SIAM membership. She believes the SIAM community must work effectively towards a more inclusive, welcoming, and diverse professional society.

Political Islands

Continued from page 1

fill cells (see Figure 2), we form a filtered simplicial complex, to which we apply PH. The resulting PH computation reveals the number of time steps required to fill a given hole. Because we evolve the entire boundary of a map with the same normal velocity, it takes longer to fill in larger holes than small ones. This allows us to track the geographical size of a given political island.

As an illustration of our methods in the context of real data, consider Tulare County in California. This county is home to Sequoia National Park and is known for the historical black farming community of Allensworth. Tulare is a strongly Republican (red) county, with a few blue and purple cities dispersed throughout. It also houses Visalia, a very large red city. Our adjacency method (see Figure 3a) captures many loops—mostly around blue and light red islands—whereas the level-set approach (see Figure 3b) successfully captures blue islands. The bar length of a given feature in the "barcodes" [4, 5] in Figure 3 corresponds to polarization strength in the adjacency construction and to hole size in the level-set construction.

We also compared tabulations of our computational results with existing Vietoris–Rips simplicial constructions [1]. These comparisons illustrate that our methods perform with speeds that are equal to or faster than standard techniques. They also yield more interpretable PH results for our quest to find political islands.

In the future, we hope to apply our methods to a longitudinal study of California precincts (and other map-based electoral data). Our adjacency and level-set constructions will be useful for applications beyond the analysis of voting islands, and we are currently utilizing them to study additional spatial systems, such as urban and biological structures. We hope that our work will inspire other researchers to begin or continue using topological tools to pursue problems in spatial networks and other spatial systems [2, 3].

At the 2019 SIAM Conference on Applications of Dynamical Systems, which took place last year in Snowbird, Utah, Michelle Feng described a project from the 2018 Voting Rights Data Institute that used an adjacency-based construction of demographic data to examine racial segregation in cities. The presentation is available from SIAM either as slides with synchronized audio[3] or as a PDF of slides only.[4]

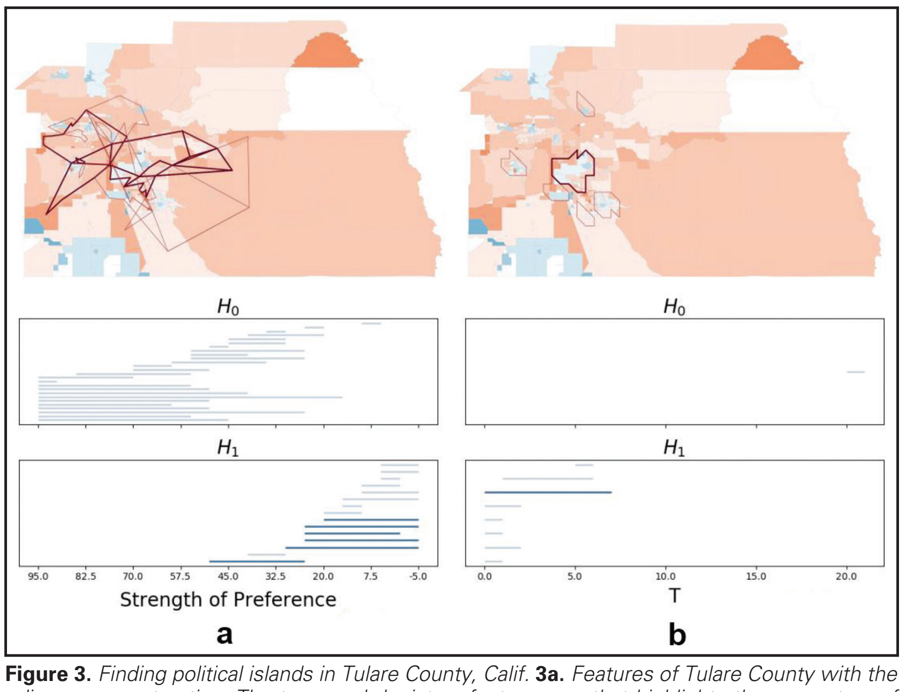

Figure 3. *Finding political islands in Tulare County, Calif.* **3a.** *Features of Tulare County with the adjacency construction. The top panel depicts a feature map that highlights the generators of our persistent homology (PH) computation's longest-persisting features. The bottom two panels show the barcode for this PH computation, where each bar in the code represents one feature. The left endpoint indicates the scale at which a feature is born (readers may ask whether the features are born at the right time), and the right endpoint indicates the scale at which it dies. All six loops (and highlighted bars) capture medium- to dark-red precincts that surround the blue precincts.* **3b.** *Feature map and barcode from the level-set construction on Tulare County. All features capture blue precincts that are surrounded by red precincts. Figure courtesy of [1].*

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[5] Topaz, C. (2016). Topological data analysis: One applied mathematician's heartwarming story of struggle, triumph, and ultimately, more struggle. *SIAM Dynamical Systems Web*. Retrieved from https://dsweb.siam.org/The-Magazine/Article/topological-data-analysis.

[3] https://www.pathlms.com/siam/courses/11697/sections/14898/video_presentations/129689

[4] https://cdn.fs.pathlms.com/Y7o7flmGRHyIdyHU7jkh?cache=true

Michelle Feng is a graduate student in the Department of Mathematics at the University of California, Los Angeles (UCLA). Her research interests include topological data analysis, network science, and mathematical applications to political and social science. Mason A. Porter is a professor in the Department of Mathematics at UCLA. His research interests include the theory, methods, and applications of complex systems, nonlinear systems, and networks.

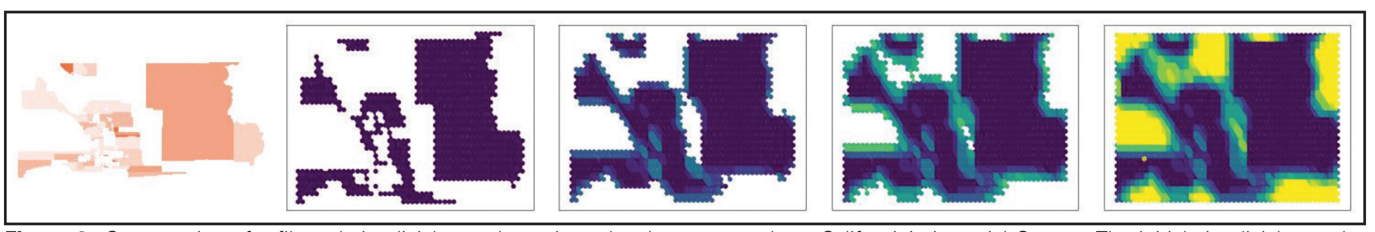

Figure 2. *Construction of a filtered simplicial complex using a level-set approach on California's Imperial County. The initial simplicial complex consists of a triangulation of a map of all red precincts. We then evolve this surface outward and color the simplices based on the order in which they enter the filtered simplicial complex. Figure courtesy of [1].*

Topological Data Analysis

Continued from page 1

data in some convenient way, yet imagine that we do not know *a priori* what about the data we would like to measure. One approach is to characterize the data through the lens of homology, i.e., by calculating its Betti numbers. But what does it mean for discrete data to have holes? To answer this question, we build a *simplicial complex* out of the data. We place an m-dimensional ball of diameter ε around each point—where ε is the *persistence scale* or *proximity parameter*—and then calculate the homology of the object formed. In fact, we do this for all $\varepsilon \geq 0$, calculating the Betti numbers at every scale. This approach not only counts topological features, but equips each one with a length scale indicative of that feature's geometry.

Collective Motion, Agent-based Models, and Data Exploration

In biological aggregations such as bird flocks, fish schools, and insect swarms, organisms interact via forces like social attraction, repulsion, and alignment. These interactions dictate the movement of individuals, and the many individual movement decisions may result in collective motion of the group. Self-organized collective motion can include vortex-like mills, ring structures, and strongly-aligned flocking groups. Agent-based models help elucidate the relationship between individual movement choices and large-scale group behavior [4, 11].

Biological aggregations can generate massive amounts of data, thus requiring tools that summarize the dynamic information arising from experimental observations or model simulations. As a thought experiment, consider the recording and imaging of 2,600 starlings during a flocking event [1]. The imaging technique provides the three-dimensional position and velocity of each bird in every video frame, recorded at 10 frames per second. Just one minute of video for a field observation creates 10^7 pieces of information. Flocking events called murmurations can last for minutes or hours; the latter could produce 10^9 pieces of information. Data analysis is essential to the study of phenomena that are not easily detectable by the human eye.

A topological approach to collective motion data encodes the Betti number b_k as a function of the scale parameter ε for a given homology dimension k and a static set of points [9]. Repeating this process for each moment in time creates a two-variable function $b_k(t, \varepsilon)$ that provides a topological signature of time-varying data across scales. If values of t and ε are selected on a grid, we can represent the Betti numbers as a matrix and visualize them as a contour plot called a CROCKER (*Contour Realization of Computed k-dimensional hole Evolution in the Rips complex*). Exploration of CROCKERs arising from the simulations of agent-based models (such as those in [4, 11]) detects phenomena that are not easily visible or noticed by commonly-used order parameters like group polarization. These phenomena include intermittent clustering of agents, lone agents, unoccupied regions of the simulation domain, and *double mills* comprising two superposed counter-rotating groups.

Model Selection

Biologists and mathematical biologists routinely face the challenge of model selection, i.e., choosing between several mathematical models that may describe a given dataset. TDA can help address this challenge. One example involves pea aphids filmed walking in an experimental arena (see Figure 1a) [8]. Researchers fed their video data into motion tracking software to obtain the coordinates of each insect during every frame. They then introduced the following modeling framework. Each aphid transitions stochastically between a moving and stationary state. Moving aphids perform an unbiased correlated random walk, which incorporates a randomly drawn step length and turning angle. A key biological question is whether the aphids interact socially. To address this query, the researchers proposed two versions of their model. The interactive model incorporates social interactions via the model parameters' dependence on distance to an aphid's nearest neighbor, while the control model ignores social interactions and uses fixed (optimized) constants for these parameters.

Which model is more faithful to the experimental data? One study adopted three approaches to answer this question [10]. The first method utilized order parameters traditionally studied in collective motion, such as group polarization and angular momentum. For these traditional order parameters, scientists compared simulated order parameter time series to experimental ones and chose the model with the statistically closer time series. Though similar to the first, the second strategy instead used order parameters that are closely related to model inputs, such as average distance from an aphid to its nearest neighbor. Because these parameters reflect *a priori* knowledge of the model, they are called *a priori order parameters*. The third method was again similar, but employed topological summaries of the data, namely CROCKERs.

While traditional order parameters offer a mixed message about model preference, *a priori* order parameters consistently indicate that the interactive model is closer to experimental data than the control model [10]. The topological approach also suggests that the interactive model is more consistent with experimental data. Given that CROCKERs do not use any *a priori* knowledge about the datasets or the models that generate them, the topological approach could be useful for describing and comparing collective motion when little is known about key model mechanisms.

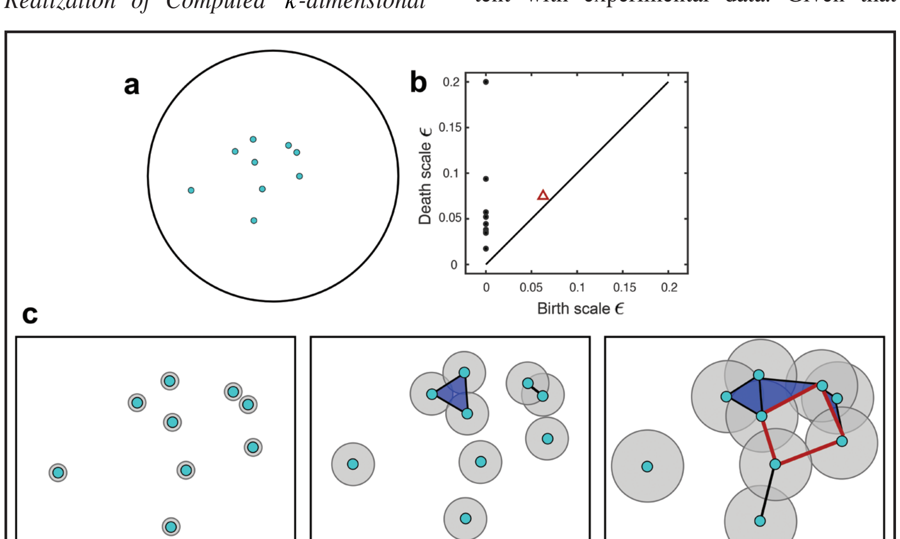

Figure 1. *Pedagogical example of persistent homology computations.* **1a.** *Initial locations of nine pea aphids in a circular domain, taken from a collective motion experiment [8] and analyzed in [10].* **1b.** *A persistence diagram summarizing the homology of the data in 1a across a range of persistence scales ε. The horizontal and vertical axes respectively specify the values of ε at which a topological feature is born and dies. Black circles represent connected components and red triangles represent flat holes in the data.* **1c.** *The Vietoris-Rips complexes of the data in 1a as the persistence scale ε takes on three increasing values. Each point is a 0-simplex and each edge is a 1-simplex, created if the $\varepsilon/2$-balls around two points intersect. Every triangle is a 2-simplex that forms if all vertices are pairwise connected by edges. Note the red quadrilateral in the last panel; because the four vertices are connected in a cyclic manner, there is a flat hole in the data. This hole manifests as the red triangle in 1b. Figure courtesy of Maria-Veronica Ciocanel.*

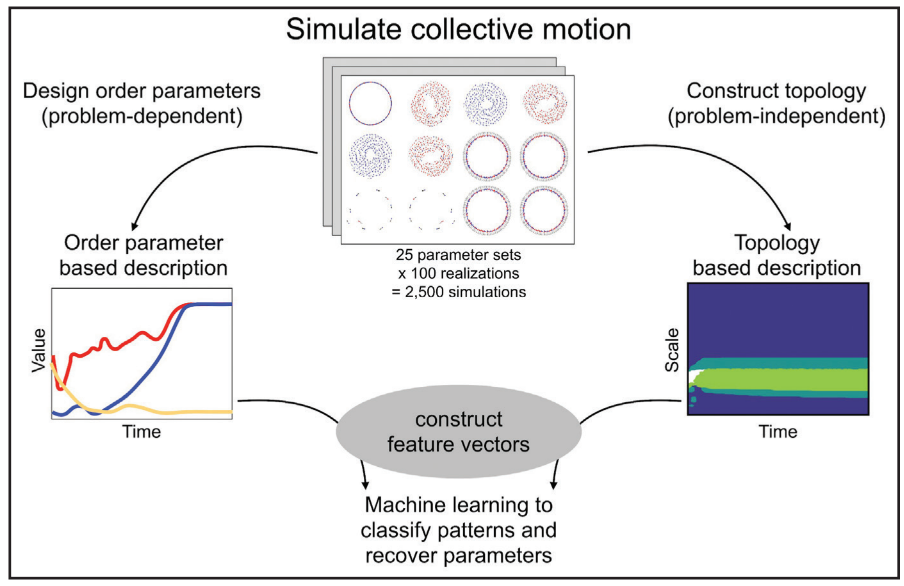

Figure 2. *Analysis pipeline [2] for recovering parameters and inferring collective motion phenotypes from data produced by simulations of an influential model of collective motion [4]. In this pipeline, one simulates the model many times and constructs a feature vector for each simulation. The feature vectors serve as input to machine learning algorithms, which identify parameters and the type of collective motion. Feature vectors arising from order parameter time series are traditionally used in studies of collective motion, such as group polarization or angular momentum. In contrast, one can utilize feature vectors that originate from topological data. Topological features yield far more accurate results than traditional features, even though they do not require contextual knowledge about model phenomena [2]. Researchers explore the limit of very low-dimensional features and attain 93.1 percent supervised classification accuracy using topological feature vectors reduced to three dimensions via principal component analysis (PCA) [2]. In contrast, three-dimensional feature vectors computed via PCA on group polarization and angular momentum achieve 46.7 percent and 30.0 percent accuracy respectively. Figure courtesy of Angelika Manhart and adapted from [2].*

Parameter Inference

Mathematical biologists are also interested in *parameter inference* — that is, deducing appropriate model parameters from an observed dataset. Scientists applied topological data analysis and machine learning [2] to the model in [4]. By nondimensionalizing the model and fixing both the number of agents and the parameters that describe self-propulsion and drag, one can reduce the model to a two-dimensional space of parameters: the ratio of the characteristic strengths of attraction and repulsion, and the ratio of their distinctive length scales. Depending on the parameters, the model might produce collective behaviors including single mills, double mills, disorganized swarms, and group dispersal.

Researchers have developed a TDA-based method for parameter recovery in this system (see Figure 2) by generating a large library of model simulations and systematically varying the two model parameters [2]. Each simulation transforms into a feature vector that summarizes the dynamics. One class of feature vectors comprises the aforementioned traditional order parameters. A second class contains vectorized CROCKER data: b_0, b_1, or the concatenation of both. One can calculate these CROCKERs from agent position or augment them with time-delayed position data to incorporate information about velocity. For every simulation, feeding each feature vector into machine learning algorithms recovers parameters and identifies a phenotypic pattern. In both supervised and unsupervised machine learning approaches, topological features give rise to far more accurate results than traditional ones, even though they do not require contextual knowledge about model phenomena (see Figure 2).

Concluding Perspectives

Persistent homology measures both a dataset's global topology and its local geometry. By using CROCKER data (or more sophisticated invariants [7]) in concert with simple data visualization, statistical tests, and machine learning techniques, we can possibly detect important events in dynamical data, choose between potential data models, and even recover parameters. These approaches have aided our understanding of collective motion in biological aggregations and may provide similar benefits in other areas of mathematical biology.

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Henry Adams is a topologist and professor of mathematics at Colorado State University. He is also the co-director of the Applied Algebraic Topology Research Network. Maria-Veronica Ciocanel is an applied mathematician at the Mathematical Biosciences Institute. She is also a President's Postdoctoral Scholar at The Ohio State University. Chad M. Topaz is an applied mathematician, data scientist, and professor of mathematics at Williams College. He is also the co-founder and director of the Institute for the Quantitative Study of Inclusion, Diversity, and Equity. Lori Ziegelmeier is an associate professor of mathematics at Macalester College. Her research interests are in geometric and topological data analysis, and she is a founder and steering committee member of Women in Computational Topology.

An Algebraic Geometry Perspective on Topological Data Analysis

By Paul Breiding

Topological data analysis (TDA) is a success story with a wide range of diverse applications. Here I will survey TDA from the point of view of *algebraic geometry*.

Algebra and algebraic geometry present certain immediate applications to TDA. The persistence module—the data structure behind *persistent homology* (PH)—is an inherently algebraic concept, and attempts to extend PH to multiple parameters utilize concepts from commutative algebra [4,10]. However, I would like to discuss some of algebraic geometry's other roles in TDA — specifically applications of numerical algebraic geometry (NAG) and enumerative algebraic geometry (EAG).

NAG concerns the computation of numerical solutions to a system of n polynomial equations $F(x)=(f_1(x),\ldots,f_n(x))=0$ in n variables $x=(x_1,\ldots,x_n)$ over the complex numbers. *Numerical homotopy continuation* is the computational paradigm in NAG. This paradigm involves the generation of a system of equations $G(x)$ whose solutions are known (a so-called *start system*), and the continuation of the solutions of $G(x)=0$ along a deformation of $G(x)$ towards $F(x)$. Such continuation leads to a *Davidenko differential equation*, an ordinary differential equation (ODE) that is solved by standard numerical methods for ODEs. The state-of-the-art implementations are Bertini,[1] HOM4PS,[2] HomotopyContinuation.jl,[3] NAG4M2,[4] and PHCPack.[5]

On the other hand, EAG counts the number of solutions to a system of polynomial equations. Although they might initially appear different, NAG and EAG are intimately related: NAG's key benefit is that one can generate initial values *for all isolated solutions* of $F(x)=0$ in $\mathbb{C}^n$. For instance, if the degree of the ith polynomial is d_i, then $G(x)=(x_i^{d_i}-a_i)_{i=1}^n$—where $a_1,\ldots,a_n\in\mathbb{C}^*$—can serve as a start system for the homotopy $(1-t)G(x)+tF(x)$, $0\leq t\leq 1$. $G(x)=0$ has $D=d_1\cdots d_n$ isolated solutions, and a theorem from algebraic geometry implies that $F(x)=0$ has *at most* D isolated solutions. Continuing the solutions of $G(x)=0$ towards $F(x)=0$ produces *all* isolated solutions of $F(x)=0$. In practice, however, $F(x)=0$ has significantly fewer solutions than D, and diverging solutions must be eliminated. EAG helps construct other start systems that are adapted to the structure of $F(x)$ and improve the algorithm's efficiency.

Returning to TDA, let us consider the situation in which $M\subset\mathbb{R}^n$ is the zero set of s polynomials in n variables $F(x)=(f_1(x),\ldots,f_s(x))$. In algebraic geometry, such an M is called a *real algebraic variety*. The conformation space of the cyclooctane molecule serves as an example. Cyclooctane is comprised of eight carbon atoms $x_1,\ldots,x_8\in\mathbb{R}^3$ aligned in a ring, such that the distances between neighboring atoms all equal $c>0$. The energy of configuration $(x_1,\ldots,x_8)$ is minimized when each angle between successive bonds amounts to $\arccos(-\frac{1}{3})\approx 109.5^\circ$. The polynomial equations in $3\cdot 8=24$ variables are

$$\|x_1-x_2\|^2=\cdots=\|x_7-x_8\|^2=\|x_8-x_1\|^2=c^2$$

and

$$\|x_1-x_3\|^2=\cdots=\|x_6-x_8\|^2=$$

$$\|x_7-x_1\|^2=\|x_8-x_2\|^2=\frac{8}{3}c^2.$$

The solution set of these equations—up to simultaneous translation and rotation—is homeomorphic to a union of the Klein bottle and a sphere, which intersect in two rings [6].

NAG can generate a sample of points from M, which may then serve as input for PH; Figure 1 (on page 1) depicts a sample from the cyclooctane variety. One idea involves sampling linear spaces L of a dimension equal to the codimension of M, and computing the points in the intersection of M and L. In another approach, researchers sample points $q\in\mathbb{R}^n$ in the ambient space and determine the point on M that minimizes the distance to q. One can cast both computational problems as a system of polynomial equations and use NAG to solve them. Investigative directions in NAG include how to sample with respect to a probability distribution on M [1] and produce samples with the desired level of density in M [8].

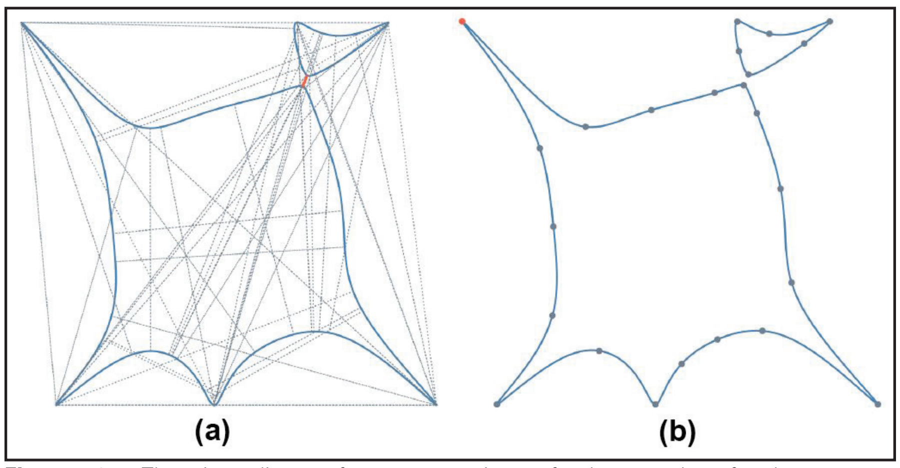

Figure 2. *The ingredients for computation of the reach of planar curve* $C=\{(x^3-xy^2+y+1)^2(x^2+y^2-1)+y^2=5\}$. **2a.** *All bottlenecks of* C. *The narrowest bottleneck is red with a width of* ≈ 0.138. **2b.** *All points of critical curvature of* C. *The red point in the uppermost left is the point of maximal curvature and is* ≈ 2097.17. *Therefore,* $\tau(C)\approx\{\frac{1}{2097.17},\frac{0.138}{2}\}=\frac{1}{2097.17}$. *Code and image courtesy of Paul Breiding and Sascha Timme.*

Researchers also use NAG and EAG to study two important numbers for TDA: the homological feature size $\mathrm{hfs}(M)$ [5] and the reach $\tau(M)$ of M [12]. Emil Horobet and Madeleine Weinstein showed that if M is an algebraic manifold (i.e., a real algebraic variety that is also a manifold) defined by polynomials over $\mathbb{Q}$, then both $\mathrm{hfs}(M)$ and $\tau(M)$ are algebraic over $\mathbb{Q}$ [11]. Therefore, one can compute both by means of NAG.

M's reach is the distance from M to its medial axis. An equivalent definition is $\tau(M)=\min\{\frac{1}{\sigma(M)},\frac{1}{2}\rho(M)\}$, where $\sigma(M)$ is the maximal curvature of a geodesic in M and $\rho(M)$ is the width of M's narrowest *bottleneck*. A bottleneck is a pair $(x,y)\in M^2$, such that $x-y$ is perpendicular to both tangent spaces T_xM and T_yM. One can formulate this as a system of polynomial equations (solved using NAG) and extract the real solutions from the complex ones. Recall that one computes the *isolated* solutions of a polynomial system in NAG; the trivial solutions for which $x=y$ are not computed. Scientists have studied bottlenecks intensely, both from the perspective of NAG [9] and EAG in terms of *polar classes* of M [7].

Equations for $\sigma(M)$ are less straightforward, but a direct formula for the curvature $\gamma(x)$ at $x\in M$ indicates that $\sigma(M)=\min_{x\in M}\gamma(x)$ for planar curves. In this case, the first-order optimality conditions for $\sigma(M)$—that the gradient of $\gamma(x)$ is perpendicular to the tangent space T_xM—generate a system of polynomial equations. Solving this yields $\sigma(M)$.

In summary, one can compute $\rho(M)$ and $\sigma(M)$ separately via NAG, and infer the reach $\tau(M)$ from those numbers. Figure 2 illustrates an example of this computation for a planar curve.[6]

One can also replace the reach with a lower bound that involves the *real condition number* of a system of polynomials [3]. This lower bound holds for both real algebraic varieties and the more general class of *semialgebraic sets*. Researchers use the condition number to derive a complexity analysis of an algorithm for computing homology.

Finally, I would like to propose three possible future directions for the use of algebraic geometry in TDA. The first is an analysis of $\sigma(M)$ in the context of NAG and EAG for bottlenecks [7, 9]. This is indispensable when computing the reach beyond planar curves. The second is PH using ellipsoids. Experiments show that this approach can greatly improve the output diagrams' quality in PH [2], yet it lacks a theoretical explanation. The third direction involves sampling. The standard approach to sampling from nonlinear objects uses *Markov Chain Monte Carlo* methods. Combining this approach with NAG seems promising.

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Paul Breiding is a postdoctoral researcher at the Technische Universität Berlin. He works in the field of applied algebraic geometry and is one of the developers of HomotopyContinuation.jl.

[1] bertini.nd.edu/
[2] hom4ps3.org
[3] juliahomotopycontinuation.org/
[4] people.math.gatech.edu/~aleykin3/NAG4M2
[5] phcpack.org

[6] View an accompanying video in the online version of this article.

A Special Funding Opportunity at the NSF/DMS

By Hans Kaper

The National Science Foundation (NSF) has updated its request for proposals for the Algorithms for Threat Detection (ATD) program. This program supports research projects to develop the next generation of mathematical and statistical algorithms for analysis of large spatiotemporal datasets, with application to quantitative models of human dynamics. The initiative is a partnership between the NSF's Division of Mathematical Sciences (DMS) and the National Geospatial Intelligence Agency (NGA). The deadline for proposal submission is March 18, 2020.

The ATD program—which is not well known in the applied mathematics and computational science community—currently supports 64 proposals, several of which represent collaborative projects. The program awarded 14 proposals during the latest funding cycle (FY19). These proposals covered a broad range of topics in mathematics and statistics, including the dynamics of content spreading in multilayer networks, real-time detection of pattern changes in networks, topological data analysis, multimodal spatiotemporal data in computer vision, and detection of time-lapse changes in imagery. The awards range from approximately $57K to just over $500K. Successful proposals from prior years addressed a similar breadth of topics, such as the dynamics of drone-based threat detection, spectral interpretations of essential subgraphs for threat discoveries, precision agriculture and satellite imaging, vector-borne diseases and weather patterns, and harmonic analysis and machine learning for emergency response.

The ATD program offers a unique opportunity for researchers to develop new mathematics and demonstrate the subject's broad applicability to issues of national security. Submitted proposals are reviewed according to the usual NSF criteria: intellectual merit and broader impacts. Successful proposals are jointly funded by the NSF and NGA. Awardees must include appropriate acknowledgment of NGA support in reports and/or publications of work performed under the award.

Details about proposal submission, processing, and review procedures are available in NSF solicitation 20-531.[1]

Hans Kaper, founding chair of the SIAM Activity Group on Mathematics of Planet Earth and editor-in-chief of SIAM News, is affiliate faculty in the Department of Mathematics and Statistics at Georgetown University.

[1] https://www.nsf.gov/pubs/2020/nsf20531/nsf20531.htm

p4est: A Parallel Software Toolbox for Efficient Mesh Refinement and Partitioning

By Carsten Burstedde

Many proven principles from traditional systems programming remain current and valuable in the development of scientific software. Three long-time favorites are (1) *Do one thing and do it well*, (2) *Keep it simple, stupid!*, and arguably (3) *Use the source, Luke*. In this article, I will review how these principles apply to the development of the `p4est` software library for adaptive mesh refinement (AMR). `p4est` adapts and partitions meshes in parallel and is used as a mesh provider for various scientific applications, as well as for general numerical mathematics libraries such as `deal.II` and `PETSc`.

Forest-of-linear-octrees AMR

In 2007, during my time as a postdoctoral researcher at the University of Texas at Austin's Center for Computational Geosciences and Optimization, my group realized that AMR is necessary for global mantle convection simulations due to the vast discrepancy of geological scales. We had learned a lot from Tiankai Tu—author of the octor code that implemented a pointer-based, distributed Cartesian octree and scaled well to several thousand message passing interface (MPI) processes—yet had no solution for spherical domains. We did consider several options, including fictitious or embedded domains and the use of multiple octrees.

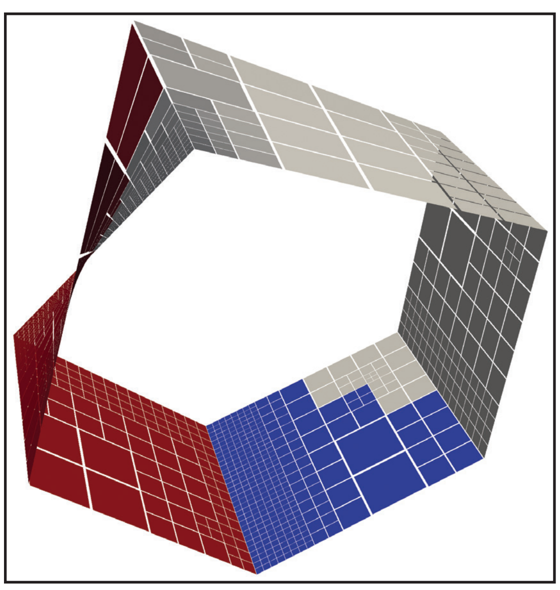

Figure 1. *An adaptive mesh for the two-dimensional Moebius strip embedded in three-dimensional space. We have executed the 2:1 balance algorithm, which limits the size difference between neighboring leaf quadrants. The color encodes a ± 1 leaf partition on three MPI ranks. Figure courtesy of [4].*

One day, I approached my fellow postdoc Lucas Wilcox (now at Naval Postgraduate School) with a proposal to reimplement octor and really understand its method of operation. Lucas immediately suggested extending the new code to a forest of octrees and building it along the algorithmic concepts of Hari Sundar and Rahul Sampath—then at the University of Pennsylvania working with George Biros—using flat arrays rather than pointers and storing only the leaves of the octree. After about a month of pair-programming, we were able to refine a two-dimensional forest manifold and write VTK files. Subsequent months saw 2:1 balance capabilities, a ghost layer algorithm, three-dimensional support, and node numbering for piecewise d-linear finite elements — scaling to 62e3 cores of the Texas Advanced Computing Center's Ranger supercomputer.

Simplicity, Correctness, and Performance

SOFTWARE AND PROGRAMMING

We were clear in our desire for a library that "just" does the meshing. For the forest, the first requirement was an encoding of the connectivity of tree roots, which themselves constitute a conforming hexahedral mesh. In two dimensions and for each tree face, we record which neighboring tree connects at which face and whether the connection is flipped. For each tree corner, we separately record which other trees and respective corners connect; there can be any number of these. In three dimensions, we have four possible rotations at a face and an arbitrary number of tree neighbors across any edge, possibly flipped [4]. This concept allows for near arbitrary domain topologies, including periodicity (see Figure 1).

The quadrant object encodes any two- or three-dimensional tree node. Its length is a (negative) power of two in relation to the root, and the coordinates of its lower left corner are integers aligned at multiples of its length. Obtaining a parent quadrant; a given child; a sibling; or a face, edge, or corner neighbor amounts to bitwise operations on this coordinate tuple (see Figure 2). Because each tuple has an equivalent interpretation as an index in a space-filling curve, an array of quadrants is sortable and searchable by the `C` library functions `qsort` and `bsearch`.

Figure 2 documents several time-tested `p4est` features, such as dimension independence. We compile both two- and three-dimensional code from the same source based on a preprocessor definition. Another feature is the favorability of clarity over optimization. A third and most underrated attribute is assertion; a lot of functions have about as many assertions as lines of actual code, which reliably catches mistakes during development and certainly helps keep the number of bugs we find to below one per year on average.

```
void
p4est_quadrant_child (const p4est_quadrant_t * q, p4est_quadrant_t * r,
                      int child_id)
{
  const p4est_qcoord_t shift = P4EST_QUADRANT_LEN (q->level + 1);

  P4EST_ASSERT (p4est_quadrant_is_extended (q));
  P4EST_ASSERT (q->level < P4EST_QMAXLEVEL);
  P4EST_ASSERT (child_id >= 0 && child_id < P4EST_CHILDREN);

  r->x = child_id & 0x01 ? (q->x | shift) : q->x;
  r->y = child_id & 0x02 ? (q->y | shift) : q->y;
#ifdef P4_TO_P8
  r->z = child_id & 0x04 ? (q->z | shift) : q->z;
#endif
  r->level = q->level + 1;
  P4EST_ASSERT (p4est_quadrant_is_parent (q, r));
}
```

Figure 2. `p4est` *quadrant child computes the* i*th child of a quadrant.*

We benefit from the use of linear arrays of leaves that are directly suitable as send and receive buffers with regard to MPI. Since we continue the space-filling curve through all trees in order—using MPI to replicate the lower left corner and tree number of the first quadrant on each rank—the `Allgather` routine can sufficiently encode the entire partition's shape. One can use top-down traversals to search for arbitrary sets of local and remote points or geometric objects [2]. `p4est` algorithms determine message pairs and sizes ahead of time, allowing us to post asynchronous point-to-point messages with known envelopes and buffer allocations. Repartitioning the mesh works in this manner, and the algorithm executes consistently in under one second (see Figure 3).

Application Interfacing

The boundary between an application and `p4est` is fairly sharp; the application indicates where to refine and when to repartition, and `p4est` builds the updated mesh in parallel — with the communication out of sight on the inside.

The application may query the mesh on several levels, trading off generality and ease of interfacing. The `p4est` ghost layer algorithm, which collects the set of all remote leaves adjacent to any local leaf, permits an application to define any type of discretization; we used this approach to create the `p4est` mesh backend for the finite element library `deal.II` [1].

See **p4est** *on page 7*

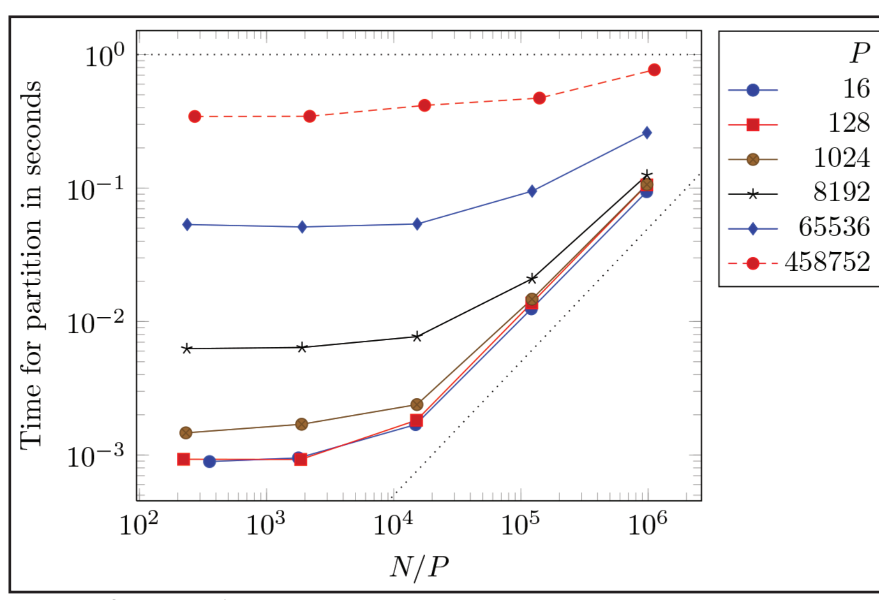

Figure 3. *Scalability of mesh repartitioning on "Juqueen." Its wall-clock time is between one second and one millisecond, revealing two regimes: one is linear in the number of local leaf quadrants N/P, and the other depends on total process count P through partition encoding. The maximum number of leaves is over .5e12. Figure courtesy of [3].*

SIAM to Sponsor Project NExT Fellows

By Kathleen Kavanagh, Lidia Mrad, and Carl Giuffre

In an effort to further support the professional development of junior faculty, particularly in the area of teaching and applied mathematics education, SIAM will now annually sponsor two Project NExT (New Experiences in Teaching) fellows. Organized by the Mathematical Association of America (MAA), Project NExT is a professional development program for new or recent Ph.D.s in the mathematical sciences that "addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, finding exciting and interesting service opportunities, and participating in professional activities," per the MAA.

Carl Giuffre, Adelphi University.

Workforce preparation and fulfillment of industry begins with teaching excellence in applied mathematics. Junior faculty play a key role in fostering an appreciation for math while empowering students from different backgrounds to solve a wide range of complex, real-world problems. However, not all recent Ph.D.s are equipped with the right resources or training to thrive in an academic setting. One of the most valuable aspects of Project NExT is its creation of a network of peers and mentors for fellows navigating their new careers.

Each cohort of fellows participates in Project NExT workshops preceding MathFest (the MAA's summer meeting) for two consecutive years. They also take part in Project NExT sessions during MathFest and additional activities throughout the meeting. In addition, fellows organize Project NExT-sponsored sessions at MathFest and the Joint Mathematics Meetings. Workshop topics include engaging students in specific mathematics courses, supporting individuals from historically underserved groups, involving undergraduates in mathematical research, writing grant proposals, and balancing teaching and research. In late July, the SIAM Conference on Applied Mathematics Education will be co-located with MathFest in Philadelphia, Penn., thus offering synergy between these two communities.

Carl Giuffre and Lidia Mrad are the first two SIAM Project NExT fellows. Giuffre received his Ph.D. in biomathematics from North Carolina State University, where he studied honey bees. His present research encompasses all social insects, including ants and termites. Giuffre uses computer vision, differential equations, and mathematical modeling to answer important questions in entomology. He is currently teaching at Adelphi University, where his Project NExT fellowship has allowed him to step outside of the traditional "college lecture" and promote a more inquiry-based learning environment. Project NExT has provided Giuffre with the tools and resources to challenge pedagogical norms while also maximizing his leadership skills and teaching potential.

Lidia Mrad, Mount Holyoke College.

Lidia Mrad earned her Ph.D. in mathematics, with a concentration in computational science, from Purdue University. She seeks to blend computational and analytical techniques to solve problems in materials science, specifically in the area of liquid crystals. Using methods from the calculus of variations, partial differential equations, and mathematical modeling, Mrad studies liquid crystal behavior relevant to optical and biological applications. After completing her postdoctoral training at the University of Arizona, she joined the Department of Mathematics and Statistics at Mount Holyoke College. Mrad's Project NExT fellowship has allowed her to connect with like-minded educators who wish to further engage students with mathematics inside and outside the classroom. She plans to use the program's support and resources to introduce more applied mathematics courses in her department and work with undergraduate students on exciting new projects.

Applications for the Project NexT fellowship require a personal statement, research statement, one-page curriculum vitae, and letter of support from the department chair. Eligibility requirements include a recent Ph.D. in mathematics, statistics, math education, or other math-intensive field, with a teaching position and experiences, attitudes, ideas, and leadership abilities that would contribute to the cohort. For consideration for the SIAM position, candidates must indicate their SIAM membership on their applications. Fellows are selected by an MAA committee. The next application deadline is April 15, 2020. See the website for further information.[1]

The impact of one exceptional faculty member undoubtedly has a far reach in the scientific community and at SIAM. SIAM is excited about this opportunity to contribute to excellence in applied mathematics education for the next generation of interdisciplinary problem-solvers.

Kathleen Kavanagh is a professor of mathematics at Clarkson University and the Vice President for Education at SIAM. Lidia Mrad is an assistant professor in the Department of Mathematics and Statistics at Mount Holyoke College. Carl Giuffre is an assistant professor in the Department of Mathematics and Computer Science at Adelphi University.

[1] https://www.maa.org/programs-and-communities/professional-development/project-next

p4est

Continued from page 6

For some common cases, we added the globally-consistent numbering of degrees of freedom as interface functions and internally queried the ghost layer. For example, the original mantle convection project calls the piecewise linear variant (see Figure 4).

To reduce the impact of $\log(N/P)$ time searches, Tobin Isaac (now at Georgia Institute of Technology) implemented an amortized top-down iteration that informs an application about every quadrant interface across faces, edges, and corners [5]. This approach supports all types of element-local discretizations and became the basis for integrating `p4est` with the `PETSc` software.

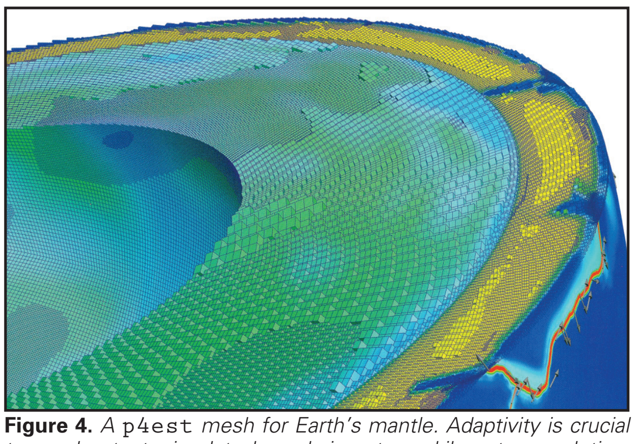

Figure 4. *A `p4est` mesh for Earth's mantle. Adaptivity is crucial to resolve tectonic plate boundaries at one-kilometer resolution; this keeps the elements coarser elsewhere for a total leaf count of only a few 100 million. Figure courtesy of [6].*

Considerations for Adopters

Given that `p4est` offers flexibility and scalability, in what aspects must a user invest? The primary answer is that `p4est` works with nonconforming, hanging-node meshes. Many discretizations can accommodate this with the addition of element-local interpolation and projection operators. Users can decide whether these additions compromise accuracy and stability.

Another attribute is `p4est`'s takeover of element ordering, which determines the partition's geometric shape. The third solution points to our encoding scheme of neighbor trees and elements, into which the application must adopt or translate. The associated authoritative documentation is still a big comment block in the `p4est` connectivity header file.

Our collection of examples in the source tree is now quite broad. In practise, users might study them and devise a thin wrapping layer around `p4est` based on their preferred conventions (a `C++` interface templated on the space dimension is one such example).

Acknowledgments: I would first like to acknowledge Lucas Wilcox, without whom `p4est` would not be what it is. Secondly, I wish to thank Tobin Isaac for his smart and deep contributions. I am grateful to Wolfgang Bangerth (Colorado State University) and Matt Knepley (University at Buffalo) for their support. Last but not least, I owe huge thanks to Omar Ghattas (University of Texas at Austin) for allowing us the opportunity to work on `p4est` and release it as free software. Finally, I would like to thank all current and future contributors and users and invite them to the `p4est` Hausdorff School, which will provide ample opportunity for technical discussion and hands-on experience. The school will be held July 20-24, 2020 in Bonn, Germany.[1]

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Carsten Burstedde is a professor of scientific computing at the Institute for Numerical Simulation at the University of Bonn, Germany.

[1] http://www.hcm.uni-bonn.de/events/eventpages/hausdorff-school/hausdorff-school-2020/p4est2020/

Texas Tech University Chapter of SIAM Holds Graduate-Level Research Competition

This fall, the Texas Tech University (TTU) Chapter of SIAM organized a series of talks titled "Graduate Student Research Days" (GSRD). GSRD was a two-day research competition that provided graduate students from the Department of Mathematics and Statistics with the opportunity to share results from their ongoing research with peers and faculty members. Students presented their work to a committee composed of three faculty members from the statistics, pure mathematics, and applied mathematics disciplines. Alex Trindade, David Weinberg, and Eugenio Aulisa served as judges for the contest.

GSRD received remarkable interest from both faculty members and graduate students. The event featured oral presentations from 10 graduate students studying various subjects in pure and applied mathematics. The judges evaluated each talk according to six criteria: identification and/or background of research topic, methodology, interpretation of results, clear presentation of material, ability to answer questions, and overall delivery. The top three speakers were awarded monetary prizes, and all presenters received certificates of participation. Given the positive feedback from both students and faculty, the TTU Chapter of SIAM has decided to make the event an annual activity.

— Texas Tech University Chapter of SIAM

Attendees of the Texas Tech University Chapter of SIAM's "Graduate Student Research Days" competition watch students present their research. Photo courtesy of Isuru Dassanayake.

The Threat of AI Comes from Inside the House

You Look Like a Thing and I Love You: How Artificial Intelligence Works and Why It's Making the World a Weirder Place. *By Janelle Shane. Voracious / Little, Brown and Company (Hachette Book Group, Inc.), New York, NY, November 2019. 272 pages, $28.00.*

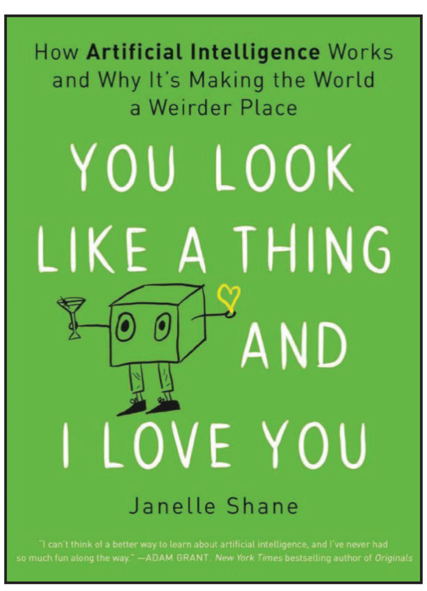
BOOK REVIEW
By Matthew R. Francis

Artificial intelligence (AI) will either destroy us or save us, depending on who you ask. Self-driving cars might soon be everywhere, if we can prevent them from running over pedestrians. Public cameras with automated face recognition technology will either avert crime or create inescapable police states. Some tech billionaires are even investing in projects that aim to determine if we are enslaved by computers in some type of *Matrix*-style simulation.

In reality, the truest dangers of AI arise from the people creating it. In her new book, *You Look Like a Thing and I Love You*, Janelle Shane describes how machine learning is often good at narrowly-defined tasks but usually fails for open-ended problems.[1]

Shane—who holds degrees in physics and electrical engineering—observes that we expect computers to be better than humans in areas where the latter often fail. This seems unreasonable, considering that we are the ones teaching the machines how to do their jobs. Problems in AI often stem from these very human failings.

As followers of her social media accounts and *AI Weirdness* blog[2] know, Shane often probes the limits of publicly-available AI algorithms for the sake of humor. Her codes produce machine-generated cat names, Dungeons & Dragons spells, Halloween costume ideas, and even complete recipes for the purpose of comedy. For instance, the title of her book comes from a set of computer-generated pickup lines: attempts to attract potential partners using clever wordplay.



You Look Like a Thing and I Love You: How Artificial Intelligence Works and Why It's Making the World a Weirder Place. By Janelle Shane. Image courtesy of Hachette Book Group, Inc.

While Shane's humorous lists make extensive cameos in her book, it is not simply a rehash of her blog. Neither is it intended as a textbook about AI. Her goal is to describe machine learning as it is actually implemented while debunking both the hype and fearmongering that surround AI. She accomplishes this objective with wit, humor, and her own (self-admitted) low-quality artwork.

My only real complaint with *You Look Like a Thing* is its mild structural awkwardness. Many of the examples that Shane returns to in detail are introduced early on, which gives rise to slight redundancies. A section on bot-human interactions and humans pretending to be AI seems a trifle abrupt in a book that is largely about the failures of AI.

However, these are minor oversights. Overall, the book offers an engaging read for those interested to learn about the reality of AI beyond the headlines and gain perspective on machine learning's true capabilities. It is replete with humor and geeky references — from *Star Trek* to Martha Wells' *Murderbot Diaries* book series.

Opening the Black Box

Most people who write scientific computer programs work in "rules-based" code: algorithms constructed to produce specific outputs. Machine learning, in contrast, takes in data and develops its own rules for producing output. This necessitates "training" the AI program on existing datasets, and Shane spends much of her book outlining the challenges of this approach.

Because AI generates its own rules, the details of its algorithms are hidden from researchers. Shane argues that studying failures in machine learning is essential for revealing the contents of the black box. In a running gag, she highlights image-description algorithms that find giraffes in completely unrelated images due to the overrepresentation of giraffe photos in the image libraries used to train them.

The disconnect between training scenarios and reality is a major theme of *You Look Like a Thing*. In one of Shane's examples, a programmer tasked AI to teach a virtual robot to go from point A to point B. While creative solutions are sometimes desirable, this AI preferred to find unworkable solutions — such as falling flat and scooting along the ground, and rewriting the rules of physics within the simulation to make the robot fly.

Shane does not spend much space detailing the various types of machine learning. She highlights one important aspect of AI as it is today: we often expect better-than-human results from a program that contains roughly as many "neurons" as a worm. It is therefore hardly surprising that tasks like image description or driving—which rely on a great deal of prior knowledge—pose difficulties for computers. AI must learn what images *are* and what a human face looks like from different angles and with varying expressions; these are tasks that people spend a lot of time learning to do with varying levels of proficiency.

Garbage in, Garbage out: AI Style

Shane also discusses "math washing" and "bias laundering," which occur when AI users claim that their algorithms must be objective because they are free of human foibles. Such users fail to recognize when bias is either built into the training data or carried over from the programmers themselves.

Science fiction aside, AI that kills deliberately is not a major concern, but Shane describes multiple cases where algorithms can *indirectly* cause harm. For example, a recent *Science* paper exposed built-in racism in a healthcare algorithm that resulted in black patients receiving far less care than their medical conditions merited [1]. This is not the first instance in which AI has reinforced systemic racism.

Additionally, some of the damage inflicted by computers arises directly from humans. Shane points out that most social media "bots" have people behind them

See **Threat of AI** *on page 10*

[1] Shane uses "AI" as shorthand for the types of machine learning implemented today, which I will also do in this review. She refers to the more common usage for human-level AI that encompasses science fiction robots and computers as "artificial general intelligence," which does not yet exist.

[2] https://aiweirdness.com/

Advocating for Science as a Science Policy Fellowship Recipient

By Sheri Martinelli

I was immediately intrigued by SIAM's call for new Science Policy Fellowship applicants, which came less than a year after the 2016 U.S. presidential election. Concerned about the view of science in the U.S.—especially within the government—I felt powerless to do anything about it. My previous experience working in a U.S. Department of Defense (DoD) laboratory, where I observed a gradual shift away from fundamental research, further piqued my interest in the program. I was therefore thrilled to be offered a fellowship, even though I was uncertain about the work I would actually be doing.

The first major event of my term as a Science Policy Fellowship recipient was the meeting of the Committee on Science Policy in spring 2018. The committee meets biannually, and the spring gathering is the main event. Fellowship recipients arrive a day early for orientation, which includes an overview of the government budget process with emphasis on science funding. We also received a primer on science advocacy that detailed SIAM's role in this area. I learned about the hierarchical structure of science advocacy, which comprises broad coalitions such as the American Association for the Advancement of Science, more specialized coalitions like large math societies, and individual organizations.

Our orientation was followed by the committee meeting on the second day, which featured an update from Lewis-Burke Associates (SIAM's legislative liaison) and invited talks by those with leadership roles in agencies that support research in applied and computational mathematics. These presenters typically include representatives from the National Science Foundation, the Department of Energy (DOE), and DoD offices with direct oversight of mathematics research. Past gatherings have also invited guests from the Office of Science and Technology Policy and the National Institutes of Health. This day-long session is an opportunity for attendees to better understand the agencies' perspectives and gain insight into their future initiatives and priorities.

Fellowship recipients spend the third day on Capitol Hill. They break into groups with committee members based on their experiences and interests, and visit U.S. congressional offices accompanied by Lewis-Burke specialists. For example, groups focus on science, technology, engineering, and mathematics (STEM) education; the DOE; and the DoD. I am routinely paired with the DoD group because of my background. At the 2018 spring meeting, my fellow group members consisted of Thomas Grandine (Boeing), Margaret Cheney (Colorado State University), John Burns (Virginia Polytechnic Institute and State University), and occasionally James Crowley (executive director of SIAM), all of whom are distinguished SIAM members with a great deal of experience with DoD research agencies. The prestige of these individuals was certainly awe-inspiring, but I viewed our interactions as my opportunity to be heard and did not refrain from speaking up. In fact, I felt I had to hold myself back much of the time! Fortunately, interacting with non-technically minded people has somewhat of a leveling effect.

My time on Capitol Hill allowed me to express my concerns about very applied research masquerading as basic research, something I have increasingly observed over the last several years. This situation

See **Science Policy** *on page 11*

Sheri Martinelli's time as a SIAM Science Policy Fellowship recipient has included meetings with staff from the U.S. House and Senate Committees on Armed Services. Photo courtesy of Margaret Cheney.

Feynman's Flying Saucer: The Second Serving

MATHEMATICAL CURIOSITIES
By Mark Levi

In the December 2019 issue of *SIAM News*, I offered an explanation[1] of Richard Feynman's observation on the wobble of a plate in free flight when launched with a spin around its central axis. Feynman noted that the plate wobbles twice as fast as it spins in the limit of small wobble (he actually states that the ratio is the other way around [3], but I think it is safe to say that he misremembered the result of his derivation). I recently came across Andy Ruina's different, direct explanation of the effect; a much more extensive discussion and list of references to later work is available on his website [5]. His analysis is more general and does not rely on axisymmetry, as mine does.

Here I provide yet another explanation of the 1:2 ratio, this one based on making the gyroscopic effect's role explicit. Since the spinning airborne plate is just a gyroscope, the key to the explanation is to first understand precisely how it feels to move a gyroscope when holding it by its axle. It will then be easy to understand the gyroscope's motion when it is released, with its axis changing direction at the moment of release. This is precisely what happens to the plate when it is launched in the air.

Gyroscope as a Particle on the Sphere Subject to Lorentz "Magnetic" Force

Instead of a flying plate, let us consider an equivalent object: a spinning wheel whose center O is fixed in space, with a massless axle of length $R=1$ (see Figure 1). The axle's endpoint P lies on the unit sphere. How does it feel trying to move this point? First, there is inertia proportional to the wheel's moment of inertia I_{diam} around the diameter. More precisely, the inertial mass is

$$m=I_{\text{diam}}R^{-2}=I_{\text{diam}}.$$

One could object to equating mass with a moment of inertia, but they *are* numerically equal if R is one unit of distance, as specified above.

Second, if the wheel is also spinning around its axis with angular velocity ω_{axial}, then point P will feel a gyroscopic force (see Figure 2). This force is normal to the velocity and of magnitude

$$F=Lv, \qquad (1)$$

where $L=I_{\text{axial}}\omega_{\text{axial}}$ is the axial angular momentum and v is the speed. It is as if P were a charged particle, with charge L moving in the magnetic field perpendicular to the sphere of magnitude $B=1$.[2] At the root of this gyroscopic effect is the fact that as we reorient OP, each particle of the wheel—while confined to a sphere—is constrained to move with a *nonzero geodesic curvature*, thus exerting a centrifugal force upon this constraint and producing a torque; the hand that moves P feels the sum of all such reaction torques. Computing this sum, i.e., integrating over all of the body's particles, results in (1). Incidentally, it is immediately clear without any calculation that the force and velocity in Figure 2 must necessarily be orthogonal to each other; otherwise, work would have been done in spinning the wheel up or down around its axle — an impossibility since the wheel bearings are perfect.

Explaining the 1:2 Ratio

The "Lorentz" force $F=Lv=\text{const.}$, since $L=\text{const.}$ and $v=\text{const.}$; the latter holds because the vectors $\mathbf{F}\perp\mathbf{v}$ in Figure 2. Every trajectory thus has constant geodesic curvature k and is therefore a circle. We can find k from Newton's second law: $mkv^2=F$. Substituting $F=Lv$ gives

$$k=L/mv,$$

an interesting observation in its own right: the geodesic curvature of circular orbits of the axle's tip is the ratio of the angular momentum to the linear momentum.

Let us now find the wobble's frequency, i.e., the angular velocity ω of P in the limit of a tight circle. Treating the small spherical cap enclosed by the tight circle as planar, the gyroscopic force in Figure 3 provides the centripetal acceleration $\omega^2 r$:

$$\omega^2 r=F/m=Lv/m=L\omega r/m.$$

Cancellation yields

$$\omega=L/m=\frac{I_{\text{axial}}}{I_{\text{diam}}}\omega_{\text{axial}}.$$

For flat disks, $I_{\text{axial}}/I_{\text{diam}}=2$, so that $\omega=2\omega_{\text{axial}}$ (in the limit of small wobble). For the other extreme of prolateness, such as a pencil spun around its longitudinal axis or a rifle bullet, $I_{\text{axial}}/I_{\text{diam}}$ is small and $\omega \ll \omega_{\text{axial}}$. This is also easily visible from the Poinsot description of the motion of free rigid bodies [4].

The Lagrange Top

The Lagrange top, i.e., the axisymmetric top, is treated with Euler's angles in most books on classical mechanics (e.g., [1, 4]). But the Lagrange top is equivalent to the particle in Figure 2 with an additional gravitational force. And this equivalence allows for a more intuitive and less cumbersome analysis of the problem than the traditional one. I may provide this analysis elsewhere.

The figures in this article were provided by the author.

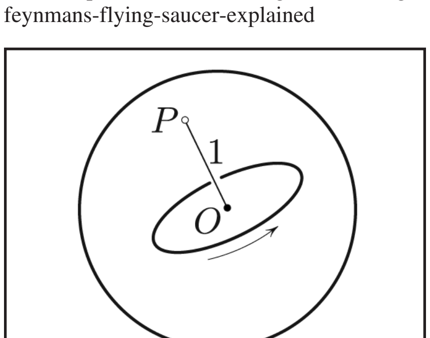


Figure 1. *Relating a gyroscope to a point mass on the sphere.*

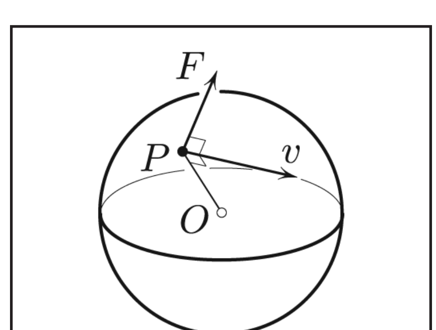


Figure 2. *Equivalence between a gyroscope and a point mass subject to a Lorentz force.*

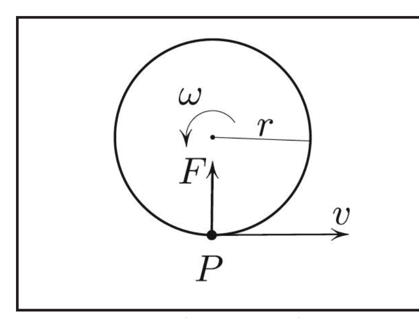


Figure 3. *Magnified view of small wobble of the axle's tip.*

[1] https://sinews.siam.org/Details-Page/feynmans-flying-saucer-explained

[2] Actually, $B=R^{-2}$ happens to be the Gaussian curvature of the sphere (I chose $R=1$ only for simplicity). In fact, Gaussian curvature plays a key role in the motion of spinning tops [2].

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Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

The Mathematics Underlying Gun Violence

By Shelby M. Scott and Louis J. Gross

Each year, gun violence is responsible for approximately 31,000 deaths and 78,000 non-fatal injuries in the U.S. [1]. These casualties cost roughly $229 billion, with rural communities and younger individuals experiencing a higher burden [11]. Exposure to gun violence is associated with a greater propensity toward both chronic health conditions and risky social behaviors [11].

The consequences of gun violence impact diverse populations across a wide variety of scales, and research therefore involves multidisciplinary perspectives. Unfortunately, work in this field is limited. In 1996, the Dickey Amendment prevented the Centers for Disease Control and Prevention from using federal funds to promote or advocate for gun control, effectively shutting down research [9].

Last spring, the National Institute for Mathematical and Biological Synthesis and the Center for the Dynamics of Social Complexity hosted 29 individuals at an investigative workshop entitled "Mathematics of Gun Violence."[1] The workshop aimed to review existing literature, identify areas that require further research, develop cross-disciplinary collaborations, and suggest data collection to assist evidence-based policy recommendations. In summarizing workshop findings, we present valuable ideas surrounding the mathematics of gun violence.

Existing Literature

While statistical methods have addressed aspects of gun crime and violence in the U.S., mathematical modeling approaches are limited (with the exception of a few existing models). To analyze the dynamics of crime hotspots, Martin B. Short and his collaborators produced a partial differential equation model that interprets supercritical or subcritical bifurcations in a crime context [8]. Other researchers have studied the efficacy of law enforcement deployment and found that the dynamics of policing have significant effects on crime distribution [7].

In the context of crime, networks may correlate with the dynamics of spread. Ben Green and his coauthors evaluated the extent to which modeling contagion on a social network can predict victimization [6]. Paul Brantingham and his group used ecological modeling to address the intergroup dynamics of gangs [3]. Sara Bastomski and her collaborators examined the way in which neighborhood-level criminal networks shape crime distribution and determined that embeddedness has a positive association with local homicide rate [2]. Violence may propagate through networks, but the impacts of intervention can spill over in similar ways [12]. Analysis of network structure could suggest improved interventions.

In addition to networks, other methods have found success in analyzing the spread of gun violence and crime. Due to the complicated relationships between individuals, systems dynamic models can be useful when investigating incarcerations and interventions [4]. Researchers have also used game theory to determine the efficacy of gun control policies [10], and developed systems of ordinary differential equations to analyze the dynamics of gun crime as it spreads throughout a population [5].

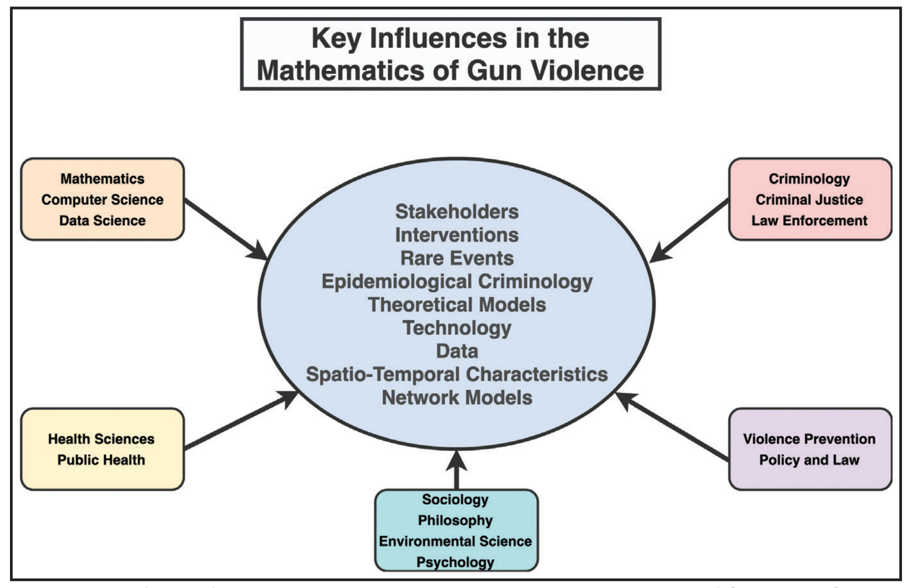
Key Takeaways

Below we identify some promising ideas that emerged from the workshop.

Collaboration with Stakeholders: A variety of relevant actors exist in situations of gun crime and violence. Therefore, identification of key players and their interactions with the affected community is important, and analysis of stakeholder network structures and their generalizability may improve interventions. Unfortunately, obtaining data to parameterize such models is difficult.

Interventions: Observing how different interventions affect various types of firearm-related events can allow the implementation of multifaceted, evidence-based approaches. It is also imperative to develop a consensus for intervention evaluation. Considering the root causes of crime and violence—rather than focusing on the outcomes—may improve response. Mathematical and statistical modeling can address both individual and population-level intervention effects.

Quantifying the Impact of Rare Events: Rarity is context-dependent, both in terms of scale and field of study. Rare events also introduce uncertainty due to outliers. Mass shootings are rare relative to interpersonal violence, and analysis of each scenario requires different methods. Borrowing from other disciplines that study rare events can help forecast the occurrence of future violent incidents and suggest appropriate reduction strategies.



A summary of the influences on gun violence research. Figure courtesy of Shelby M. Scott.

Epidemiological Criminology: Applying tenets from epidemiology to criminology is common, and not just in the formal paradigm of "epidemiological criminology." When employing multiscale models that connect the individual scale to the population and community scales (as in disease ecology), one must consider the nuances and limitations of the analogy between disciplines.

Theoretical Models: Investigating the factors and interactions that push an individual from nonviolence to violence, even in the absence of data, can offer insight about crime interruption. Ideas from sociology and psychology—when combined with quantitative models—may provide information about generalizable concepts and their interactions for violence analysis.

Technology: The introduction of new technologies that are relevant to gun violence and crime requires that one determine the technology's specific purpose, evaluation, and impact — not only on the social system, but also for the individuals who are subject to the new tools. Practitioners must consider ethical issues and concerns of bias when working with collected data and conclusions drawn from technological advances.

Data Collection and Use: There is a dearth of data quality and quantity pertaining to crime assessment. Tradeoffs also occur in the applicability of available datasets between precision, realism, and generalizability. In light of this, interdisciplinary collaborations are necessary to fill gaps in the data using theory and methods from various fields. As with technology, it is important to address ethical matters and the data's overall purpose.

Spatiotemporal Characteristics: Many quantitative models have contemplated the spatial, temporal, and spatiotemporal aspects of gun violence, but it is important to account for the differing data scales and

See **Gun Violence** *on page 12*

[1] http://www.nimbios.org/workshops/WS_gunviolence

Threat of AI

Continued from page 8

(though their biographies may be automatically generated), simply because algorithms are not yet sophisticated enough to mimic human behavior online. Similarly, repressive governments claim to use AI-driven facial recognition, but evidence indicates that most analysis of surveillance footage is typically done by humans.

Shane pulls no punches without losing sight of the fun inherent in the field. *You Look Like a Thing* is therefore a good choice for people who are either fearful of AI or excessively optimistic about it. And as with all good science fiction, the story ends up being as much about *us* as about the machines. If it takes imaginary giraffes and murderbots to expose the truth about our own limitations, so be it.

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Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

Science Policy

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poses a challenge to students who find it difficult to publish their work and invariably sacrifice long-term global leadership in basic science in favor of near-term deployment of technology.

Fellowship recipients and committee members typically meet with staffers; elected officials participate on occasion, especially if any of the SIAM members happen to be constituents. The spring meeting is scheduled to not only accommodate as many regular committee members as possible, but also to coincide with a stage of the budget process (e.g., drafting of appropriations or authorizations) at which SIAM members can be most influential. While attendees still hear from the agencies at the fall meeting, it is not as productive a time to effectively insert our priorities into the budget. Therefore, Capitol Hill visits do not occur in the fall.

In addition to mandatory meeting attendance, Science Policy Fellowship recipients must complete a project in an area of interest. Given my DoD experience, I chose a project that would draw from that familiarity. An oft-quoted speech by Michael Griffin, the Under Secretary of Defense for Research and Engineering, lays out the top priorities of future DoD research. Using these stated priorities as a guide, I wrote a white paper for SIAM—with much help from Lewis-Burke—on the contributions of previous and ongoing support for fundamental research in applied and computational mathematics to advances in these areas.

By writing this document and attending agency presentations at committee meetings, I began to truly understand the challenges of communicating the value of mathematics to an increasingly myopic society. Unfortunately, it takes years—if not decades—for purely mathematical results to morph into technology, or a "thing" whose importance most people can grasp. For this reason, involvement in policy or other less technical aspects of our profession is immensely valuable — we must get better at promoting the importance of our work to nonspecialists.

Despite media reports about hyper-partisanship and varying degrees of gridlock—not to mention troubling rhetoric from politicians themselves—the government largely continues to function. Staffers perform most of the groundwork, and simply being present and meeting with them, regardless of political affiliation, allows us to influence policy. Although the president's recent budget requests have sought to make large cuts to scientific research, congressional members who actually control the budget are more pragmatic. We have had fruitful discussions with staffers from both major parties. We have also had rushed, unproductive interactions with staffers from both major parties. The meetings with majority and minority staff from the U.S. House and Senate Committees on Armed Services have been of particular interest.

Ultimately, I've found my experience as a SIAM Science Policy Fellowship recipient to be extremely valuable. It has provided a wonderful opportunity to contribute to SIAM, learn a great deal about communicating the value of applied and computational mathematics, and gain more confidence in the system. The fellowship has reassured me that those with direct influence are aware of and working on the concerns that I raised about the state of support for fundamental research in mathematics within the DoD.

Sheri Martinelli is an assistant research professor at Pennsylvania State University's Applied Research Laboratory. Her research is primarily in the areas of computational and ocean acoustics. She also holds a faculty appointment in Penn State's Graduate Program in Acoustics.

Reflections from the SIAM Texas-Louisiana Section Meeting

The SIAM Texas-Louisiana Section held its second annual meeting jointly with Southern Methodist University's (SMU) Department of Mathematics in Dallas, Texas, last November. SIAM contributions—along with two grants from the National Science Foundation (NSF)—helped support the conference and enabled the attendance of students and early career mathematicians.

The meeting attracted over 200 attendees, largely from Texas and Louisiana. Topics spanned a wide variety of fields, ranging from imaging to geometry — with two "tracks" devoted to applications of topology and machine learning. The conference consisted of nearly 40 minisymposia, an undergraduate session, and a poster session with over 30 posters (including those from undergraduate participants). A panel also focused on the future of applied mathematics in business, industry, and government.

Three plenary speakers covered a breadth of important mathematical applications. Steve Jiang (University of Texas Southwestern Medical Center) described his group's research in medical functions of artificial intelligence. Susan E. Minkoff (University of Texas at Dallas) presented an overview of mathematical problems in geosciences, including micro-seismic events (extremely small earthquakes) generated during the process of fracking for hydrocarbons on land. And Peter Kuchment (Texas A&M University) surveyed three recent techniques in medical and homeland security imaging: hybridization of physical modalities in the imaging process to simultaneously achieve acceptable contrast and resolution, invoking of "internal information" in solving inverse problems, and "cone transforms" and Compton-type camera imaging for emission imaging with low signal-to-noise data.

An NSF Research Training Group grant allowed approximately 15 undergraduate students to attend a presentation on mathematics and economics by Cullum Clark (George W. Bush Presidential Center), as well as a lecture on linear algebra's applications to imaging science by Prasanna Rangarajan (SMU). Additionally, a minisymposium focused on research opportunities for undergraduates.

Thanks to the efforts of the organizing committee, the SIAM Texas-Louisiana Section's annual meeting was successful and almost all reviews were positive. Many attendees indicated that they gained new ideas and collaborations as a direct result of the conference.

— *Officers of the SIAM Texas-Louisiana Section*

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SIAM Conferences and Programs Roar into the Twenties

By Richard Moore

January marks the start of a new decade and the end of my first year as SIAM's director of Programs and Services. Both occasions inspire reflection on the success of SIAM conferences[1] and programs over the last year, and the challenges and opportunities that lie ahead.

The 2019 SIAM Conference on Computational Science and Engineering (CSE19), which took place last February in Spokane, Wash., had the distinction of being SIAM's biggest-ever meeting with 1,895 attendees. The 2019 SIAM Conference on Applied Algebraic Geometry (AG19), held last July in Bern, Switzerland, and the 2019 SIAM Conference on Applications of Dynamical Systems (DS19), held last May in Snowbird, Utah, also experienced significant jumps in attendance. Indeed, the meeting commonly known as "Snowbird" has grown so large that its next iteration in May 2021 will take place in Portland, Ore., to better accommodate attendees. In addition to providing more hotel capacity and meeting space, the move to sea level from the high Utah mountains will allow individuals with altitude-related health issues to attend DS21.

2019 also saw the quadrennial International Congress for Industrial and Applied Mathematics (ICIAM),[2] which took place last July in Valencia, Spain. While this is not strictly a SIAM meeting, SIAM is a contributing society to its organization and supported ICIAM 2019 in various ways. SIAM organized the presentation of the Peter Henrici, John von Neumann, and AWM-SIAM Sonia Kovalevsky Prize Lectures,[3] and administered a National Science Foundation grant that provided 63 student and early-career travel awards. SIAM Activity Groups also organized 48 minisymposia.

In 2020, SIAM will partner with other societies at two of its meetings: the 2020 SIAM Annual Meeting (AN20)—to be held this July in Toronto, Canada, in conjunction with the annual meeting of the Canadian Applied and Industrial Mathematics Society[4]—and the SIAM Conference on Applied Mathematics Education (ED20), to be co-located with the Mathematical Association of America's MathFest[5] in Philadelphia, Penn., later that month.

Perhaps the most exciting event on our conference horizon is the launch of the new SIAM Conference on Mathematics of Data Science,[6] whose first instantiation—MDS20—will occur jointly with the 2020 SIAM International Conference on Data Mining (SDM20) in Cincinnati, Ohio, this May. MDS20/SDM20 will include a career fair (in addition to the traditional career fair to be held at AN20), thus marking the increasingly important role of our conferences in developing our workforce and launching the careers of young scientists.

2020 will also bring about some changes to modernize SIAM conferences, including the discontinuation of full printed programs;[7] instead, attendees are encouraged to use the mobile app or the continuously updated online program (at-a-glance programs will still be provided on site, and full program PDFs will be available for download). SIAM is also retooling its child care grants to make them more flexible and better aligned with conference duration.[8] Lastly, adaptations to SIAM's conference management system will accommodate requests by conference organizing committees more efficiently. Experimenting with scheduling and poster judging tools are on my to-do list for early 2020.

As director of Programs and Services, I am also involved in the implementation of special programs old and new, such as those funded via a generous grant by Philippe and Claire-Lise Tondeur.[9] One such initiative is the BIG (Business, Industry, and Government) Jobs Live Interview Series, during which graduate students in the mathematical sciences have the opportunity to interview mathematicians working in industry. The series launched in November 2019, and the second installment took place this January. The interviewers were selected by a committee[10] based on video submissions and trained at WHYY-FM, Philadelphia's public radio station.

A longer-standing program is the Gene Golub SIAM Summer School (G2S3),[11] now entering its second decade. G2S3 2019 took place last June in Aussois, France, with a focus on high performance data analytics, and G2S3 2020—on the theory and practice of deep learning—will occur this July in Muizenberg, South Africa.

SIAM's conferences and programs are only possible due to the hard work and dedication of countless individuals, from conference co-chairs and committee members to the SIAM staff with whom I have the pleasure of working. SIAM wishes to extend special thanks to Cynthia Phillips (Sandia National Laboratories), outgoing Vice President for Programs, for her tireless work in support of SIAM's activities. We look forward to working with James Nagy (Emory University), whose tenure as VP for Programs begins this month.

Richard Moore is the director of Programs and Services at SIAM.

[1] https://www.siam.org/conferences/calendar

[2] http://www.iciam.org/

[3] https://iciam2019.org/index.php/scientific-program/highlighted-speakers/siam-prize-lectures

[4] https://caims.ca/

[5] https://www.maa.org/meetings/mathfest

[6] https://www.siam.org/conferences/cm/conference/mds20

[7] https://sinews.siam.org/Details-Page/updates-from-the-december-siam-board-meeting

[8] https://sinews.siam.org/Details-Page/changes-to-siams-child-care-grant-program

[9] https://www.siam.org/students-education/programs-initiatives/tondeur-initiatives

[10] https://sinews.siam.org/Details-Page/siam-ams-student-media-fellows-announced

[11] https://www.siam.org/students-education/programs-initiatives/gene-golub-siam-summer-school

Attendees of the 2019 SIAM Conference on Computational Science and Engineering—which took place last February in Spokane, Wash., and was SIAM's largest meeting to date—mix and mingle between sessions. SIAM photo.

Gun Violence

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appropriateness of analysis methods based on the situation. One must note measurement error—common in spatiotemporal data—when drawing conclusions or making actionable recommendations.

Network Models: Networks of individuals often perpetuate gun crime and violence, but these networks can also be instrumental in interrupting it. Determining the structure and influence of violence that perpetuates and interrupts networks could lead to improved interventions.

Concluding Thoughts

Common discussion topics at the workshop included the need for more cross-disciplinary and interdisciplinary work. Leveraging diverse expertise across a wide range of fields can improve methods to study gun violence. Model verification, intervention programs, and technologies also require better evaluation techniques. Finally, workshop participants emphasized the importance of data improvements. The quality of available data is unreliable and necessitates the collection of many additional datasets. Enhanced opportunities for broadening the pool of researchers depends on increased funding agency support for projects in this major area of concern.

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Shelby M. Scott is a Ph.D. candidate and National Defense Science and Engineering Graduate Fellow in the Department of Ecology and Evolutionary Biology at the University of Tennessee, Knoxville, where she is also pursuing a master's degree in statistics. Her research interests include applying mathematical and statistical modeling to topics in public policy, ecology, and healthcare, as well as quantitative biology education and outreach. Louis J. Gross is Chancellor's Professor of Ecology and Evolutionary Biology and Mathematics at the University of Tennessee, Knoxville, where he also directs the National Institute for Mathematical and Biological Synthesis, a synthesis center supported by the National Science Foundation. His research focuses on applications of mathematics and computational methods in disease ecology, landscape ecology, spatial control for natural resource management, climate change, and the development of quantitative curricula for life sciences students.

Participants of the "Mathematics of Gun Violence" workshop, organized by the National Institute for Mathematical and Biological Synthesis and the Center for the Dynamics of Social Complexity. Photo courtesy of Catherine Crawley.